Appendix A
The Source of Stochastic Models in Conceptual Climate Dynamics

The variability of global climate patterns for the last decades has received overwhelming interest recently. The impact human activities might have on the current terrestrial climate balance underlines the need for reliable climate modeling and simulation. The mathematical models underlying modern simulations are very complex and high dimensional. The closer to reality the resulting virtual pictures are, the closer our understanding of their contents is to our understanding of real climate. This possibly just means that it may be equally poor. In addition, climatology is a science without experiments or empirical inference in the usual sense, apart from the reproduction of past climate patterns by statistical inference from paleo-climatic data. The cross-validation of simulation output with these data is usually rather difficult. As a consequence, there certainly is the danger of too much confidence in the simulation output from the models, and the virtual world they create. And it is certainly wrong to consider computer experiments as acceptable compensation for lack of real experiments and empirical data. Therefore a physical or analytical understanding of the phenomena both in the real as in the virtual world of model simulations through conceptual insight is of central importance. It can be provided by considering conceptual, analytically accessible stochastic reductions of the complex models. Accordingly, stochastic model reduction in climate dynamics is of paramount importance.

A.1 A Conceptual Approach to Low-Dimensional Climate Dynamics

One of the main obstacles of climate modeling is the substantial variability on spatial and temporal scales ranging over many orders of magnitude. It reaches from turbulent eddies in the ocean surface due to breaking waves, through mid-latitude cyclonic storms hundreds of kilometers in extent and lasting for days, to millennial scale shifts in ice cover and ocean circulation. The low-lying physical

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description behind imposes important mutual dependencies of quantities on these highly different time scales, which in general cannot be resolved entirely. This spread poses major challenges for any quantitatively accurate and computationally feasible representation.

To account for this variety of effects on very different scales, climatologists developed a big collection of models which are commonly classified into three groups. On the top level of quantitative accuracy are the comprehensive General Circulation Models (GCMs). These are the quantitatively most ambitious models, which attempt to represent the climate system in as much detail as computational resources and conceptional reasoning allow. Earth System Models of Intermediate Complexity (EMICs) instead are models of a more restrained resolution, which attempt to represent some subsystem of Earth's climate in detail, such as the ocean, the land surface or the atmosphere, while the interaction with other subsystems as well as external forcing remains parameterized. At the bottom of the model hierarchy according to [CMW$^+$02] are low dimensional ones such as for instance energy balance models, that ignore almost all quantities and their interactions, except for a few. They are studied under highly idealized conditions, such that they are hardly of quantitative relevance. Their interest lies in their accessibility for mathematical analysis. Very often they are completely solvable and entirely understandable. They may qualitatively predict phenomena encountered in more complex models. Their reduced complexity can help to develop conceptual qualitative paradigms capable to interpret and understand simulations obtained on the basis of EMICs or GCMs. Classical examples of this are the prediction of multiple states of the thermohaline circulation by Stommel [Sto61], of the phenomenon of sensitivity to initial conditions by Lorenz [Lor63], and of glacial metastability.

In the lower levels of climate modeling it is crucial to decide which processes to represent explicitly, which to parameterize, and how to justify or even construct the parametrization. Following [IM02], in an updated version of the traditional approach an analogy with thermodynamic limit theorems is used: by taking the proportion of scales to an infinite limit, a complete separation of micro and macro scales is obtained. In a first step, averaging of small scale processes produces deterministic dynamics for the large scale processes. In a second step, the fluctuation of the large scale variables around the averaged values of the small scale quantities is expressed by stochastic differential or partial differential equations, in which the large scale variables are driven by random processes representing the small scale components. The mathematically rigorous derivation of such equations by Khasminskii [Kha66] leads to *linear systems*, however.

A.1.1 Hasselmann's Unfinished Program

There have been serious attempts to derive simple *non-linear* climate models with stochastic forcing from idealized GCMs. This project is labeled "Hasselmann's program" after an article by Arnold [Arn01], in which the ideas by the climatologist [Has76] dating back to the mid-seventies are translated into modern mathematical

language. Hasselmann's work is explicitly aimed at increasing the mathematical and physical understanding of more resolved climate models.

We shall briefly sketch the main ideas. In a first step an idealized GCM is considered as a large system of coupled ordinary (or partial) differential equations, in which for $0 < \varepsilon \ll 1$ the climate state $z = (x^\varepsilon, y^\varepsilon)$ can be separated into "slow" $x^\varepsilon(t, y^\varepsilon)$ and "fast" variables $y^\varepsilon(\frac{1}{\varepsilon}t, x^\varepsilon)$. Such a system can be formally described by

$$\dot{x}^\varepsilon = f(x^\varepsilon, y^\varepsilon),$$
$$\dot{y}^\varepsilon = \frac{1}{\varepsilon} g(x^\varepsilon, y^\varepsilon).$$

The scale separation should be described by a small parameter ε corresponding to the "response time" of the scales of slow and fast variables. Now define in physical jargon $u^\varepsilon(t) := \langle x^\varepsilon(t, \cdot)\rangle, t \geqslant 0$, as an "average" of the slow variables with respect to an invariant measure of the subsystem of the fast ones. This should lead to an averaged ordinary or partial differential equation

$$\dot{u}^\varepsilon = F(u^\varepsilon),$$

where $F(u^\varepsilon) := \langle f(x^\varepsilon, \cdot)\rangle$. The first mathematically rigorous proof of such a procedure was given by Bogolyubov and Mitropolskii [BM61], establishing that under appropriate assumptions $\lim_{\varepsilon \to 0+} x^\varepsilon(t) = u^0(t)$.

In a second step, the fluctuation $x^\varepsilon(t) - u^0(t)$ of the solution around the averaged one is studied. Khasminskii [Kha66] discovered that for $t \in [0, T]$

$$L^\varepsilon(t) = \frac{1}{\sqrt{\varepsilon}} \left(x^\varepsilon(t) - u^0(t)\right)$$

has a limiting Gaussian law as $\varepsilon \to 0+$. This way, he obtains linear differential equations for the slow variables with a stochastic term replacing the fast ones on finite intervals. In the framework of diffusion limits, deviations from averaged behavior produce non-linear (partial) differential equations with stochastic forcing (see [AK01] and [MTE99]). In this reduction, an assumption is crucial that is usually very hard to rigorously establish: mixing properties of the fast components, which lead to a decay of correlations viewed by an equilibrium measure. Even in simple ocean models studied in [Maa94] coupled to a Lorenz equation as atmospheric component, different regimes of the fast motion that are only partially chaotic, complicate the mathematical treatment.

Yet many qualitative phenomena could not be captured by these methods, since they happen on ε-dependent time scales, that tend to be large for small ε, i.e. on intervals $[0, T(\varepsilon)]$, where $T(\varepsilon) \to \infty, \varepsilon \to 0+$. Among these are for example the Markovian transitions between stable states of the deterministic system that become metastable by the action of noise.

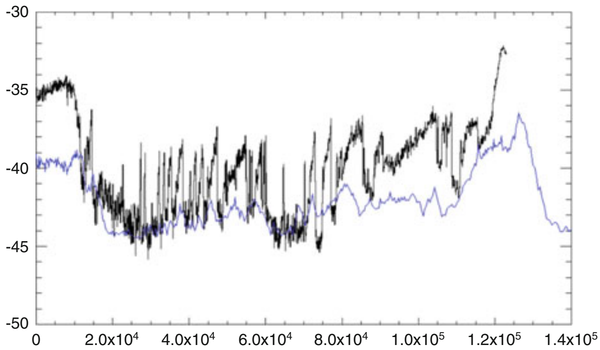

Fig. A.1 Greenland ice core $\delta^{18}O$ temperature proxies ([NGR04] core data, *black line*), 50 year average, from 120,000 years before present until now. The higher the values the warmer the average temperature [Public domain figure]

The systematic mathematical deduction of these stochastically forced equations from deterministic models remains a challenge some 35 years after their heuristic derivation by Hasselmann.

A.1.2 Energy Balance Models Perturbed by Noise of Small Intensity

An alternative approach for obtaining relevant conceptual models in climate dynamics short-circuits the derivation according to Hasselmann's program. It consists in the explicit study of given paleoclimatic time series, and the selection of the best fitting dynamical model through statistical inference. Assume that the data in the time series are realized by one of a family of deterministic dynamical systems perturbed by additive stochastic noise. Assume further that the noise is parameterized by a parameter located in a set in Euclidean space. To choose the best fitting one among the dynamical models, one has to develop a statistical test for instance for the noise parameter—often a rather hard task.

For a paleoclimatic time series from the Greenland ice shelf (Fig. A.1) providing proxies for the yearly average temperatures of the last glacial period, climatologists around [Dit99] proposed an energy balance model perturbed by heavy-tailed α-stable noise of small intensity. A statistical analysis on a physical level of rigor was used to estimate the best fitting α.

Recently this conclusion has been supported strongly by mathematical studies. In [HIP09, GHIP11] the model selection problem for the Greenland temperature time series was carried out successfully. The class of models considered is given by a dynamical system driven by a one dimensional additive α-stable process. Based on a path-wise roughness analysis using the power variations of trajectories an estimator

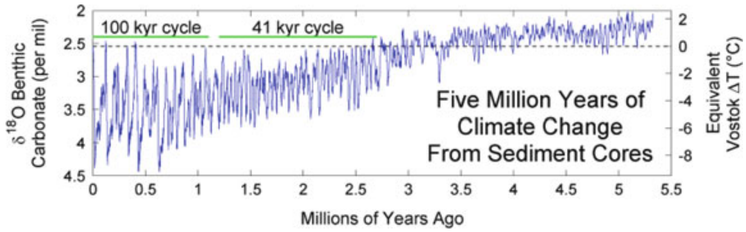

Fig. A.2 Temperature proxy for the last five million years [LR05,PJR$^+$99] [Public domain figure]

for $\alpha \in (0, 2)$ is established. The convergence quality of this method to a unique parameter gives at least a good indication that such a signal is observed in the time series.

A.1.3 The Motivating Phenomenon: Paleoclimatic Warming Events

In the literature the term "ice age" has different meanings. In this part we adopt the following convention. *Ice age* denotes a period of lower temperature of Earth's surface and atmosphere on a scale corresponding roughly to Earth's age, i.e. on a billion to hundred million year scale. During an ice age, frequent expansions and retreats of continental ice sheets, polar ice caps and alpine glaciers are observed. These episodes of extra cold climate are called *glacial periods*. See [IR10].

Since 2.58 million years before present polar ice shields appear to reemerge, resulting in the current Quaternary Ice Age, during which around 47 glacial periods have taken place so far (See Fig. A.2).

The eventual causes for the onset of an ice age are not very clear yet. Instead, the succession of glaciation periods at least during the current ice age is closely linked to the periodic behavior of some of Earth's orbital parameters, the so-called Milankovich cycles.

The theory of climate variability due to the change in planetary orbital parameters goes back to the Serbian civil engineer M. Milankovich (1879–1958). In collaboration with W. Köppen, a German meteorologist, he recognized that the decrease of summer insolation at high latitudes may be responsible for the growth of glaciers. He expresses Earth's incoming solar radiation at a given point on the surface and time as a function of the orbital parameters, but is unsure about the critical latitude to trigger a glaciation period.

If we suppose that Earth's orbit around the sun lies approximately in a plane, it can be decomposed into three major components. The *eccentricity* of the elliptic annual trajectories of Earth around the sun vary regularly over time with periodic components of about 100,000 years. Earth's axis of rotation has an *inclination* with respect to the normal of the orbital plane, the angle of which varies between

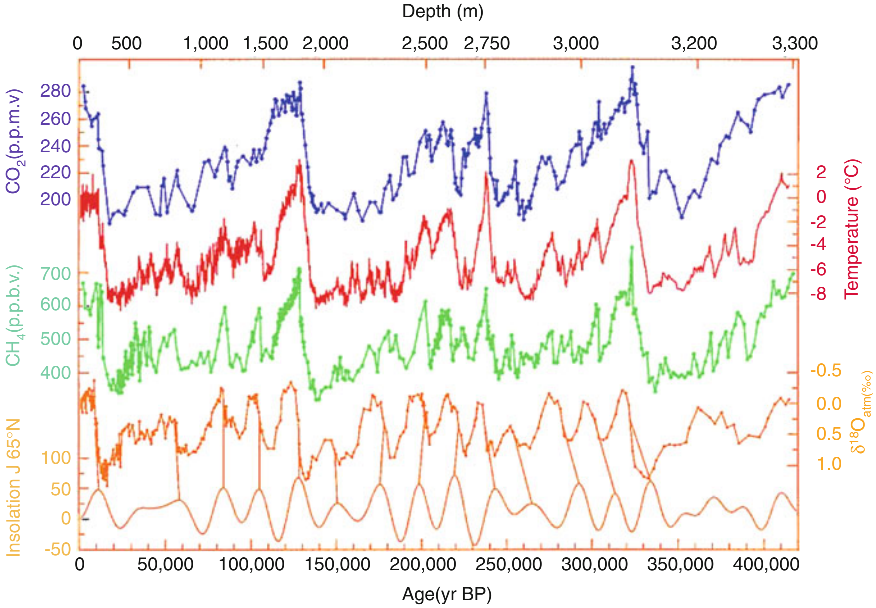

Fig. A.3 420,000 years of ice core data from [PBL+97, PJR+99], Antarctica research station, from *bottom* to *top*: Solar variation at 65° due to Milankovich cycles (connected to $1^{18}O$), ^{18}O isotope of oxygen, levels of methane, relative temperature [Public domain figure]

22.1 and 24.5° with an approximate period of 41,000 years. It influences the solar radiation influx at high latitudes, see [Har94]. A third component is contributed by the periodic *precession* of the equinoxes, i.e. the gyration of Earth's rotation axis around the normal of the orbital plane with major periods of 19,000 and 23,000 years.

The combined effect of these three components accounts for up to 30 % of incoming solar radiation at high latitudes. The diagram of Fig. A.3 exhibits a fairly good correspondence of the summer insolation at 65° North calculated on the basis of this orbital forcing.

In the long-range data plot in Fig. A.2 one recognizes the dominant periodicity of 41,000 years until one million years ago which is replaced by the 100,000 year periodicity since then. For a recent discussion of this phenomenon see [Dit09].

The present work is motivated by a phenomenon observed during the last glacial period, about 100,000–10,000 years before present. Temperature proxies in the Greenland ice core indicate that the orbital forcing discussed above does not have a major effect within this period, and temperatures do not stay uniformly low. Instead one can recognize at least 21 major spikes, indicating abrupt extraordinary local increases by up to 8° within less than 30 years, followed by a gradual decline during several centuries (see [IR10]). The distribution of the spikes in Fig. A.1 is rather regular over the whole period.

The origin of these patterns is not quite clear. In the literature the spikes are classified into two categories. The first one consists of so-called Heinrich events. They are thought to be caused by ice sheet instabilities with a huge discharge of icebergs, i.e. enormous fresh water influx into the Atlantic. Between three and six rapid coolings are considered to be of Heinrich type. The remaining spikes are named Dansgaard–Oeschger events after their discoverers. There is so far no good explanation for their emergence. Some authors, for instance [GR01, Rah03] and [DKA06], suggest a superposition of short periodic signals of solar radiation, leading to temperature evolutions with periodic intervals that determine the Dansgaard–Oeschger events. If in this case the system retains several stable states separated by temperature thresholds those may be overcome by random perturbations. Such a phenomenon ist often referred to as *stochastic resonance*.

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