

Bibliographical Remarks

Chapters 1–5 of Part I of the book address the asymptotics of Green’s functions for boundary value problems for the Laplacian.

The analysis of uniform asymptotic approximations for Green’s functions for Dirichlet problems in multi-dimensional domains with small perforations is included in Chap. 1, which is based on the papers [23, 24].

Chapter 2 incorporates the results of the paper [26], which deals with Neumann and mixed boundary value problems, with Neumann boundary conditions on the boundaries of small holes. The analysis of [26] includes uniform asymptotics of Green’s kernels in two- and three-dimensional domains containing a small hole.

Chapters 3 and 4 address uniform asymptotics of Green’s kernels in domains with several perforations and the numerical simulations. The material of these chapters is based on the results of [32]. The paper [25] shows other examples of uniform approximations of Green’s functions in singularly perturbed domains, such as thin bodies, truncated cones and domains with small grooves on the exterior boundaries—this material is discussed in Chap. 5.

Part II of the book, incorporating Chaps. 6, 7 and 8 presents the asymptotic approach for uniform approximations of Green’s kernels in vector problems of elasticity in two- and three-dimensional elastic bodies with small holes. Chapter 6 discussing the case of a domain with a single inclusion is based on the paper [33], and Chap. 7 addressing the case of multiply-perforated elastic bodies includes the results of [32].

In Part III, we consider the case when the number of perforations becomes large. A new method of meso-scale asymptotic approximations is introduced in Chaps. 9 and 10. Chapter 9 on meso-scale approximations for solutions of Dirichlet problems uses the results [27], and the case of mixed boundary value problems in multiply-perforated domains of Chap. 10 is discussed in [29].

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