Bibliography

1. S. S. Abhyankar, Local rings of high embedding dimension, Amer. J. Math. **89** (1967), 1073–1077.
2. R. Apéry, Sur les branches superlinéaires des courbes algébriques, C. R. Acad. Sci. Paris **222** (1946), 1198–1200.
3. C. Arf, Une interprétation algébrique de la suite des ordres de multiplicité d'une branche algébrique, Proc. London Math. Soc. (2) **50** (1948), 256–287.
4. V. Barucci, Numerical semigroup algebras, in Multiplicative Ideal Theory in Commutative Algebra, pp. 39–53, Springer, New York, 2006, Editors: James W. Brewer, Sarah Glaz, William J. Heinzer and Bruce M. Olberding.
5. V. Barucci, D. E. Dobbs, M. Fontana, Maximality properties in numerical semigroups and applications to one-dimensional analytically irreducible local domains, Memoirs of the Amer. Math. Soc. **598** (1997).
6. J. Bertin, P. Carbonne, Semi-groupes d'entiers et application aux branches, J. Algebra **49** (1977), 81–95.
7. M. Bras-Amorós, Improvements to evaluation codes and new characterizations of Arf semigroups, in Applied Algebra, Algebraic Algorithms and Error-Correcting Codes (Toulouse, 2003), pp. 204–215, Lecture Notes in Comput. Sci., 2643, Springer, Berlin, 2003, Editors: M. Fossorier, T. Høholdt and A. Poli.
8. M. Bras-Amorós, Acute semigroups, the order bound on the minimum distance, and the Feng-Rao improvements, IEEE Trans. Inform. Theory **50** (2004), no. 6, 1282–1289.
9. M. Bras-Amorós, A note on numerical semigroups, IEEE Trans. Inform. Theory **53** (2007), no. 2, 821–823.
10. M. Bras-Amorós, Fibonacci-like behavior of the number of numerical semigroups of a given genus, Semigroup Forum **76** (2008), 379–384.
11. M. Bras-Amorós, P. A. García-Sánchez, Patterns on numerical semigroups, Linear Algebra Appl. **414** (2006), 652–669.
12. A. Brauer, On a problem of partitions, Amer. J. Math. **64** (1942), 299–312.
13. A. Brauer, J. E. Shockley, On a problem of Frobenius, J. Reine Angew. Math. **211** (1962), 215–220.
14. H. Bresinsky, On prime ideals with generic zero $x_i = t^{n_i}$, Proc. Amer. Math. Soc. **47** (1975), 329–332.
15. W. C. Brown, F. Curtis, Numerical semigroups of maximal and almost maximal length, Semigroup Forum **42** (1991), 219–235.
16. W. C. Brown, J. Herzog, One-dimensional local rings of maximal and almost maximal length, J. Algebra **151** (1992), 332–347.
17. A. Campillo, On saturation of curve singularities (any characteristic), Proc. of Symp. in Pure Math. **40** (1983), 211–220.

J.C. Rosales, P.A. García-Sánchez, *Numerical Semigroups,*
Developments in Mathematics 20, DOI 10.1007/978-1-4419-0160-6,

18. A. Campillo, J. I. Farrán, C. Munuera, On the parameters of algebraic-geometry codes related to Arf semigroups, IEEE Trans. Inform. Theory **46** (2000), no. 7, 2634–2638.
19. J. Castellanos, A relation between the sequence of multiplicities and the semigroups of values of an algebroid curve, J. Pure Appl. Algebra **43** (1986), 119–127.
20. S. T. Chapman, P. A. García-Sánchez, D. Llena, V. Ponomarenko, J. C. Rosales, The catenary and tame degree in finitely generated cancellative monoids, Manuscripta Math. **120** (2006), 253–264.
21. F. Curtis, On formulas for the Frobenius number of a numerical semigroup, Math. Scand. **67** (1990), no. 2, 190–192.
22. J. L. Davison, On the linear Diophantine problem of Frobenius, J. Number Theory **48** (1994), 353–363.
23. M. Delgado, P. A. García-Sánchez, J. Morais, "numericalsgps": a GAP package on numerical semigroups. (`http://www.gap-system.org/Packages/numericalsgps.html`).
24. M. Delgado, P. A. García-Sánchez, J. C. Rosales, J. M. Urbano-Blanco, Systems of proportionally modular Diophantine inequalities, Semigroup Forum **76** (2008), 469–488.
25. M. Delgado, J. C. Rosales, On the Frobenius number of a proportionally modular Diophantine inequality, Port. Math. (N.S.) **63** (2006), no. 4, 415–425.
26. F. Delgado, A. Núñez, Monomial rings and saturated rings, in Géométrie Algébrique et Applications, I (La Rábida, 1984), pp. 23–34, Travaux en Cours, 22, Hermann, Paris, 1987.
27. C. Delorme, Sous-monoïdes d'intersection complète de $\mathbb{N}$, Ann. Scient. École Norm. Sup. (4) **9** (1976), 145–154.
28. M. Djawadi, G. Hofmeister, Linear Diophantine problems, Arch. Math. (Basel) **66** (1996), 19–29.
29. L. Fel, Frobenius problem for semigroups $S(d_1, d_2, d_3)$, Functional Analysis and Other Mathematics **1** (2006), 119–157.
30. K. G. Fischer, W. Morris, J. Shapiro, Affine semigroup rings that are complete intersections, Proc. Amer. Math. Soc. **125** (1997), 3137–3145.
31. P. Freyd, Rédei's finiteness theorem for commutative semigroups, Proc. Amer. Math. Soc. **19** (1968), 1003.
32. R. Fröberg, C. Gottlieb, R. Häggkvist, On numerical semigroups, Semigroup Forum **35** (1987), 63–83.
33. P. A. García-Sánchez, J. C. Rosales, Numerical semigroups generated by intervals, Pacific J. Math. **191** (1999), no. 1, 75–83.
34. P. A. García-Sánchez, J. C. Rosales, Every positive integer is the Frobenius number of an irreducible numerical semigroup with at most four generators, Arkiv Mat. **42** (2004), 301–306.
35. The GAP Group. GAP - Groups, Algorithms, and Programming, Version 4.4, 2004. (`http://www.gap-system.org`).
36. A. Geroldinger, F. Halter-Koch, Non-Unique Factorizations: Algebraic, Combinatorial and Analytic Theory, Pure and Applied Mathematics, 278, Chapman & Hall/CRC, Boca Raton, 2006.
37. R. Gilmer, Commutative Semigroup Rings, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, 1984.
38. R. L. Graham, D. E. Knuth, O. Patashnik, Concrete Mathematics: A Foundation for Computer Science, Addison-Wesley Publishing Company, Advanced Book Program, Reading, MA, 1989.
39. P. A. Grillet, A short proof of Rédei's theorem, Semigroup Forum **46** (1993), 126–127.
40. G. H. Hardy, E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, Oxford, 1979.
41. J. Herzog, Generators and relations of abelian semigroups and semigroup rings, Manuscripta Math. **3** (1970), 175–193.
42. S. M. Johnson, A linear Diophantine problem, Canad. J. Math. **12** (1960), 390–398.

43. C. Kirfel, R. Pellikaan, The minimum distance of codes in an array coming from telescopic semigroups, IEEE Trans. Inform. Theory **41** (1995), no. 6, part 1, 1720–1732. Special issue on algebraic geometry codes.
44. E. Kunz, The value-semigroup of a one-dimensional Gorenstein ring, Proc. Amer. Math. Soc. **25** (1970), 748–751.
45. E. Kunz, Über die Klassifikation numerischer Halbgruppen, Regensburger Mathematische Schriften, 11, Universität Regensburg, Fachbereich Mathematik, Regensburg, 1987.
46. R. G. Levin, On commutative, nonpotent Archimedean semigroups, Pacific J. Math. **27** (1968), 365–371.
47. J. Lipman, Stable ideals and Arf rings, Amer. J. Math. **93** (1971), 649–685.
48. D. Narsingh, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall Series in Automatic Computation, Prentice Hall, Englewood Cliffs, NJ, 1974.
49. A. Núñez, Algebro-geometric properties of saturated rings, J. Pure Appl. Algebra **59** (1989), 201–214.
50. F. Pham, B. Teissier, Fractions lipschitziennes et saturation de Zariski des algèbres analytiques complexes, Centre Math. École Polytech., Paris, 1969, in Actes du Congrès International des Mathématiciens (Nice, 1970), Tome 2, pp. 649–654, Gauthier-Villars, Paris, 1971.
51. G. B. Preston, Rédei's characterization of congruences on finitely generated free commutative semigroups, Acta Math. Acad. Sci. Hungar. **26** (1975), 337–342.
52. J. L. Ramírez Alfonsín, The Diophantine Frobenius Problem, Oxford University Press, Oxford, 2005.
53. L. Rédei, The Theory of Finitely Generated Commutative Semigroups, Pergamon, Oxford-Edinburgh-New York, 1965.
54. A. M. Robles-Pérez, J. C. Rosales, Equivalent proportionally modular Diophantine inequalities, Archiv der Mathematik **90** (2008), 24–30.
55. J. C. Rosales, Semigrupos numéricos, Tesis Doctoral, Universidad de Granada, Spain, 2001.
56. J. C. Rosales, Function minimum associated to a congruence on integral n-tuple space, Semigroup Forum **51** (1995) 87–95.
57. J. C. Rosales, On numerical semigroups, Semigroup Forum **52** (1996), 307–318.
58. J. C. Rosales, On symmetric numerical semigroups, J. Algebra **182** (1996), no. 2, 422–434.
59. J. C. Rosales, An algorithmic method to compute a minimal relation for any numerical semigroup, Internat. J. Algebra Comput. **6** (1996), no. 4, 441–455.
60. J. C. Rosales, On presentations of subsemigroups of $\mathbb{N}^n$, Semigroup Forum **55** (1997), 152–159.
61. J. C. Rosales, Numerical semigroups with Apéry sets of unique expression, J. Algebra **226** (2000), no. 1, 479–487.
62. J. C. Rosales, Symmetric numerical semigroups with arbitrary multiplicity and embedding dimension, Proc. Amer. Math. Soc. **129** (2001), no. 8, 2197–2203.
63. J. C. Rosales, Principal ideals of numerical semigroups, Bull. Belg. Math. Soc. Simon Stevin **10** (2003), 329–343.
64. J. C. Rosales, Fundamental gaps of numerical semigroups generated by two elements, Linear Algebra Appl. **405** (2005), 200–208.
65. J. C. Rosales, Adding or removing an element from a pseudo-symmetric numerical semigroup, Boll. Unione Mat. Ital. **9-B** (2006), 681–696.
66. J. C. Rosales, Subadditive periodic functions and numerical semigroups, J. Algebra Appl. **6** (2007), no. 2, 305–313.
67. J. C. Rosales, Families of numerical semigroups closed under finite intersections and for the Frobenius number, Houston J. Math. **34** (2008), 339–348.
68. J. C. Rosales, Numerical semigroups that differ from a symmetric numerical semigroup in one element, Algebra Colloquium **15** (2008), 23–32.
69. J. C. Rosales, One half of a pseudo-symmetric numerical semigroup, Bull. London Math. Soc. **40** (2008), 347–352.
70. J. C. Rosales, M. B. Branco, Decomposition of a numerical semigroup as an intersection of irreducible numerical semigroups, Bull. Belg. Math. Soc. Simon Stevin **9** (2002), 373–381.

71. J. C. Rosales, M. B. Branco, Numerical semigroups that can be expressed as an intersection of symmetric numerical semigroups, J. Pure Appl. Algebra **171** (2002), nos. 2–3, 303–314.
72. J. C. Rosales, M. B. Branco, Three families of free numerical semigroups with arbitrary embedding dimension, Int. J. Commutative Rings **1** (2002), no. 4, 195–201. Also in the book: Ayman Badawi, Commutative Rings, pp. 151–157, Nova Science Publishers, New York, 2002.
73. J. C. Rosales, M. B. Branco, Irreducible numerical semigroups, Pacific J. Math. **209** (2003), 131–143.
74. J. C. Rosales, M. B. Branco, Irreducible numerical semigroups with arbitrary multiplicity and embedding dimension, J. Algebra **264** (2003), 305–315.
75. J. C. Rosales, J. I. García-García, Hereditarily finitely generated commutative monoids, J. Algebra **221** (1999), no. 2, 723–732.
76. J. C. Rosales, J. I. García-García, Generalized N-semigroups, in Proceedings of the International Conference on Semigroups, pp. 171–180, World Scientific, Singapore, 2000, Editors: Paula Smith, Emília Giraldes and Paula Martins.
77. J. C. Rosales, P. A. García-Sánchez, On complete intersection affine semigroups, Comm. Algebra **23** (1995), 5395–5412.
78. J. C. Rosales, P. A. García-Sánchez, On Cohen-Macaulay and Gorenstein simplicial affine semigroups, Proc. Edinburgh Math. Soc. **41** (1998), 517–537.
79. J. C. Rosales, P. A. García-Sánchez, On numerical semigroups with high embedding dimension, J. Algebra **203** (1998), no. 2, 567–578.
80. J. C. Rosales, P. A. García-Sánchez, Finitely Generated Commutative Monoids, Nova Science Publishers, New York, 1999.
81. J. C. Rosales, P. A. García-Sánchez, Numerical semigroups with embedding dimension three, Archiv der Mathematik **83** (2004), 488–496.
82. J. C. Rosales, P. A. García-Sánchez, Pseudo-symmetric numerical semigroups with three generators, J. Algebra **291** (2005), 46–54.
83. J. C. Rosales, P. A. García-Sánchez, Every numerical semigroup is one half of infinitely many symmetric numerical semigroups, Comm. Algebra **36** (2008), 2910–2916.
84. J. C. Rosales, P. A. García-Sánchez, Numerical semigroups having a Toms decomposition, Canadian Math. Bull. **51** (2008), 134–139.
85. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, Every positive integer is the Frobenius number of a numerical semigroup with three generators, Math. Scand. **94** (2004), 5–12.
86. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, *k*-factorized elements in telescopic numerical semigroups, in Proceedings of the Chapel Hill Conference on Arithmetical Properties of Commutative Rings and Monoids, pp. 260–271, Lecture Notes in Pure Appl. Math., 241, CRC Press, Boca Raton, 2005, Editor: S. T. Chapman.
87. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, M. B. Branco, Systems of inequalities and numerical semigroups, J. London Math. Soc. **65** (2002), no. 3, 611–623.
88. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, M. B. Branco, Arf numerical semigroups, J. Algebra **276** (2004), 3–12.
89. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, M. B. Branco, Saturated numerical semigroups, Houston J. Math. **30** (2004), 321–330.
90. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, J. A. Jimenez-Madrid, The oversemigroups of a numerical semigroup, Semigroup Forum **67** (2003), 145–158.
91. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, J. A. Jiménez-Madrid, Fundamental gaps in numerical semigroups, J. Pure Appl. Alg. **189** (2004), 301–313.
92. J. C. Rosales, P. A. García-Sánchez, J. I. García-García, J. M. Urbano-Blanco, Proportionally modular Diophantine inequalities, J. Number Theory **103** (2003), 281–294.
93. J. C. Rosales, P. A. García-Sánchez, J. M. Urbano-Blanco, On presentations of commutative monoids, Internat. J. Algebra Comput. **9** (1999), no. 5, 539–553.
94. J. C. Rosales, P. A. García-Sánchez, J. M. Urbano-Blanco, Modular Diophantine inequalities and numerical semigroups, Pacific J. Math. **218** (2005), no. 2, 379–398.
95. J. C. Rosales, P. A. García-Sánchez, J. M. Urbano-Blanco, The set of solutions of a proportionally modular Diophantine inequality, J. Number Theory. **128** (2008), 453–467.

96. J. C. Rosales, P. Vasco, The smallest positive integer that is solution of a proportionally modular Diophantine inequality, Math. Inequal. Appl. **11** (2008), 203–212.
97. J. C. Rosales, J. M. Urbano-Blanco, Opened modular numerical semigroups, J. Algebra **306** (2006), 368–377.
98. J. C. Rosales, J. M. Urbano-Blanco, Proportionally modular Diophantine inequalities and full semigroups, Semigroup Forum **72** (2006), 362–374.
99. J. D. Sally, On the associated graded ring of a local Cohen-Macaulay ring, J. Math. Kyoto Univ. **17** (1977), 19–21.
100. J. D. Sally, Cohen-Macaualy local rings of maximal embedding dimension, J. Algebra **56** (1979), 168–183.
101. E. S. Selmer, On a linear Diophantine problem of Frobenius, J. Reine Angew. Math. **293/294** (1977), 1–17.
102. J. J. Sylvester, Mathematical questions with their solutions, Educational Times **41** (1884), 21.
103. T. Tamura, Nonpotent Archimedean semigroups with cancellative law, I, J. Gakugei Tokushima Univ. **8** (1957), 5–11.
104. J. M. Urbano-Blanco, Semigrupos numéricos proporcionalmente modulares, Tesis Doctoral, Universidad de Granada, Spain, March 2005.
105. B. Teissier, Appendice à “Le probléme des modules pour le branches planes”, cours donné par O. Zariski au Centre de Math. de L’École Polytechnique, Paris (1973).
106. A. Toms, Strongly perforated K_0-groups of simple C^*-algebras, Canad. Math. Bull. **46** (2003), 457–472.
107. K. Watanabe, Some examples of one dimensional Gorenstein domains, Nagoya Math. J. **49** (1973), 101–109.
108. H. S. Wilf, A circle-of-lights algorithm for the “money-changing problem”, Amer. Math. Monthly **85** (1978), 562–565.
109. O. Zariski, General theory of saturation and saturated local rings I, II, III, Amer. J. Math. **93** (1971), 573–684, 872–964, **97** (1975), 415–502.

List of symbols

$\langle n_1,\dots,n_p\rangle$ - submonoid generated by $\{n_1,\dots,n_p\}$, p. 1.
Arf(S) - Arf closure of the numerical semigroup S, p. 25.
Ap(S,m) - Apéry set of the element $m \neq 0$ in S, p. 8.
Ch(S) - chain associated to the numerical semigroup S, p. 92.
Ch(X) - union of the chains associated to the numerical semigroups in X, p. 98.
Cong(ρ) - congruence generated by ρ, p. 108.
$\mathrm{d}_A(a)$ - greatest common divisor of the elements in A less than or equal to $a \in A$, p. 28.
D(X) - set of all positive divisors of the elements of X, p. 51.
$\Delta(X) = \{(x,x) \mid x \in X\}$ - the diagonal of $X \times X$, p. 108.
e(S) - embedding dimension of the numerical semigroup S, p. 9.
F(S) - Frobenius number of the numerical semigroup S, p. 9.
FG(S) - set of fundamental gaps of S, p. 52.
Free(X) - free monoid on X, p. 107.
$\mathscr{F}(X)$ - Frobenius variety generated by the set of numerical semigroups X, p. 98.
g(S) - genus (or degree of singularity) of the numerical semigroup S, p. 9.
$\mathscr{G}(\mathscr{V})$ - graph associated to the Frobenius variety $\mathscr{V}$, pp. 92, 101.
$\mathscr{I}(S)$ - set of irreducible numerical semigroups containing the numerical semigroup S, p. 47.
im(f) - image of the homomorphism f, p. 106.
Irr(σ) - set of irreducible elements of σ, p. 109.
J(S) - for a numerical semigroup minimally generated by $n_1 < \cdots < n_e$ is the set $\{\lambda_2 x_2 + \cdots + \lambda_e x_e \mid \lambda_2 n_2 + \cdots + \lambda_e n_e \notin \mathrm{Ap}(S,n_1)\}$, p. 138.
ker(f) the kernel congruence of the monoid homomorphism f, p. 106.
m(S) - multiplicity of the numerical semigroup S, p. 9.
M(f) - monoid associated to the subadditive function f, p. 58.
$\mathrm{Maximals}_{\leq}(X)$ - maximal elements of X with respect to the ordering $\leq$, p. 13.
$\mathrm{Minimals}_{\leq}(X)$ - minimal elements of X with respect to the ordering $\leq$, p. 13.
$a \bmod b$ - is the quotient of the division of a by b;
$a \equiv b \bmod m$ means $(a-b) \bmod m = 0$, p. 20.
n(S) - cardinality of the set of elements in S less than its Frobenius number, p. 15.

$\mathrm{N}(S)$ - set of elements in S less than its Frobenius number, p. 15.

$\mathbb{N}$ - set of nonnegative integers, p. 1.

$\mathrm{o}(g)$ - order of an element g in a group G, p. 167.

$\mathscr{O}(S)$ - set of numerical semigroups containing the numerical semigroup S, p. 44.

$\mathrm{PF}(S)$ - set of pseudo-Frobenius numbers of the numerical semigroup S, p. 13.

$\mathbb{Q}_0^+$ - set of nonnegative rational numbers, p. 59.

$\mathrm{Q}(S)$ - quotient group of S, p. 163.

$\mathscr{S}$ - set of all numerical semigroups, p. 47.

$\mathscr{S}_m$ - set of numerical semigroups with multiplicity m, p. 58.

$\mathscr{S}(g_1,\ldots,g_t)$ - set of numerical semigroups not cutting $\{g_1,\ldots,g_t\}$, p. 47.

$\mathscr{S}(P)$ - set of numerical semigroups admitting a pattern P, p. 96.

$\mathrm{S}(a,b,c)$ - set of integer solutions to $ax \bmod b \leq cx$, p. 58.

$\mathrm{S}(A)$ - with $A \subset \mathbb{Q}_0^+$, the set of integers of the submonoid $\langle A\rangle$, p. 59.

$\mathrm{Sat}(S)$ - saturated closure of the numerical semigroup S, p. 30.

$\mathscr{S}\mathscr{F}_m$ - set of m-periodic subadditive functions, p. 58.

$\mathrm{SG}(S)$ - set of special gaps of S, p. 44.

R - relation defining the R-classes of the expressions of a given element in a numerical semigroup, p. 111.

$\mathrm{t}(S)$ - type of the numerical semigroup S, p. 13.

$\mathscr{V}$ - a Frobenius variety, p. 99.

$\mathscr{V}(A)$ - $\mathscr{V}$-monoid generated by A, p. 99. Used also as a prefix to denote systems of generators and monoids relative to this variety; see Chapter 6.

$\mathbb{Z}$ - set of integers, p. 13.

$\mathrm{Z}(n)$ - set of factorizations or expression of n in a numerical semigroup S, p. 111.

$\mathrm{Z}_B(n)$ - set of factorizations or expression of n in a numerical semigroup S relative to the set of generators B, p. 111.

Index