Appendix A
Transform Pairs and Properties

See Tables A.1, A.2, A.3, A.4, A.5, A.6, A.7, and A.8.

Table A.1 DFT pairs

$x(n)$,Period $= N$	$X(k)$, Period $= N$
$\delta(n)$	1
1	$N\delta(k)$
$e^{j\left(\frac{2\pi}{N}k_0 n\right)}$	$N\delta(k-k_0)$
$\cos\left(\frac{2\pi}{N}k_0 n+\theta\right)$	$\frac{N}{2}(e^{j\theta}\delta(k-k_0)+e^{-j\theta}\delta(k-(N-k_0)))$
$\cos\left(\frac{2\pi}{N}k_0 n\right)$	$\frac{N}{2}(\delta(k-k_0)+\delta(k-(N-k_0)))$
$\sin\left(\frac{2\pi}{N}k_0 n\right)$	$\frac{N}{2}(-j\delta(k-k_0)+j\delta(k-(N-k_0)))$
$x(n)=\begin{cases}1 & \text{for } n=0,1,\ldots,L-1\\ 0 & \text{for } n=L,L+1,\ldots,N-1\end{cases}$	$e^{\left(-j\frac{\pi}{N}(L-1)k\right)}\frac{\sin(\frac{\pi}{N}kL)}{\sin(\frac{\pi}{N}k)}$

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Table A.2 DFT properties

Property	$x(n)$, $h(n)$, Period = N	$X(k)$, $H(k)$, Period = N
Linearity	$ax(n) + bh(n)$	$aX(k) + bH(k)$
Duality	$\frac{1}{N}X(N \mp n)$	$x(N \pm k)$
Time-shifting	$x(n \pm n_0)$	$e^{\pm j\frac{2\pi}{N}n_0 k}X(k)$
Frequency-shifting	$e^{\mp j\frac{2\pi}{N}k_0 n}x(n)$	$X(k \pm k_0)$
Time-convolution	$\sum_{k=0}^{N-1} x(k)h(n-k)$	$X(k)H(k)$
Frequency-convolution	$x(n)h(n)$	$\frac{1}{N}\sum_{v=0}^{N-1} X(v)H(k-v)$
Time-expansion	$h(an) = \begin{cases} x(n) & \text{for } n = 0, 1, \ldots, N-1 \\ 0 & \text{otherwise} \end{cases}$ where a is any positive integer	$H(k) = X(k \bmod N)$, $k = 0, 1, \ldots, aN-1$
Time-reversal	$x(N-n)$	$X(N-k)$
Conjugation	$x^*(N \pm n)$	$X^*(N \mp k)$
Parseval's theorem	$\sum_{n=0}^{N-1} \|x(n)\|^2$	$\frac{1}{N}\sum_{k=0}^{N-1} \|X(k)\|^2$

Table A.3 FS pairs

$x(t)$, Period = T	$X_{fs}(k),\ \omega_0 = \frac{2\pi}{T}$
$\begin{cases} 1 & \text{for } \|t\| < a \\ 0 & \text{for } a < \|t\| \leq \frac{T}{2} \end{cases}$	$\frac{\sin(k\omega_0 a)}{k\pi}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{1}{T}$
$e^{jk_0\omega_0 t}$	$\delta(k - k_0)$
$\cos(k_0\omega_0 t)$	$0.5(\delta(k + k_0) + \delta(k - k_0))$
$\sin(k_0\omega_0 t)$	$0.5j(\delta(k + k_0) - \delta(k - k_0))$

Table A.4 FS properties

Property	$x(t), h(t),$ Period $= T$	$X_{fs}(k), H_{fs}(k),\ \omega_0 = \frac{2\pi}{T}$
Linearity	$ax(t) + bh(t)$	$aX_{fs}(k) + bH_{fs}(k)$
Time-shifting	$x(t \pm t_0)$	$e^{\pm jk\omega_0 t_0} X_{fs}(k)$
Frequency-shifting	$x(t)e^{\pm jk_0\omega_0 t}$	$X_{fs}(k \mp k_0)$
Time-convolution	$\int_0^T x(\tau)h(t-\tau)d\tau$	$TX_{fs}(k)H_{fs}(k)$
Frequency-convolution	$x(t)h(t)$	$\sum_{l=-\infty}^{\infty} X_{fs}(l)H_{fs}(k-l)$
Time-scaling	$x(at), a > 0,$ Period $= \frac{T}{a}$	$X_{fs}(k),\ \omega_0 = a\frac{2\pi}{T}$
Time-reversal	$x(-t)$	$X_{fs}(-k)$
Time-differentiation	$\frac{d^n x(t)}{dt^n}$	$(jk\omega_0)^n X_{fs}(k)$
Time-integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X_{fs}(k)}{jk\omega_0}$, if $(X_{fs}(0) = 0)$
Parseval's theorem	$\frac{1}{T}\int_0^T \lvert x(t)\rvert^2 dt$	$\sum_{k=-\infty}^{\infty} \lvert X_{fs}(k)\rvert^2$
Conjuagte symmetry	$x(t)$ real	$X_{fs}(k) = X_{fs}^*(-k)$
Even symmetry	$x(t)$ real and even	$X_{fs}(k)$ real and even
Odd symmetry	$x(t)$ real and odd	$X_{fs}(k)$ imaginary and odd

Table A.5 DTFT pairs

$x(n)$	$X(e^{j\omega})$, Period $= 2\pi$
$\begin{cases} 1 \text{ for } -N \le n \le N \\ 0 \text{ otherwise} \end{cases}$	$\frac{\sin(\omega\frac{(2N+1)}{2})}{\sin(\frac{\omega}{2})}$
$\frac{\sin(an)}{\pi n},\ 0 < a \le \pi$	$\begin{cases} 1 \text{ for } \lvert\omega\rvert < a \\ 0 \text{ for } a < \lvert\omega\rvert \le \pi \end{cases}$
$a^n u(n),\ \lvert a\rvert < 1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u(n),\ \lvert a\rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$a^{\lvert n\rvert},\ \lvert a\rvert < 1$	$\frac{1-a^2}{1-2a\cos(\omega)+a^2}$
$a^n \sin(\omega_0 n)u(n),\ \lvert a\rvert < 1$	$\frac{(a)e^{-j\omega}\sin(\omega_0)}{1-2(a)e^{-j\omega}\cos(\omega_0)+(a)^2e^{-j2\omega}}$
$a^n \cos(\omega_0 n)u(n),\ \lvert a\rvert < 1$	$\frac{1-(a)e^{-j\omega}\cos(\omega_0)}{1-2(a)e^{-j\omega}\cos(\omega_0)+(a)^2e^{-j2\omega}}$
$\delta(n)$	1
$\sum_{k=-\infty}^{\infty} \delta(n-kN)$	$\frac{2\pi}{N}\sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi}{N}k)$
$u(n)$	$\pi\delta(\omega) + \frac{1}{1-e^{-j\omega}}$
1	$2\pi\delta(\omega)$
$sgn(n)$	$\frac{2}{1-e^{-j\omega}}$
$e^{j\omega_0 n}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 n)$	$\pi(\delta(\omega+\omega_0)+\delta(\omega-\omega_0))$
$\sin(\omega_0 n)$	$j\pi(\delta(\omega+\omega_0)-\delta(\omega-\omega_0))$

Table A.6 DTFT properties

Property	$x(n), h(n)$	$X(e^{j\omega}), H(e^{j\omega})$
Linearity	$ax(n) + bh(n)$	$aX(e^{j\omega}) + bH(e^{j\omega})$
Time-shifting	$x(n \pm n_0)$	$e^{\pm j\omega n_0} X(e^{j\omega})$
Frequency-shifting	$x(n)e^{\pm j\omega_0 n}$	$X(e^{j(\omega \mp \omega_0)})$
Time-convolution	$\sum_{k=-\infty}^{\infty} x(k)h(n-k)$	$X(e^{j\omega})H(e^{j\omega})$
Frequency-convolution	$x(n)h(n)$	$\frac{1}{2\pi}\int_0^{2\pi} X(e^{jv})H(e^{j(\omega-v)})dv$
Time-expansion	$h(n)$ $h(an) = x(n),\ a >$ 0 is an integer and $h(n) = 0$ zero otherwise	$H(e^{j\omega}) = X(e^{ja\omega})$
Time-reversal	$x(-n)$	$X(e^{-j\omega})$
Conjugation	$x^*(\pm n)$	$X^*(e^{\mp j\omega})$
Difference	$x(n) - x(n-1)$	$(1 - e^{-j\omega})X(e^{j\omega})$
Summation	$\sum_{l=-\infty}^{n} x(l)$	$\frac{X(e^{j\omega})}{(1-e^{-j\omega})} + \pi X(e^{j0})\delta(\omega)$
Frequency-differentiation	$(n)^k x(n)$	$(j)^k \frac{d^k X(e^{j\omega})}{d\omega^k}$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} \|x(n)\|^2$	$\frac{1}{2\pi}\int_0^{2\pi} \|X(e^{j\omega})\|^2 d\omega$
Conjuagte symmetry	$x(n)$ real	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Even symmetry	$x(n)$ real and even	$X(e^{j\omega})$ real and even
Odd symmetry	$x(n)$ real and odd	$X(e^{j\omega})$ imaginary and odd

Table A.7 FT pairs

$x(t)$	$X(j\omega)$
$u(t+a) - u(t-a)$	$2\frac{\sin(\omega a)}{\omega}$
$\frac{\sin(\omega_0 t)}{\pi t}$	$u(\omega+\omega_0) - u(\omega-\omega_0)$
$e^{-at}u(t),\ \mathrm{Re}(a) > 0$	$\frac{1}{a+j\omega}$
$te^{-at}u(t),\ \mathrm{Re}(a) > 0$	$\frac{1}{(a+j\omega)^2}$
$e^{-a\|t\|},\ \mathrm{Re}(a) > 0$	$\frac{2a}{a^2+\omega^2}$
$\frac{1}{a}((t+a)u(t+a) - 2tu(t) + (t-a)u(t-a))$	$a\left(\frac{\sin(\omega\frac{a}{2})}{\omega\frac{a}{2}}\right)^2$
$e^{-at}\sin(\omega_0 t)u(t),\ \mathrm{Re}(a) > 0$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
$e^{-at}\cos(\omega_0 t)u(t),\ \mathrm{Re}(a) > 0$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
$\delta(t)$	1
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - k\frac{2\pi}{T})$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
1	$2\pi\delta(\omega)$
$sgn(t)$	$\frac{2}{j\omega}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$	$\pi(\delta(\omega+\omega_0) + \delta(\omega-\omega_0))$
$\sin(\omega_0 t)$	$j\pi(\delta(\omega+\omega_0) - \delta(\omega-\omega_0))$

Table A.8 FT properties

Property	$x(t), h(t)$	$X(j\omega), H(j\omega)$
Linearity	$ax(t) + bh(t)$	$aX(j\omega) + bH(j\omega)$
Duality	$X(\pm t)$	$2\pi x(\mp j\omega)$
Time-shifting	$x(t \pm t_0)$	$X(j\omega)e^{\pm j\omega t_0}$
Frequency-shifting	$x(t)e^{\pm j\omega_0 t}$	$X(j(\omega \mp \omega_0))$
Time-convolution	$x(t) * h(t)$	$X(j\omega)H(j\omega)$
Frequency-convolution	$x(t)h(t)$	$\frac{1}{2\pi}(X(j\omega) * H(j\omega))$
Time-scaling	$x(at)$, $a \neq 0$ and real	$\frac{1}{\lvert a\rvert}X(j\frac{\omega}{a})$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(\pm t)$	$X^*(\mp j\omega)$
Time-differentiation	$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(j\omega)$
Time-integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$
Frequency-differentiation	$t^n x(t)$	$(j)^n \frac{d^n X(j\omega)}{d\omega^n}$
Parseval's theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2 d\omega$
Autocorrelation	$x(t) * x(-t) =$ $\int_{-\infty}^{\infty} x(\tau)x(\tau - t)d\tau$	$\lvert X(j\omega)\rvert^2$
Conjuagte symmetry	$x(t)$ real	$X(j\omega) = X^*(-j\omega)$
Even symmetry	$x(t)$ real and even	$X(j\omega)$ real and even
Odd symmetry	$x(t)$ real and odd	$X(j\omega)$ imaginary and odd

Appendix B
Useful Mathematical Formulas

Trigonometric Identities

Pythagorean Identity

$$\sin^2 x + \cos^2 x = 1$$

Addition and Subtraction Formulas

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

Double-angle Formulas

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

Product Formulas

$$2 \sin x \cos y = \sin(x - y) + \sin(x + y)$$

$$2 \cos x \sin y = -\sin(x - y) + \sin(x + y)$$

$$2 \sin x \sin y = \cos(x - y) - \cos(x + y)$$

$$2 \cos x \cos y = \cos(x - y) + \cos(x + y)$$

Sum and Difference Formulas

$$\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$$

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$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Other Formulas

$$\sin(-x) = \sin(2\pi - x) = -\sin x$$

$$\cos(-x) = \cos(2\pi - x) = \cos x$$

$$\sin(\pi \pm x) = \mp \sin x$$

$$\cos(\pi \pm x) = -\cos x$$

$$\cos\left(\frac{\pi}{2} \pm x\right) = \mp \sin x$$

$$\sin\left(\frac{\pi}{2} \pm x\right) = \cos x$$

$$\cos\left(\frac{3\pi}{2} \pm x\right) = \pm \sin x$$

$$\sin\left(\frac{3\pi}{2} \pm x\right) = -\cos x$$

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{j2}$$

Series Expansions

$$e^{jx} = 1 + (jx) + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \frac{(jx)^4}{4!} + \cdots + \frac{(jx)^r}{(r)!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^r\frac{x^{2r}}{(2r)!} - \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^r\frac{x^{2r+1}}{(2r+1)!} - \cdots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{(1)(3)}{(2)(4)}\frac{x^5}{5} + \frac{(1)(3)(5)}{(2)(4)(6)}\frac{x^7}{7} + \cdots, \quad |x| < 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x, \quad |x| < 1$$

Summation Formulas

$$\sum_{k=0}^{N} k = \frac{N(N+1)}{2}$$

$$\sum_{k=0}^{N-1} (a + kd) = \frac{N(2a + (N-1)d)}{2}$$

$$\sum_{k=0}^{N-1} ar^k = \frac{a(1 - r^N)}{1 - r}, \quad r \neq 1$$

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}, \quad |r| < 1$$

$$\sum_{k=0}^{\infty} kr^k = \frac{r}{(1 - r)^2}, \quad |r| < 1$$

$$1 + \cos(t) + \cos(2t) + \cdots + \cos(Nt) = \frac{1}{2} + \frac{\sin(0.5(2N+1)t)}{2\sin(0.5t)}$$

Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{at} dt = \frac{e^{at}}{a}$$

$$\int te^{at} dt = \frac{e^{at}}{a^2}(at - 1)$$

$$\int e^{bt} \sin(at) dt = \frac{e^{bt}}{a^2 + b^2}(b \sin(at) - a \cos(at))$$

$$\int e^{bt} \cos(at) dt = \frac{e^{bt}}{a^2 + b^2}(b \cos(at) + a \sin(at))$$

$$\int \sin(at)dt = -\frac{1}{a}\cos(at)$$

$$\int \cos(at)dt = \frac{1}{a}\sin(at)$$

$$\int t\sin(at)dt = \frac{1}{a^2}(\sin(at) - at\cos(at))$$

$$\int t\cos(at)dt = \frac{1}{a^2}(\cos(at) + at\sin(at))$$

$$\int \sin^2(at)dt = \frac{t}{2} - \frac{1}{4a}\sin(2at)$$

$$\int \cos^2(at)dt = \frac{t}{2} + \frac{1}{4a}\sin(2at)$$

$$\int \frac{1}{a^2+t^2}dt = \frac{1}{a}\tan^{-1}(\frac{t}{a})$$

$$\int \sin(at)\sin(bt)dt = \frac{\sin((a-b)t)}{2(a-b)} - \frac{\sin((a+b)t)}{2(a+b)}, \quad a^2 \neq b^2$$

$$\int \sin(at)\cos(bt)dt = -\left(\frac{\cos((a-b)t)}{2(a-b)} + \frac{\cos((a+b)t)}{2(a+b)}, \quad a^2 \neq b^2\right)$$

$$\int \cos(at)\cos(bt)dt = \frac{\sin((a-b)t)}{2(a-b)} + \frac{\sin((a+b)t)}{2(a+b)}, \quad a^2 \neq b^2$$

Differentiation Formulas

$$\frac{d(uv)}{dt} = u\frac{dv}{dt} + v\frac{du}{dt}$$

$$\frac{d(f(u))}{dt} = \frac{d(f(u))}{du}\frac{du}{dt}$$

$$\frac{d(\frac{u}{v})}{dt} = \frac{v\frac{du}{dt} - u\frac{dv}{dt}}{v^2}$$

$$\frac{d(x^n)}{dt} = nx^{n-1}$$

$$\frac{d(e^{at})}{dt} = ae^{at}$$

$$\frac{d(\sin(at))}{dt} = a\cos(at)$$

$$\frac{d(\cos(at))}{dt} = -a\sin(at)$$

L'Hôpital's Rule

$$\text{If } \lim_{x\to a} f(x) = 0 \text{ and } \lim_{x\to a} g(x) = 0, \text{ or}$$

$$\text{If } \lim_{x\to a} f(x) = \infty \text{ and } \lim_{x\to a} g(x) = \infty, \text{ then}$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{\frac{df(x)}{dx}}{\frac{dg(x)}{dx}}$$

The rule can be applied as many times as necessary.

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Answers to Selected Exercises

Chapter 1

1.1.2 0.
1.2.3

$$x(n) = -2\delta(n) - \delta(n-2) + 3\delta(n+2) + 3\delta(n+3)$$

1.3.3

$$\{-1.4575, -1.7292, -1.7320, -1.7321, -1.7321\}$$

1.4.4

$$371.0329$$

1.5.1

$$x(n) = \begin{cases} -2 \text{ for } n = 1, 0, -1 \\ 0 \quad \text{otherwise} \end{cases}$$

1.6.5

$$\{2.5981, 0.7765, -1.5000, -2.8978, -2.5981, -0.7765, 1.5000, 2.8978\}$$

$$x(n) = 2.5981 \cos\left(\frac{\pi}{4}n\right) - 1.5 \sin\left(\frac{\pi}{4}n\right)$$

1.7.3

$$c(n) = 3.8639 \cos\left(\frac{\pi}{4}n + 0.7945\right)$$

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The samples of the sinusoid $a(n)$ are

$$\{2.0000, -1.0353, -3.4641 - 3.8637, -2.0000, 1.0353, 3.4641, 3.8637\}$$

The samples of the sinusoid $b(n)$ are

$$\{0.7071, 1.0000, 0.7071, 0.0000, -0.7071, -1.0000, -0.7071, -0.0000\}$$

The samples of the sinusoid $c(n) = a(n) + b(n)$ are

$$\{2.7071, -0.0353, -2.7570, -3.8637, -2.7071, 0.0353, 2.7570, 3.8637\}$$

1.8.4 The waveform is periodic with period 6.
1.9.4

$$x(n) = (-j)(e^{j(\frac{2\pi}{8}n-\frac{\pi}{6})} - e^{-j(\frac{2\pi}{8}n-\frac{\pi}{6})})$$

The samples of $x(n)$ are

$$\{-1.0000, 0.5176, 1.7321, 1.9319, 1.0000, -0.5176, -1.7321, -1.9319\}$$

1.10.3 The samples of $x(n)$ are

$$\{0.8660, 0.2588, -0.5000, -0.9659, -0.8660, -0.2588, 0.5000, 0.9659\}$$

$$x(-4) = x(4) = -0.8660$$

1.11.4

$$x_e(-3) = 0.5, x_e(-2) = 0.5, x_e(-1) = 0, x_e(0) = 0,$$
$$x_e(1) = 0, x_e(2) = 0.5, x_e(3) = 0.5$$

$$x_o(-3) = -0.5, x_o(-2) = -0.5, x_o(-1) = 0,$$
$$x_o(0) = 0, x_o(1) = 0, x_o(2) = 0.5, x_o(3) = 0.5$$

1.12.3 Neither.
1.13.3 Average power is 2.
1.14.3 $x(an + k) = \sin(\frac{\pi}{8}n - \frac{\pi}{3})$.

$$\{-0.8660, 0, -0.2588, 0, 0.5000, 0, 0.9659, 0, 0.8660,$$
$$0, 0.2588, 0, -0.5000, 0, -0.9659, 0\}$$

1.15.1 $x(an + k) = \{x(1) = 1, x(2) = 2\}$, and 0 otherwise.
1.16.2 $(-2 + j4)$.

1.17.3 $0.3231 + j0.5846$.
1.18.1 $17.0294e^{(-j1.6296)}$.
1.19.3 $5 - j2, 5 + j2$.
1.20.3

$$\{-0.8090 + j0.5878, -0.8090 - j0.5878, 0.3090 + j0.9511, 0.3090 - j0.9511, 1\}$$

Chapter 2

2.1.2 $2^6 2^9 = 2^{15} = 32768$.
2.2.2 $x(n) = e^{j(\frac{\pi}{4}n+\frac{\pi}{6})} + e^{-j(\frac{\pi}{4}n+\frac{\pi}{6})} = (0.5\sqrt{3} + j0.5)e^{j\frac{\pi}{4}n} + (0.5\sqrt{3} - j0.5)e^{-j\frac{\pi}{4}n}$
The samples are

$$\{1.7321, 0.5176, -1.0000, -1.9319, -1.7321, -0.5176, 1.0000, 1.9319\}$$

2.3.5 $\{10 + j2, 4 + j4, -2 + j6, -8\}$.
2.4.2

$$x(n) = \{-0.2679, 3.0000, -3.7321, 5.0000\}$$

$$X(k) = \{4, 3.4641 + j2, -12, 3.4641 - j2\}$$

Chapter 3

3.1.2

$$X(k) = \{10, -2 + j2, -2, -2 - j2\}, Y(k) = \{0, 1 + j3, -10, 1 - j3\},$$
$$Z(k) = \{30, -8, 14, -8\}$$

3.2.3

$$X(k) = \{11, 1, 3, 1\}, \quad X(-7) = 1, \quad x(23) = 2$$

3.3.1

$$\{11, -2 - j1, -3, -2 + j1\}$$

3.4.2

$$\{-3 - j1, 4, -3 + j1, -6\}$$

3.5.2

$$\{2, -4 - j2, -6, -4 + j2\}$$

3.6.3

$$\{16, 4, 8, -8\}$$

3.7.3

$$X(k) = \{-5 + j1, 2 + j4, 3 - j1, 4 - j4\},$$
$$X^*(4 - k) = \{-5 - j1, 4 + j4, 3 + j1, 2 - j4\}$$

3.8.1

$$y(n) = \{16, 15, 6, 13\}$$

3.9.2

$$Y(k) = \{5, -1 + j10, 9, -1 - j10\}$$

3.10.2

$$r_{xh}(n) = \{14, 6, 5, 3\}$$

3.11.3

$$r_{xx}(n) = \{18, 5, 8, 5\}$$

3.12.1

$$X(k) = \{7, -1 - j2, -1, -1 + j2\}$$

3.13.3

$$\{1, 7, 1, 7\}$$

3.14.3

$$\{1, 0, -5, 0\}$$

3.15.1

$$X(k) = \{3, -1\}, \quad XZ(k) = \{3, 1 - j2, -1, 1 + j2\}$$

3.16.5 Real and odd

$$X(k) = \{0, j6, 0, -j6\}$$

3.17.2

$$X(k) = \{14, 3 + j1, 0, 3 - j1\}$$

The power over one period is 54.

Chapter 4

4.1.1

$$x(m, n) = 2 - 1.5e^{j\left(\frac{2\pi}{4}(m+n)+\frac{\pi}{6}\right)} - 1.5e^{-j\left(\frac{2\pi}{4}(m+n)+\frac{\pi}{6}\right)}$$
$$- 0.5je^{j\left(\frac{2\pi}{4}(2m+n)+\frac{\pi}{3}\right)} + 0.5je^{-j\left(\frac{2\pi}{4}(2m+n)+\frac{\pi}{3}\right)}$$

The samples of $x(m, n)$ are

$$x(m,n) = \begin{bmatrix} 0.2679 & 4.0000 & 3.7321 & -0.0000 \\ 2.6340 & 4.0981 & 1.3660 & -0.0981 \\ 5.4641 & 1.0000 & -1.4641 & 3.0000 \\ -0.3660 & -1.0981 & 4.3660 & 5.0981 \end{bmatrix}$$

The DFT is

$$X(k,l) = \begin{bmatrix} 32 & 0 & 0 & 0 \\ 0 & -20.7846 - j12 & 0 & 0 \\ 0 & 6.9282 - j4 & 0 & 6.9282 + j4 \\ 0 & 0 & 0 & -20.7846 + 12 \end{bmatrix}$$

4.2.1

$$X(k,l) = \begin{bmatrix} 40 & -3 + j3 & -2 & -3 - j3 \\ 1 + j1 & -2 + j2 & -7 + j3 & j2 \\ 2 & 5 - j7 & 4 & 5 + j7 \\ 1 - j1 & -j2 & -7 - j3 & -2 - j2 \end{bmatrix}$$

The average power is 122.

4.3.3

$$X(k,l) = \begin{bmatrix} 1 & -j1 & -1 & j1 \\ j1 & 1 & -j1 & -1 \\ -1 & j1 & 1 & -j1 \\ -j1 & -1 & j1 & 1 \end{bmatrix}$$

4.4.2

$$X(k,l) = \begin{bmatrix} 26.00 + j0.00 & 0.00 + j8.00 & -6.00 + j0.00 & 0.00 - j8.00 \\ 6.00 + j0.00 & -4.00 - j4.00 & 2.00 - j4.00 & -4.00 - j4.00 \\ 2.00 + j0.00 & 0.00 + j0.00 & -6.00 + j0.00 & 0.00 + j0.00 \\ 6.00 + j0.00 & -4.00 + j4.00 & 2.00 + j4.00 & -4.00 + j4.00 \end{bmatrix}$$

$$H(k,l) = \begin{bmatrix} 18.00 + j0.00 & 3.00 - j3.00 & 0.00 + j0.00 & 3.00 + j3.00 \\ 3.00 + j5.00 & -6.00 + j2.00 & -5.00 - j1.00 & 0.00 + j2.00 \\ 0.00 + j0.00 & -1.00 - j1.00 & -6.00 + j0.00 & -1.00 + j1.00 \\ 3.00 - j5.00 & 0.00 - j2.00 & -5.00 + j1.00 & -6.00 - j2.00 \end{bmatrix}$$

$$X(k,l)H(k,l) = \begin{bmatrix} 468.00 + j0.00 & 24.00 + j24.00 & -0.00 + j0.00 & 24.00 - j24.00 \\ 18.00 + j30.00 & 32.00 + j16.00 & -14.00 + j18.00 & 8.00 - j8.00 \\ 0.00 + j0.00 & 0.00 + j0.00 & 36.00 + j0.00 & 0.00 + j0.00 \\ 18.00 - j30.00 & 8.00 + j8.00 & -14.00 - j18.00 & 32.00 - j16.00 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 40 & 25 & 24 & 37 \\ 23 & 24 & 19 & 36 \\ 29 & 23 & 33 & 23 \\ 37 & 33 & 29 & 33 \end{bmatrix}$$

$$X(k,l)H^*(k,l) = \begin{bmatrix} 468.00 + j0.00 & -24.00 + j24.00 & -0.00 + j0.00 & -24.00 - j24.00 \\ 18.00 - j30.00 & 16.00 + j32.00 & -6.00 + j22.00 & -8.00 + j8.00 \\ 0.00 + j0.00 & -0.00 + j0.00 & 36.00 + j0.00 & -0.00 + j0.00 \\ 18.00 + j30.00 & -8.00 - j8.00 & -6.00 - j22.00 & 16.00 - j32.00 \end{bmatrix}$$

$$r_{xh}(m,n) = \begin{bmatrix} 31 & 24 & 35 & 36 \\ 20 & 32 & 36 & 44 \\ 26 & 24 & 34 & 24 \\ 28 & 25 & 24 & 25 \end{bmatrix}$$

$$H(k,l)X^*(k,l) = \begin{bmatrix} 468.00 + j0.00 & -24.00 - j24.00 & -0.00 + j0.00 & -24.00 + j24.00 \\ 18.00 + j30.00 & 16.00 - j32.00 & -6.00 - j22.00 & -8.00 - j8.00 \\ 0.00 + j0.00 & 0.00 + j0.00 & 36.00 + j0.00 & -0.00 + j0.00 \\ 18.00 - j30.00 & -8.00 + j8.00 & -6.00 + j22.00 & 16.00 + j32.00 \end{bmatrix}$$

$$r_{hx}(m,n) = \begin{bmatrix} 31 & 36 & 35 & 24 \\ 28 & 25 & 24 & 25 \\ 26 & 24 & 34 & 24 \\ 20 & 44 & 36 & 32 \end{bmatrix}$$

$$X(k,l)X^*(k,l) = \begin{bmatrix} 676 & 64 & 36 & 64 \\ 36 & 32 & 20 & 32 \\ 4 & 0 & 36 & 0 \\ 36 & 32 & 20 & 32 \end{bmatrix} \quad r_{xx}(m,n) = \begin{bmatrix} 70 & 40 & 38 & 40 \\ 50 & 42 & 34 & 42 \\ 40 & 36 & 40 & 36 \\ 50 & 42 & 34 & 42 \end{bmatrix}$$

Chapter 5

5.1.3

The input sequences are zero padded.

$$X(k) = \{10, -0.1213 - j6.5355, -1 + j3, 4.1213 - j0.5355, 0, 4.1213 + j0.5355, -1 - j3, -0.1213 + j6.5355\}$$

$$H(k) = \{-2, 2.4142 + j1.2426, -2 - j2, -0.4142 + j7.2426, 10, -0.4142 - j7.2426, -2 + j2, 2.4142 - j1.2426\}$$

$$Y(k) = X(k)H(k) = \{-20, 7.8284 - j15.9289, 8 - j4, 2.1716 + j30.0711, 0, 2.1716 - j30.0711, 8 + j4, 7.8284 + j15.9289\}$$

$$y(n) = \mathrm{IDFT}(Y(k)) = \{2, -3, 7, -7, -3, 0, -16, 0\}$$

5.2.1 The input sequences are zero padded.

$$X(k,l) = \begin{bmatrix} 1 & 2+j1 & 3 & 2-j1 \\ -2-j3 & -1+j2 & 4+j & 3-j4 \\ -5 & j5 & 5 & -j5 \\ -2+j3 & 3+j4 & 4-j1 & -1-j2 \end{bmatrix}$$

$$H(k,l) = \begin{bmatrix} 4 & -j4 & -4 & j4 \\ 5+j1 & 1-j1 & -1+j3 & 3+j5 \\ 6 & 4-j2 & 2 & 4+j2 \\ 5-j1 & 3-j5 & -1-j3 & 1+j1 \end{bmatrix}$$

$$Y(k,l) = X(k,l)H(k,l) = \begin{bmatrix} 4 & 4-j8 & -12 & 4+j8 \\ -7-j17 & 1+j3 & -7+11 & 29+j3 \\ -30 & 10+j20 & 10 & 10-j20 \\ -7+j17 & 29-j3 & -7-j11 & 1-j3 \end{bmatrix}$$

$$y(m,n) = \mathrm{IDFT}(Y(k,l)) = \begin{bmatrix} 2 & -3 & -9 & 0 \\ 0 & 14 & 3 & 0 \\ -2 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5.3.2 The input sequences are zero padded.

$$X(k) = \{5, -0.4142 - j2.2426, 3 + j2, 2.4142 - j6.2426,$$
$$-7, 2.4142 + j6.2426, 3 - j2, -0.4142 + j2.2426\}$$

$$H^*(k) = \{6, -0.4142 + j3.2426, 2 - j2, 2.4142 + j5.2426,$$
$$-6, 2.4142 - j5.2426, 2 + j2, -0.4142 - j3.2426\}$$

$$Y(k) = X(k)H^*(k) = \{30, 7.4437 - j0.4142, 10 - j2, 38.5563 - j2.4142,$$
$$42, 38.5563 + j2.4142, 10 + j2, 7.4437 + j0.4142\}$$

$$r_{xh}(n) = \mathrm{IDFT}(Y(k)) = \{23, -6, 6, 4, 0, 4, 7, -8, \}$$

5.4.1 The input sequences are zero padded.

$$X(k,l) = \begin{bmatrix} 1 & 2+j1 & 3 & 2-j1 \\ -4-j5 & -3 & 2-j1 & 1-j6 \\ -9 & -4+j5 & 1 & -4-j5 \\ -4+j5 & 1+j6 & 2+j1 & -3 \end{bmatrix}$$

$$H^*(k,l) = \begin{bmatrix} 2 & -2+j4 & -6 & -2-j4 \\ 4-j2 & 0 & -2-j4 & 2-j6 \\ 6 & 4+j2 & 2 & 4-j2 \\ 4+j2 & 2+j6 & -2+j4 & 0 \end{bmatrix}$$

$$Y(k,l) = X(k,l)H^*(k,l) = \begin{bmatrix} 2 & -8+j6 & -18 & -8-j6 \\ -26-j12 & 0 & -8-j6 & -34-j18 \\ -54 & -26+j12 & 2 & -26-j12 \\ -26+j12 & -34+j18 & -8+j6 & 0 \end{bmatrix}$$

$$r_{xh}(m,n) = \mathrm{IDFT}(Y(k,l)) = \begin{bmatrix} -17 & -9 & 0 & 0 \\ 9 & 2 & 0 & 9 \\ 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & -1 \end{bmatrix}$$

5.5.2

$$X(k) = \{8, 1+j3, -2, 1-j3\}, \quad H(k) = \{0, -j, 0, j\}$$

$$x_h(n) = \{1.5, 0.5, -1.5, -0.5\} \quad \mathrm{DFT}(x(n) + jx_h(n))\{8, 2+j6, -2, 0\}$$

Chapter 6

6.1.1

$$x_0(n) = x_4(n) = x_8(n) = \{1 + j1.7321, 1 + j1.7321, 1 + j1.7321, 1 + j1.7321\}$$

$$x_1(n) = x_5(n) = \{1 + j1.7321, -1.7321 + j1, -1 - j1.7321, 1.7321 - j1\}$$

$$x_2(n) = x_6(n) = \{1 + j1.7321, -1 - j1.7321, 1 + j1.7321, -1 - j1.7321\}$$

$$x_3(n) = x_7(n) = \{1 + j1.7321, 1.7321 - j1, -1 - j1.7321, -1.7321 + j1\}$$

6.2.2

$$x_0(n) = x_8(n) = \{-1.7321, -1.7321, -1.7321, -1.7321, \\ -1.7321, -1.7321, -1.7321, -1.7321\}$$

$$-2\cos\left(\frac{2\pi}{8}1n + \frac{\pi}{6}\right) = -2\cos\left(\frac{2\pi}{8}7n - \frac{\pi}{6}\right) = -2\cos\left(\frac{2\pi}{8}9n + \frac{\pi}{6}\right)$$

$$= \{-1.7321, -0.5176, 1.0000, 1.9319, 1.7321, 0.5176, -1.0000, -1.9319\}$$

$$-2\cos\left(\frac{2\pi}{8}2n + \frac{\pi}{6}\right) = -2\cos\left(\frac{2\pi}{8}6n - \frac{\pi}{6}\right) = -2\cos\left(\frac{2\pi}{8}10n + \frac{\pi}{6}\right)$$

$$= \{-1.7321, 1, 1.7321, -1, -1.7321, 1, 1.7321, -1\}$$

$$-2\cos\left(\frac{2\pi}{8}3n + \frac{\pi}{6}\right) = -2\cos\left(\frac{2\pi}{8}5n - \frac{\pi}{6}\right) = -2\cos\left(\frac{2\pi}{8}11n + \frac{\pi}{6}\right)$$

$$= \{-1.7321, 1.9319, -1, -0.5176, 1.7321, -1.9319, 1, 0.5176\}$$

$$x_4(n) = x_{12}(n) = \{-1.7321, 1.7321, -1.7321, 1.7321, -1.7321, 1.7321, -1.7321, 1.7321\}$$

6.3.3

$$w_r(n) = \{1, 1, 1, 1, 1, 0, 0, 0\}$$

$$x(n) = \{1, -0.7071, -0, 0.7071, -1, 0.7071, 0, -0.7071\}$$

$$X(k) = \{0, 0, 0, 4, 0, 4, 0, 0\}$$

$$x_t(n) = \{1, -0.7071, -0, 0.7071, -1, 0, 0, 0\}$$

$$X_t(k) = \{0, 1, j1.4142, 3, 0, 3, -j1.4142, 1\}$$

$$|X_t(k)| = \{0, 1, 1.4142, 3, 0, 3, 1.4142, 1\}$$

6.4.4

$$w_{han}(n) = \{0, 0.3455, 0.9045, 0.9045, 0.3455, 0, 0, 0\}$$

$$x(n) = \{0.5, 0.9659, 0.8660, 0.2588, -0.5, -0.9659, -0.8660, -0.2588\}$$

$$X(k) = \{0, 2 - j3.4641, 0, 0, 0, 0, 2 + j3.464\}$$

$$x_t(n) = \{0, 0.3337, 0.7833, 0.2341, -0.1727, 0, 0, 0\}$$

$$X_t(k) = \{1.1784, 0.2432 - j1.1848, -0.9561 - j0.0996, 0.1023 + j0.3818,$$

$$0.0428, 0.1023 - j0.3818, -0.9561 + j0.0996, 0.2432 + j1.1848\}$$

$$|X_t(k)| = \{1.1784, 1.2095, 0.9612, 0.3953, 0.0428, 0.3953, 0.9612, 1.2095\}$$

Chapter 7

7.1.3

$$x(t) = e^{j(\pi 0t)} + j2e^{j(2t+\frac{\pi}{6})} - j2e^{-j\left(2t+\frac{\pi}{6}\right)} + 4e^{j\left(3t-\frac{\pi}{3}\right)} + 4e^{-j\left(3t-\frac{\pi}{3}\right)}$$

The coefficients are

$$X_{fs}(0) = 1, X_{fs}(\pm 1) = 0, X_{fs}(\pm 2) = -1 \pm j\sqrt{3}, X_{fs}(\pm 3) = 2 \mp j2\sqrt{3}$$

The samples are

$$\{3.0000, -2.5300, 1.3822, 8.3258, -10.5999, 9.2096, -1.7876\}$$

$\omega_0 = 1$.
7.2.2 After canceling common factors, the frequency of the waveforms is $\frac{1}{2}$ and $\frac{2}{9}$. The LCM of the denominators (2,9) is 18. The GCD of the numerators (1,2) is 1. Therefore, the fundamental frequency is $\omega_0 = \frac{1}{18}$ rad/s. The fundamental period is $T = \frac{2\pi}{\omega_0} = \frac{2\pi 18}{1} = 36\pi$. The first sinusoid is the 9th harmonic. The second sinusoid is the 4th harmonic.

$$X_{fs}(0) = 1, X_{fs}(\pm 4) = \mp 0.5j, X_{fs}(\pm 9) = \pm j$$

7.3.1

$$\begin{aligned} X_{fs}(k) &= \frac{1}{T}\int_{-a}^{a} e^{-jk\omega_0 t}dt \\ &= \frac{1}{T}\frac{-1}{jk\omega_0}e^{-jk\omega_0 t}\,\Big|_{-a}^{a} = \frac{1}{T}\frac{-1}{jk\omega_0}(e^{-jk\omega_0 a} - e^{jk\omega_0 a}) \\ &= \frac{2}{T}\frac{1}{j2k\omega_0}(e^{jk\omega_0 a} - e^{-jk\omega_0 a}) = \frac{2}{T}\frac{\sin(k\omega_0 a)}{k\omega_0} \end{aligned}$$

With $\omega_0 = \pi/5, a = 2, T = 10$.

$$X_{fs}(k) = \frac{\sin(0.4k\pi)}{k\pi}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin(0.4k\pi)}{k\pi} e^{jk0.2\pi t}$$

7.4.1

$$x(t) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{k=1}^{\infty} \left(\frac{1}{1-4k^2} \right) \cos\left(2k\left(t - \frac{\pi}{2}\right)\right) =$$

$$\frac{2}{\pi} - \frac{4}{3\pi} \cos\left(2\left(t - \frac{\pi}{2}\right)\right) - \frac{4}{15\pi} \cos\left(4\left(t - \frac{\pi}{2}\right)\right) - \frac{4}{35\pi} \cos\left(6\left(t - \frac{\pi}{2}\right)\right) + \cdots$$

$$= \frac{2}{\pi} + \frac{4}{3\pi}\cos(2t) - \frac{4}{15\pi}\cos(4t) + \frac{4}{35\pi}\cos(6t) + \cdots$$

7.5.2 With $T_s = 1$, the samples are

$$\{3.3660, 3, 4.3660, 1.2679\}$$

With 4 samples, the DFT coefficients are

$$\{12, -1 - j1.7321, 3.4641, -1 + j1.7321\}$$

With $T_s = 0.8$, the samples are

$$\{3.3660, 3.2624, 3.7056, 3.6386, 1.0273\}$$

With 5 samples, the DFT coefficients are

$$\{15, -1.25 - j2.1651, 2.1651 - j1.25, 2.1651 + j1.25, -1.25 + j2.1651\}$$

Chapter 8

8.2

$$j2\,(\sin(2\omega)) = (e^{j2\omega} - e^{-j2\omega}) \leftrightarrow \delta(n+2) - \delta(n-2)$$

8.4

$$X(e^{j\omega}) = 3 + e^{-j\omega} + 2e^{-j2\omega} \quad \text{and} \quad H(e^{j\omega}) = 1 + 2e^{-j\omega} + 4e^{-j2\omega}$$

$$X(e^{j\omega})H(e^{j\omega}) = 3 + 7e^{-j\omega} + 16e^{-j2\omega} + 8e^{-j3\omega} + 8e^{-j4\omega}$$

$$\{3, 7, 16, 8, 8\}$$

8.6

$$H(e^{j\frac{2\pi}{8}}) = \frac{e^{j\frac{2\pi}{8}}}{e^{j\frac{2\pi}{8}} + 0.7} = 0.6350\angle(0.3197)$$

$$y(n) = 0.6350 \sin\left(\frac{2\pi}{8}n - \frac{\pi}{6} + 0.3197\right)$$

8.8

$$y(n) = \left(-3(-0.6)^n + 4(-0.8)^n\right) u(n)$$

8.10.1

$$X(e^{j\omega}) = 2e^{j2\omega} + e^{j\omega} + 3 + 4e^{-j\omega}$$

$$xp(n) = \{x(0) = 3, x(1) = 4, x(2) = 2, x(3) = 1\}$$

$$xp\_t(n) = \{x(0) = 3, x(1) = 4, x(2) = 2, x(3) = 0\}$$

The DFT of $xp\_t(n)$ is

$$\{9, 1 - j4, 1, 1 + j4\}$$

The DFT of $xp(n)$ is

$$\{10, 1 - j3, 0, 1 + j3\}$$

The DFT of the window $w(n) = \{1, 1, 1, 0\}$ is

$$\{3, -j1, 1, j1\}$$

The linear convolution of the two spectra, divided by 4, is

$$\{7.5, 0.75 - j4.75, 1.75 - j0.25, 1 + j4, 1.5, 0.25 + j0.75, -0.75 + j0.25\}$$

The circular convolution output can be obtained by adding the last three terms with the first three terms.

Chapter 9

9.2

$$\frac{2 + j\omega}{(2 + j\omega)^2 + 9}$$

$$X(j0) = 0.1538, \quad X(j\pi) = 0.2727 - j0.0912$$

9.5.1

$$X_{fs}(0) = 0.5 \quad \text{and} \quad X_{fs}(k) = \frac{j}{2k\pi}, \quad k \neq 0$$

9.6.2

$$\frac{1}{2\pi}\left(\pi\delta(t) + \frac{1}{jt}\right) \leftrightarrow u(-\omega)$$

9.7.3

$$X(j\omega) = -j\left(\frac{\sin((\omega-3)2)}{(\omega-3)} - \frac{\sin((\omega+3)2)}{(\omega+3)}\right)$$

9.8.2

$$\frac{1}{5+j\omega}\frac{1}{4+j\omega}$$

9.9.1

$$\frac{\pi}{2}(\delta(\omega-3) + \delta(\omega+3)) - \frac{j\omega}{(\omega^2-3^2)}$$

9.10.3

$$e^{-j2\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

9.11.3

$$X(j\omega) = -2\frac{e^{-j2\omega}}{j\omega} - \frac{1}{\omega^2} + \frac{e^{-j2\omega}}{\omega^2}$$

The derivative of $x(t)$ is

$$j\omega X(j\omega) = -2e^{-j2\omega} + \frac{1}{j\omega} - \frac{e^{-j2\omega}}{j\omega}$$

9.12.3

$$0.5\pi\delta(\omega) + \frac{0.5}{j\omega} - \frac{0.5}{2+j\omega}$$

9.14.1

$$\sin\left(\frac{2\pi}{8}t + \frac{\pi}{6}\right) = \cos\left(\frac{2\pi}{8}t - \frac{\pi}{3}\right) \leftrightarrow \pi\left(e^{-j\frac{\pi}{3}}\delta\left(\omega - \frac{2\pi}{8}\right) + e^{j\frac{\pi}{3}}\delta\left(\omega + \frac{2\pi}{8}\right)\right)$$

9.15.1

$$u(t+5) - u(t-5) \leftrightarrow 2\frac{\sin(5\omega)}{\omega}$$

$T_s = 1$.

$$x_s(t) = \sum_{n=-\infty}^{\infty} (u(n+5) - u(n-5))\delta(t-n) \leftrightarrow$$
$$X_s(j\omega) = \sum_{k=-\infty}^{\infty} 2\frac{\sin(5(\omega+2k\pi))}{\omega+2k\pi}$$

$T_s = 0.5$.

$$x_s(t) = \sum_{n=-\infty}^{\infty} (u(0.5n+5) - u(0.5n-5))\delta(t-0.5n) \leftrightarrow$$
$$X_s(j\omega) = 2\sum_{k=-\infty}^{\infty} 2\frac{\sin(5(\omega+4k\pi))}{\omega+4k\pi}$$

The other form for $X_s(j\omega)$, with $T_s = 1$, is

$$0.5e^{j5\omega} + e^{j4\omega} + e^{j3\omega} + e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + 0.5e^{-j5\omega}$$

For $T_s = 0.5$, 21 terms are required. With $T_s = 1$, the samples of $X(j\omega)$ and those of the 2 sampled versions are, respectively,

$$\{10, -1.8006, 1.2732, -0.6002, 0\}$$

$$\{10, -1.7125, 1.0151, -0.3090, 0\}$$

$$\{10, -1.7071, 1, -0.2929, 0\}$$

With $T_s = 0.5$, the samples of the 2 sampled versions are, respectively,

$$\{19.9999, -3.5575, 2.4218, -1.0663, 0\}$$

$$\{19.9999, -3.5575, 2.4218, -1.0663, 0\}$$

9.16.3

$$x_s(t) = \sum_{n=-\infty}^{\infty} 2\cos\left(3\frac{2\pi}{8}(0.5n) + \frac{\pi}{3}\right)\delta(t-0.5n) \leftrightarrow$$
$$X_s(j\omega) = \frac{2\pi}{(0.5)(16)} \sum_{m=-\infty}^{\infty} \left(16e^{j\frac{\pi}{3}}\delta\left(\omega - 3\frac{2\pi}{8} - 4m\pi\right) + 16e^{-j\frac{\pi}{3}}\delta\left(\omega + 3\frac{2\pi}{8} - 4m\pi\right)\right)$$

9.19

$$y(t) = (0.25e^{-t} + 0.75e^{-3t} - 3.5te^{-3t})u(t)$$

9.20.1

$$X(j\omega) = \frac{1}{1 + j\omega}$$

The sample values of the signal are

$$\{0.5000, 0.2865, 0.0821, 0.0235\}$$

The DFT of these values, after scaling by $T_s = 1.25\,\mathrm{s}$, is

$$\{1.1151, 0.5224 - j0.3287, 0.3401, 0.5224 + j0.3287\}$$

The corresponding 4 samples of the FT are

$$\{1, 0.3877 - j0.4872, 0.1367 - j0.3435, 0.0657 - j0.2478\}$$

9.21.1

$$X(j\omega) = \frac{1}{1.5 + j\omega}$$

The sample values of the spectrum are

$$\{0.6667, 0.0235 - j0.1228, 0.0060 - j0.0631, 0.0027 - j0.0423, 0.0027 + j0.0423, 0.0060 + j0.0631, 0.0235 + j0.1228\}$$

The IDFT of these values, after scaling by $1/T_s = 1/0.2\,\mathrm{s}$, is

$$\{0.5222, 0.7430, 0.5482, 0.5151, 0.3860, 0.3786, 0.2404\}$$

The first 7 samples of $x(t)$ are

$$\{1, 0.7408, 0.5488, 0.4066, 0.3012, 0.2231, 0.1653\}$$

Chapter 10

10.1.2

$$xe(n) = \{0, 1, 0, 1\}, xo(n) = \{0, 0, 0, 0\}$$

$$X(k) = \{2, 0, -2, 0\}, Xe(k) = \{2, 0, -2, 0\}, Xo(k) = \{0, 0, 0, 0\}$$

10.2.3

$$x(n) = \{1.3660 - j1.3660, -0.3660 - j0.3660, 1.3660 - j1.3660, -0.3660 - j0.3660,$$

$$1.3660 - j1.3660, -0.3660 - j0.3660, 1.3660 - j1.3660, -0.3660 - j0.3660\}$$

$$X(k) = \{4 - j6.9282, 0, 0, 0, 6.9282 - j4, 0, 0, 0\}$$

10.3.5

$$x(n) = \{0.7071, 0.7071, -0.7071, -0.7071, 0.7071, 0.7071, -0.7071, -0.7071\}$$

$$y(n) = \{0.5000, -0.9659, 0.8660, -0.2588, -0.5000, 0.9659, -0.8660, 0.2588\}$$

$$x(n) + jy(n) =$$

$$\{0.7071 + j0.5000, 0.7071 - j0.9659, -0.7071 + j0.8660, -0.7071 - j0.2588,$$

$$0.7071 - j0.5000, 0.7071 + j0.9659, -0.7071 - j0.8660, -0.7071 + j0.2588\}$$

$$X(k) + jY(k) = \{0, 0, 2.8284 - j2.8284, -3.4641 + j2, 0, 3.4641 + j2, 2.8284 + j2.8284, 0\}$$

$$X(k) = \{0, 0, 2.8284 - j2.8284, 0, 0, 0, 2.8284 + j2.8284, 0\}$$

$$Y(k) = \{0, 0, 0, 2 + j3.4641, 0, 2 - j3.4641, 0, 0\}$$

10.4.4

$$x(n) = \{0.7071, 0.0000, -0.7071, 1.0000, -0.7071, -0.0000, 0.7071, -1.0000\}$$

$$X(k) = \{0, 0, 0, 2.8284 - j2.8284, 0, 2.8284 + j2.8284, 0, 0\}$$

Index

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