Appendix A

A.1 Useful Formalae

Förster-type Nonradiative Energy Transfer (FRET) Rate:

$$\gamma_{trans} = \frac{2}{\hbar}\mathrm{Im}\left[\int dV\left(\frac{\varepsilon_A(\omega)}{4\pi}\right)\mathbf{E}_{in}(\mathbf{r})\cdot\mathbf{E}^*_{in}(\mathbf{r})\right] \tag{2.1}$$

FRET Rate in the Long Distance Approximation:

1. NP → NP

$$\gamma_{\alpha,trans} = \frac{2}{\hbar}b_\alpha\left(\frac{ed_{exc}}{\varepsilon_{eff_{NP_D}}}\right)^2\frac{R^3_{NP_A}}{d^6}\left|\frac{3\varepsilon_0}{\varepsilon_{NP_A}(\omega_{exc})+2\varepsilon_0}\right|^2\mathrm{Im}[\varepsilon_{NP_A}(\omega_{exc})] \tag{2.14}$$

2. NW → NP

$$\gamma_{\alpha,trans} = \frac{2}{\hbar}b_\alpha\left(\frac{ed_{exc}}{\varepsilon_{eff_{NW}}}\right)^2\frac{R^3_{NP_A}}{d^6}\cos^6(\theta_0)\left|\frac{3\varepsilon_0}{\varepsilon_{NP_A}(\omega_{exc})+2\varepsilon_0}\right|^2\mathrm{Im}[\varepsilon_{NP_A}(\omega_{exc})] \tag{2.15}$$

3. QW→NP

$$\gamma_{\alpha,trans} = \frac{2}{\hbar}b_\alpha\left(\frac{ed_{exc}}{\varepsilon_{eff_{QW}}}\right)^2\frac{R^3_{NP_A}}{d^6}\cos^6(\theta_0)\left|\frac{3\varepsilon_0}{\varepsilon_{NP_A}(\omega_{exc})+2\varepsilon_0}\right|^2\mathrm{Im}[\varepsilon_{NP_A}(\omega_{exc})] \tag{2.16}$$

4. NP → NW

$$\gamma_{\alpha,trans} = \frac{2}{\hbar}\left(\frac{ed_{exc}}{\varepsilon_{eff_{NP}}}\right)^2\left(\frac{3\pi}{32}\right)\frac{R^2_{NW_A}}{d^5}\left(a_\alpha + b_\alpha\left|\frac{2\varepsilon_0}{\varepsilon_{NW_A}(\omega_{exc})+\varepsilon_0}\right|^2\right)\mathrm{Im}[\varepsilon_{NW_A}(\omega_{exc})] \tag{2.28}$$

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5. NW → NW

$$\gamma_{\alpha,trans} = \frac{2}{\hbar}\left(\frac{ed_{exc}}{\varepsilon_{eff_{NW_D}}}\right)^2\left(\frac{3\pi}{32}\right)\frac{R_{NW_A}^2}{d^5}\left(a_\alpha + b_\alpha\left|\frac{2\varepsilon_0}{\varepsilon_{NW_A}(\omega_{exc})+\varepsilon_0}\right|^2\right)Im[\varepsilon_{NW_A}(\omega_{exc})] \tag{2.29}$$

6. QW → NW

$$\gamma_{\alpha,trans} = \frac{2}{\hbar}\left(\frac{ed_{exc}}{\varepsilon_{eff_{QW}}}\right)^2\left(\frac{3\pi}{32}\right)\frac{R_{NW_A}^2}{d^5}\cos^5(\theta_0)\left(a_\alpha + b_\alpha\left|\frac{2\varepsilon_0}{\varepsilon_{NW_A}(\omega_{exc})+\varepsilon_0}\right|^2\right)\mathrm{Im}[\varepsilon_{NW_A}(\omega_{exc})] \tag{2.30}$$

7. NP, NW, QW → QW

$$\gamma_{\alpha,trans} = \frac{2}{\hbar}b_\alpha\left(\frac{ed_{exc}}{\varepsilon_{eff_D}}\right)^2\frac{1}{d^4}\left|\frac{2\varepsilon_0}{\varepsilon_{QW_A}+\varepsilon_0}\right|^2\mathrm{Im}[\varepsilon_{QW_A}(\omega_{exc})] \tag{2.35}$$

8. NP, NW, QW → 1D NP Assembly

$$\gamma_\alpha = \frac{2}{\hbar}b_\alpha\left(\frac{ed_{exc}}{\varepsilon_{eff_D}}\right)^2\left(\frac{3\pi R_{NP_A}^3}{8}\right)\frac{\lambda_{NP}}{d^5}(c_D)^5\left|\frac{3\varepsilon_0}{\varepsilon_{NP_A}(\omega)+2\varepsilon_0}\right|^2\mathrm{Im}|\varepsilon_{NP_A}(\omega)| \tag{3.8}$$

$c_D = 1, \cos(\theta_0)$ for NP and QW, respectively, and $\left(1+\tan^2\theta_0\sin^2\alpha\right)^{-1/2}$ for a NW.

9. NP, NW, QW → 2D NP Assembly

$$\gamma_\alpha = \frac{2}{\hbar}b_\alpha\left(\frac{ed_{exc}}{\varepsilon_{eff_D}}\right)^2\left(\frac{\pi R_{NP_A}^3}{2}\right)\frac{\sigma_{NP}}{d^4}\left|\frac{3\varepsilon_0}{\varepsilon_{NP_A}(\omega)+2\varepsilon_0}\right|^2\mathrm{Im}|\varepsilon_{NP_A}(\omega)| \tag{3.12}$$

10. NP, NW, QW → 3D NP Assembly

$$\gamma_\alpha = \frac{2}{\hbar}b_\alpha\left(\frac{ed_{exc}}{\varepsilon_{eff_D}}\right)^2\left(\frac{\pi R_{NP_A}^3}{6}\right)\frac{\rho_{NP}}{d^3}\left|\frac{3\varepsilon_0}{\varepsilon_{NP_A}(\omega)+2\varepsilon_0}\right|^2\mathrm{Im}|\varepsilon_{NP_A}(\omega)| \tag{3.16}$$

11. NP, NW, QW → 1D NW Assembly

$$\gamma_\alpha = \frac{2}{\hbar}\left(\frac{ed_{exc}}{\varepsilon_{eff_D}}\right)^2\left(\frac{\pi R_{NW_A}^2}{8}\right)\left(\frac{\lambda_{NW}}{d^4}\right)\left(a_\alpha + b_\alpha\left|\frac{2\varepsilon_0}{\varepsilon_{NW_A}(\omega)+\varepsilon_0}\right|^2\right)\mathrm{Im}|\varepsilon_{NW_A}(\omega)| \tag{3.20}$$

12. NP, NW, QW → 2D NW Assembly

$$\gamma_\alpha = \frac{2}{\hbar}\left(\frac{ed_{exc}}{\varepsilon_{eff_D}}\right)^2\left(\frac{\pi R_{NW_A}^2}{24}\right)\left(\frac{\sigma_{NW}}{d^3}\right)\left(a_\alpha + b_\alpha\left|\frac{2\varepsilon_0}{\varepsilon_{NW_A}(\omega)+\varepsilon_0}\right|^2\right)\mathrm{Im}|\varepsilon_{NW_A}(\omega)| \tag{3.24}$$

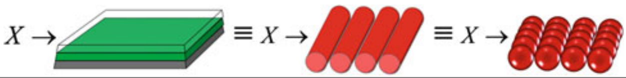
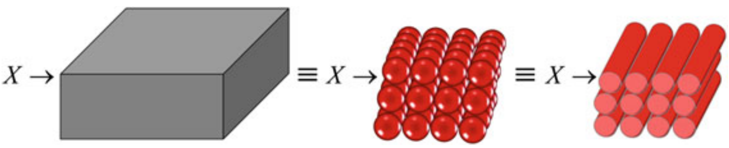
A.2 Effective Dielectric Constant

See Tables A.1 and A.2.

Table A.1 Effective dielectric constant expressions for NP, NW, and QW cases in the long distance approximation [Reprinted (adapted) with permission from Ref. [1] (Copyright 2013 American Chemical Society)]

α-direction	NP	NW	QW
x	$\varepsilon_{eff_D} = \frac{\varepsilon_{NP_D}+2\varepsilon_0}{3}$	$\varepsilon_{eff_D} = \frac{\varepsilon_{NW}+\varepsilon_0}{2}$	$\varepsilon_{eff_D} = \varepsilon_0$
y	$\varepsilon_{eff_D} = \frac{\varepsilon_{NP_D}+2\varepsilon_0}{3}$	$\varepsilon_{eff_D} = \varepsilon_0$	$\varepsilon_{eff_D} = \varepsilon_0$
z	$\varepsilon_{eff_D} = \frac{\varepsilon_{NP_D}+2\varepsilon_0}{3}$	$\varepsilon_{eff_D} = \frac{\varepsilon_{NW}+\varepsilon_0}{2}$	$\varepsilon_{eff_D} = \varepsilon_0$

Table A.2 Generic distance dependency for the FRET rates, with equivalent cases of arrayed nanostructures in term of d dependence [Reprinted (adapted) with permission from Ref. [2] (Copyright 2014 American Chemical Society)]

Generic distance dependence	FRET Donor (D) → Acceptor (A)
$\gamma \propto \frac{1}{d^6}$	$X \rightarrow$ 
$\gamma \propto \frac{1}{d^5}$	
$\gamma \propto \frac{1}{d^4}$	
$\gamma \propto \frac{1}{d^3}$	
$X =$ 	d: separation distance between D and A ≡ : equivalent

References

1. P.L. Hernández-Martínez, A.O. Govorov, H.V. Demir, Generalized theory of Förster-type nonradiative energy transfer in nanostructures with mixed dimensionality. J. Phys. Chem. C **117**, 10203–10212 (2013)
2. P.L. Hernández-Martínez, A.O. Govorov, H.V. Demir, Förster-type nonradiative energy transfer for assemblies of arrayed nanostructures: confinement dimension vs. stacking dimension. J. Phys. Chem. C **118**(9), 4951–4958 (2014)