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Notation Index

$\mathbf{R}$	set of real numbers
$\mathcal{N}$	set of nonnegative real numbers
ϕ	null or empty set
$\mathbf{x} = (x_i)$	the vector whose i^{th} component is x_i
$\mathbf{1}$	the sum vector containing all 1's
$\mathbf{0}$	the null vector/matrix containing all 0's
$\mathbf{I}_n$	identity matrix of order n
$\mathbf{e}_i$	i^{th} unit column vector
$\mathbf{x}'$	transpose of a vector
$\mathbf{x} \cdot \mathbf{y}$	scalar product of vectors $\mathbf{x}$, $\mathbf{y}$
$\Vert \cdot \Vert$	Euclidean norm
$\mathbf{x}/\Vert \mathbf{x} \Vert$	direction of vector $\mathbf{x}$
$\Vert \mathbf{x} - \mathbf{y} \Vert$	distance between points $\mathbf{x}$, $\mathbf{y}$
$\mathbf{A}$	an $(m \times n)$ matrix $\mathbf{A}$
$\rho(\mathbf{A})$	rank of a matrix $\mathbf{A}$
$\vert \mathbf{A} \vert$	determinant of an n^{th} order matrix $\mathbf{A}$
$x \in \mathcal{S}$	x is an element of a set $\mathcal{S}$
$x \notin \mathcal{S}$	x is not an element of a set $\mathcal{S}$
$\mathcal{A} \subseteq \mathcal{B}$	set $\mathcal{A}$ is a subset of set $\mathcal{B}$ (possibly $\mathcal{A} = \mathcal{B}$)
$\mathcal{A} \subset \mathcal{B}$	set $\mathcal{A}$ is a proper subset of set $\mathcal{B}$
$\mathcal{V}_n(\mathcal{F})$	n-dimensional vector space over field $\mathcal{F}$
$\mathcal{V}_n(\mathbf{R}) = \mathbf{R}^n$	n-space or the set of all real n-component vectors
$dim(\mathcal{V})$	dimension of a vector space $\mathcal{V}$
$\mathcal{M}$	subspace or linear manifold
$\ell(\mathcal{M})$	linear hull of subspace $\mathcal{M}$
$\mathcal{M}_1 \oplus \mathcal{M}_2$	direct sum of subspaces $\mathcal{M}_1, \mathcal{M}_2$
$\mathcal{M}^{\perp}$	orthogonal complement or dual space of $\mathcal{M}$

$d(\mathcal{S})$	diameter of a set $\mathcal{S}$
$d(\mathbf{x},\mathcal{S})$	distance between vector $\mathbf{x}$ and set $\mathcal{S}$
$d(\mathcal{S},\mathcal{T})$	distance between sets $\mathcal{S},\mathcal{T}$
$B(\mathbf{x}_0,\delta)$	δ-neighborhood of a point $\mathbf{x}_0 \in \mathbf{R}^n$ (or an open ball or sphere of radius δ about $\mathbf{x}_0$)
$\mathcal{C}$	cone
$\mathcal{C}^+$	polar cone
$\mathcal{C}^*$	dual cone
$(\mathbf{a})$	ray or half-line
$(\mathbf{a})^\perp$	orthogonal cone to $(\mathbf{a})$
$(\mathbf{a})^+$	polar of $(\mathbf{a})$
$(\mathbf{a})^*$	dual of $(\mathbf{a})$
$\mathcal{S}^o$	interior of set $\mathcal{S}$
$\mathcal{S}^c$	complementary set of $\mathcal{S}$
$\overline{\mathcal{S}}$	closure of set $\mathcal{S}$
$\partial(\mathcal{S})$	boundary of set $\mathcal{S}$
$coni(\mathcal{S})$	conical hull of set $\mathcal{S}$
$0^+(\mathcal{S})$	recession cone of set $\mathcal{S}$
$aff(\mathcal{S})$	affine hull of set $\mathcal{S}$
$co(\mathcal{S})$	convex hull of set $\mathcal{S}$
$ri(\mathcal{S})$	relative interior of convex set $\mathcal{S}$
$r\partial(\mathcal{S})$	relative boundary of set $\mathcal{S}$
$\mathcal{S}_1+\mathcal{S}_2$	linear sum of convex sets $\mathcal{S}_1,\mathcal{S}_2$
$cone(\mathcal{S})$	convex cone generated by $\mathcal{S}$
$ray(\mathcal{S})$	union of the origin and the set of recession half-lines
sup	supremum or least upper bound
inf	infimum or greatest lower bound
$\mathcal{A}\times\mathcal{B}$	Cartesian product of sets $\mathcal{A},\mathcal{B}$
$[\mathbf{x}_1,\mathbf{x}_2]$	closed line segment joining points $\mathbf{x}_1,\mathbf{x}_2$

$\{\mathbf{x}_k\}$	sequence of points
$\{\mathbf{x}_{k_j}\}, j=1,2,$	subsequence of points
$\{\mathbf{x}_k\}_{h\in\mathcal{K}}$	subsequence of points, where $\mathcal{K}$ is a subset of positive integers
$\mathcal{H}$	hyperplane
$[\mathcal{H}^+], [\mathcal{H}^-]$	closed half-planes
$(\mathcal{H}^+), (\mathcal{H}^-)$	open half-planes
σ^k	k-dimensional simplex
$a \equiv b \,(\mathrm{mod}\, m)$	$a-b$ is divisible by m
$f: \mathcal{S}_1 \to \mathcal{S}_2$	point-to-point mapping from $\mathcal{S}_1$ to $\mathcal{S}_2$
$f \circ g$	composition of f with g
$\mathcal{D}_f$	domain of f
$\mathcal{R}_f$	range of f
$\mathcal{G}_f$	graph of f
f^{-1}	single-valued inverse mapping
$2^{\mathcal{Y}}$	power set of $\mathcal{Y}$
$F: \mathcal{S}_1 \to \mathcal{S}_2$	point-to-set mapping from $\mathcal{S}_1$ to $\mathcal{S}_2$
F^{-1}	inverse point-to-set mapping
$F(\mathbf{x})$	image set of $\mathbf{x}$
$F^{+1}(\mathcal{B})$	upper inverse image of $\mathcal{B}$ under F
$F^{-1}(\mathcal{B})$	lower inverse image of $\mathcal{B}$ under F
$G \circ F$	composition correspondence
$F \cap G$	intersection correspondence
$F \times G$	product correspondence
$F + G$	sum correspondence

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