Appendix 1: Descartes, Mydorge and Beeckman: The Evolution of Cartesian Lens Theory 1627–1637

A.1 Introduction

In Sect. 4.5.2 it was foreshadowed that the analysis of the trajectory of Descartes' lens theory can provide crucial supporting evidence for my reconstruction of Descartes' discovery of the law and his attempts at its physico-mathematical rationalization. In particular, it can do this by allowing us quite firmly to date the material in the Mydorge letter to the period 1626/1627 shortly after the law of refraction was discovered, in cosecant form, by Descartes and Mydorge.

In this Appendix we are going to canvass the following points: [1] In constructing his lens theory Mydorge begins with the cosecant form of the law and only finds a sine formulation in the course of elaborating the theory. [2] His synthetic proofs of the anaclastic properties of plano-hyperbolic and sphero-elliptical lenses are similar to, but clearly pre-date those offered by Descartes later in the *Dioptrique* of 1637. Moreover, [3] Descartes' own synthetic lens theory demonstrations in the *Dioptrique* differ from those of Mydorge in another historically revealing way, the matter turning on a technical and aesthetic issue which Descartes seems to have learned from Beeckman in October 1628. In other words Descartes' lens theory developed during three moments between 1626/1627 and the publication of the *Dioptrique*: First, we have the earliest lens theory of Descartes and Mydorge in the Mydorge letter, whose content dates from 1626/1627; second, we shall see some consequential shifts and articulations in Descartes' theory as a result of consultations and negotiations with Isaac Beeckman in 1628; and, finally, we have the synthetic lens theory of the *Dioptrique* of 1637.

All these facts will therefore suggest that the Mydorge letter contains Mydorge and Descartes' *earliest lens theory*, and arguably *their first form of the law*, the cosecant form. The *material in the letter,* if not the artifact itself, pre-dates October 1628, certainly predates composition of the *Dioptrique* and very plausibly is as early as 1626/1627. So, this dating points to the cosecant form of the law as the first form Mydorge and Descartes possessed. This, as we have seen, is the key to reconstructing how they obtained it, because the other independent discovers first obtained

J. Schuster, *Descartes-Agonistes: Physico-mathematics, Method & Corpuscular-Mechanism 1618-33*, Studies in History and Philosophy of Science 27,
DOI 10.1007/978-94-007-4746-3, © Springer Science+Business Media Dordrecht 2013

it in the same *unequal radius form*.[1] We start, therefore, by returning to the Mydorge letter, intending to analyze all those parts of it not examined in our earlier discussion in Chap. 4. Having already looked only at Mydorge's Proposition 1, his statement and geometrical illustration of the cosecant law of refraction, we begin with his second proposition.

A.2 Mydorge's Refractive Index Instrument: Cosecants Not Sines

In Proposition 2 of his letter, Mydorge explains a device used to determine the refractive index of a given medium, in this case the glass Descartes and Mydorge apparently intended to use in the fabrication of lenses (Fig. A.1.1). Mydorge sends a ray, FG, through the triangular prism of glass ABC. The ray enters the prism normal to AB and is refracted at AC to E. DIH is the normal to AC at I.[2] The geometry of the device is elegant. The angle of incidence FID is equal to the angle BAC and hence is known in advance. The angle of refraction HIE is equal to the sum of angles FID and IEC.

Only one measurement, that of angle IEC need be made in order to determine the refractive index (RI) for

$$\frac{\sin \mathrm{FID}}{\sin(\mathrm{FID}+\mathrm{IEC})}=\mathrm{RI}$$

Curiously, however, Mydorge does not exploit the device in this manner, by taking the sines of the angles of incidence and refraction. Instead, he relies upon the radius form of the law taught in Proposition 1 (Fig. A.1.2). Around I he draws the arc of the circle of radius FI. He constructs FK parallel to AC cutting the arc FK at K; then from K he drops a line parallel to DIH cutting the refracted ray IE at L. The ratio IL:FI is the index sought.[3]

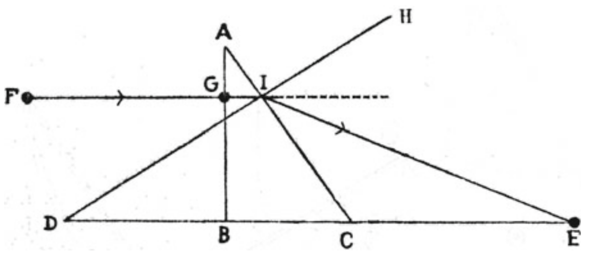

Fig. A.1.1 Simplified version of Mydorge, illustration to Proposition II, Mersenne (1938–1988) I, p.406

[1] Lohne (1959), (1963), Vollgraff (1913), (1936), deWaard (1935–36); Buchdahl (1972).

[2] Mersenne (1932–88) I. p.405

[3] *Ibid*. pp. 406–7. That is, the constant ratio IL:FI and the construction technique used in Proposition 1 will yield the paths of all other refracted rays. Cf. Above Sect. 4.5.1 and Fig. 4.5.

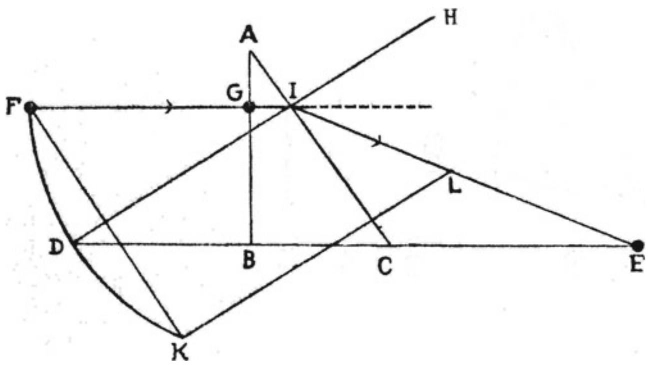

Fig. A.1.2 Mydorge, illustration to Proposition II, Mersenne (1938–1988) I, p.406

Now, had Mydorge discovered the law in what we have called sine form, he could have greatly simplified the whole discussion and diagram. A likely interpretation is that the radius form of the law was prior to the sine form. In seeking to exploit the law in an experimental situation, Mydorge apparently reached for the only form of it with which he was acquainted, the radius form. There is further evidence for this interpretation in the remainder of the letter.

A.3 Mydorge's Synthetic Propositions 3 and 4 on Anaclastic Surfaces: An 'Antique' Version of the Sine Law

Propositions 3 and 4 of Mydorge's letter are devoted to lens theory proper.[4] Applying the law of refraction to an hyperbola (prop. 3) and to an ellipse (prop. 4), Mydorge shows that if an incident ray, parallel to the transverse axis (t.a) of either of these conics, is refracted at its point of incidence with the section to the appropriate focus, then,

$$\frac{\sin i}{\sin r} = \frac{\text{t.a.}}{\text{f.d}}$$

where f.d. denotes the focal distance of either of these conic sections.

The case of the hyperbola is illustrated in Fig. A.1.3. CBA is the left branch of the hyperbola, D and E its foci, BF the transverse axis, CH the tangent to the section at C, and CI the normal to the section at C. The incident ray GC is refracted at C to the distant focus D. Mydorge uses the sine form of the law of refraction, representing the sine of the angle of incidence GCI by GL and the sine of the angle of refraction ICK by KM. We should note especially here for later reference the odd, or as

[4] *Ibid*. pp. 408–9

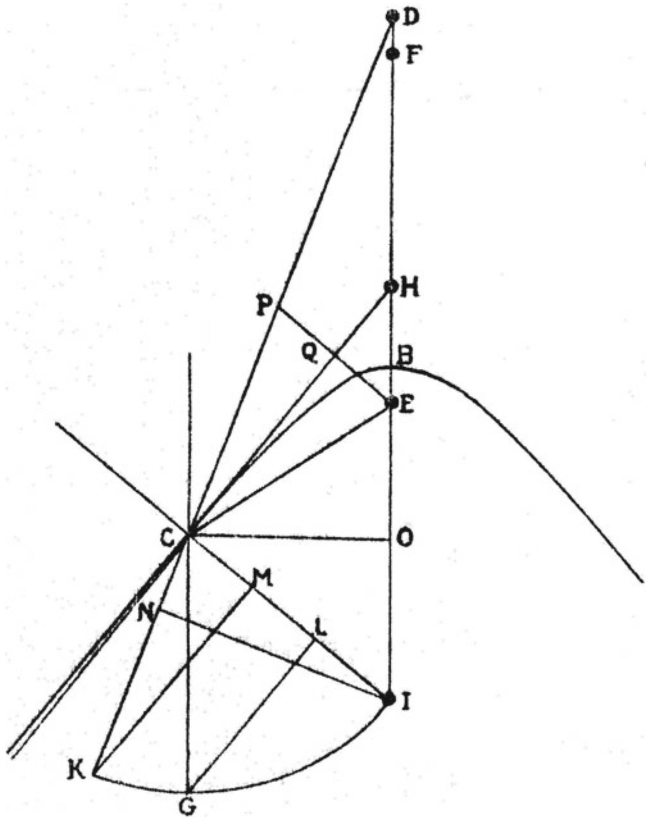

Fig. A.1.3 Mydorge, Proposition III, hyperbola as anaclastic curve, Mersenne (1938–1988) I, p.408

I shall call it 'antique', representation of the sine law, with the sines of incidence and refraction inscribed on the same side of the refracting surface.

Mydorge easily demonstrates that (GL:KM)=(BF:DE), the proportion sought between the ratio of the sine of the angle of incidence to the sine of the angle of refraction, and the ratio of the transverse axis to focal distance of the hyperbola.[5] Since ray GC and point C were selected at random, the demonstration applies to any such ray parallel to the transverse axis and refracted by the section to the distant focus. The relevance to lens theory is clear, although it is not spelled out by Mydorge in these propositions: If an hyperbola, defined by the ratio of transverse axis to focal distance of BF:DE, were embodied in a convex plano-hyperbolic lens made of a

[5] The proof proceeds easily and in routine fashion based on well known properties of the conics, chiefly by means of deduction through a sequence of equal and similar triangles inscribed in the figure. The proof tactics in these routine *concluding stages* are identical to those Mydorge uses in his Proposition 5 discussed below, and in Descartes' corresponding proof for the plano-hyperbolic lens in the *Dioptrique.* Here we are concentrating on *opening stages* of these proofs, where various representations of the law of refraction are adduced and further manipulated.

transparent material the index of refraction from which into air were equal to BF:DE, that lens would focus all rays incident parallel to its transverse axis to its distant focus—it would embody an anaclastic surface.

Mydorge's use of a sine form of the law of refraction, albeit in the unusual 'antique' form, might seem to undermine the claim that he and Descartes first discovered the law in radius form. This, however, is not the case, as we can see by placing Propositions 3 and 4 in the two relevant contexts which facilitate their accurate interpretation. The first of these contexts is the subsequent history of Propositions 3 and 4 down to their publication by Descartes in the *Dioptrique* of 1637. We shall see in the next Section that Mydorge's proofs sit at the very beginning of this history, during which the 'antique' version of the sine law was transformed into our familiar, let us say 'natural' form, in which the sines of incidence and refraction are assigned to their respective sides of the refracting interface. All this will strongly reinforce our earlier conjecture that the material in the Mydorge letter dates from 1626/1627, the very period of the initial discovery of the law of refraction. With that conclusion in hand, we will then turn in Sect. A.5 to the second context of Propositions 3 and 4, which is the surrounding text of Mydorge's letter itself; that is, Propositions 1 and 2, which we have discussed, and Proposition 5, his final proposition, which we will have to examine with great care. Proposition 5 shows how Mydorge connected the putatively original, radius or cosecant form of the law to a sine form of the law, but only in its first or 'antique' version. Additionally, because this material is quite early, we will be able to detect in Proposition 5 echoes of Mydorge's (and Descartes') earliest analysis of the anaclastic problem, the very beginning of their research on lens theory with a law of refraction in hand. *We will conclude in Sect. A.6 that the 'antique' sine form was evolved out of the radius form of the law during the course of this analysis.* In other words, we shall see that the sine law in its initial, 'antique' form was discovered during the course of an analysis of the anaclastic problem initially launched on the basis of the newly discovered radius form of the law. The 'antique' sine form, after having been uncovered in this way, was then deployed in the more synthetic Propositions 3 and 4, with Descartes' more 'natural' representation of the law—only unveiled in the *Dioptrique*—nowhere in sight (until Beeckman suggested it in October 1628).

A.4 Relating Mydorge's Propositions 3 and 4 to Descartes' Analogues in the *Dioptrique*: From 'Antique' to 'Natural' Representation of the Sines, Thanks to Isaac Beeckman in October 1628

First let us consider the place of Mydorge's propositions 3 and 4 in the development of Cartesian lens theory between 1627 and 1637. In the *Dioptrique* Descartes proves propositions identical to those of Mydorge, but they differ in one historically revealing way. Instead of setting up the sines of the angles of incidence and refraction by reference to a semi-circle on one side of the interface, as Mydorge had done, Descartes

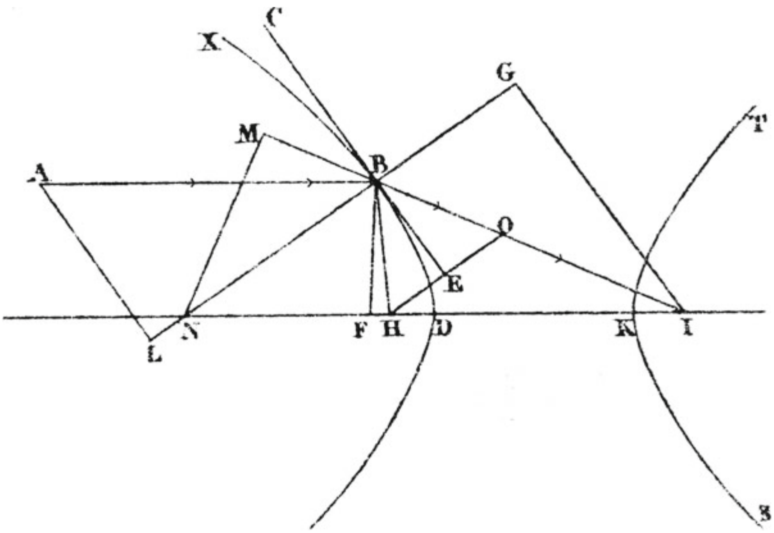

Fig. A.1.4 Descartes' hyperbola proof in the *Dioptrique,* AT VI, p.179

directly relates the sines to their respective rays. Consider Figs. A.1.3 and A.1.4 where this point is illustrated using Mydorge's and Descartes' figures for the case of the hyperbola. (Similar considerations would apply in the cases of their figures for the ellipse).

In Descartes' diagram (Fig. A.1.4) H and I are the foci of the hyperbola, D and K its vertices; ray AB is refracted at B to I; CBE is tangent to the left branch at B; LNBG is normal to CBE at B; BA is taken equal to BI so that AL and GI represent respectively the sine of the angle of incidence and the sine of the angle of refraction in the 'natural' representation of the law, familiar since that time. The proof that the ratio of the sines of the angles of incidence and refraction [AL:GI] equals the ratio of the transverse axis to the focal distance [DK:HI] again follows easily on routine knowledge of the conics, utilizing a sequence of relations amongst equal and similar triangles in the figure.[6] In Mydorge's diagram (Fig. A.1.3), we recall GL and KM are the sines of the angle of incidence and the angle of refraction respectively, given in 'antique' form as we have already seen.

Now, we can actually pin down the likely source of Descartes' later 'natural' representation. Descartes' friend Isaac Beeckman seems to have been the author of Descartes' mature representation of the sines in the context of lens theory. In 1628 Descartes asked Beeckman to provide a proof of the refractive properties Descartes had claimed for the hyperbola. Beeckman's proof omits several steps and does not fully specify the construction. But geometrically it is identical to Fig. A.1.4 and was 'approved' by Descartes.[7] At the same time, in 1628, Descartes showed to Beeckman

[6] *Dioptrique*, AT VI p. 178.

[7] AT x. 341-2; Beeckman (1939–53) fol. 338r.

an elegant proof for the case of the ellipse.[8] However, he did not subsequently use that proof in the *Dioptrique*, probably because the lines representing the sines of incidence and refraction are not related to their respective rays in the intuitively obvious way displayed in Fig. A.1.4. One can conclude that Descartes elected to use Beeckman's more 'natural' representation of the sines in both cases, ellipse and hyperbola, in the synthetic proofs in the *Dioptrique*, thus superseding his own elegant ellipse proof and Mydorge's early 'one sided' representation of the sines in Propositions 3 and 4. This episode with Beeckman, which we may imagine to have been in the nature of a negotiation (and set of mutual challenges, as befitted their previous interactions in 1618–1619) marks the second moment in the evolution of Descartes' lens theory (see Chap. 3).

To sum up so far: The development of the lens theory proofs places Mydorge's Propositions 3 and 4 very early in his and Descartes' researches. In terms of proof content and diagrammatic representation, Propositions 3 and 4 are the earliest proofs in their lens theory of which we have any record; and they are clearly the starting point for Beeckman's and Descartes' later improvements. Mydorge's demonstrations obviously pre-date Descartes' and Beeckman's discussions of lens theory in 1628, and hence they arguably date from the very period of the discovery of the law of refraction. This, accordingly, aids in our dating of all of the material in the Mydorge letter from 1626/1627. The dating becomes even more likely when one considers that by 1632 the Cartesian sine form of the law was well known to several of Descartes' associates, including Golius and Mersenne, in addition to Beeckman. In informing Golius about his optical work Descartes mentioned only the sine from of the law.[9] But in his letter Mydorge, Descartes' closest associate, does not initially use the sine form, and when he does introduce it, in his lens theory, he produces an early 'one-sided' version soon superseded in Beeckman's and Descartes' proofs. It is therefore most unlikely that the material in the letter was initially composed in 1631 or later, the possibility left open by De Waard when he tried to date the letter. All the evidence points toward the conclusion that the material in the Mydorge letter was an early and rather unsystematic and undigested report on his and Descartes' researches of 1626/1627.

[8] *Ibid*.

[9] Descartes to Golius, 2 February 1632, AT I. p.239ff. When Descartes met Beeckman in October 1628 he offered him a striking and very important mechanical analogy for the law-like refraction of light, appealing to a bent arm balance supporting identical weights, whose arms are immersed in media (upper and lower) of differing specific gravities. Section 4.7.4 above and Schuster (2000), 290–295, show how this analogy directly bespeaks Descartes' dynamical thinking about the absolute force of light and its determinations, before and after refraction. The bent arm balance, however, is presented to Beeckman using representations of the sines of the incidence and refraction of the arms, on their respective sides of the interface—the 'natural' representation of the sine law we are talking about. The issue is that whilst Descartes had by 1628 worked out this model for his dynamics of light, the representation of the sine law embodied in it was not applied back into lens theory until Beeckman suggested it. Presumably, Descartes still had to hand proofs resembling those of Mydorge from 1626/1627.

A.5 Decoding Mydorge's Proposition 5: The Cosecant Form Leads to 'Discovery' of the 'Antique'Sine Form Then Used Synthetically in Propositions 3 and 4

With these conclusions about the early date of the Mydorge letter in mind, we can now proceed to the second context of Propositions 3 and 4, the surrounding text of the letter, and in particular Proposition 5. What we are after is an explanation of Mydorge's use of the sine form of the law in Proposition 3 and 4, an explanation grounded in an understanding of the surrounding portions of Mydorge's text and framed by our now strong conviction that the material in the letter is indeed of very early provenance, dating back to the period of the discovery of the law of refraction.

Proposition 5 deals with the specification of hyperbolic and elliptical anaclastic curves in actual empirical cases. It amounts to a linking of Proposition 2 with Propositions 3 and 4. Mydorge starts by showing how to measure the index of refraction for rays passing into the air from the glass out of which the lenses are to be fashioned. Exactly as in Proposition 2, the index is determined by passing one ray through a triangular glass prism, and the index is expressed as a ratio of radii (not as a ratio of sines) by applying the radius form of the law of refraction to the given ray. That is, in Fig. A.1.5, which shows the first few steps in Mydorge's fifth proposition, the glass:air index is given as IL:FI.

Next, the empirically determined index is used to set the ratio of transverse axis: focal distance for the hyperbolas and ellipses in question. This construction is effected by exactly repeating a construction Mydorge had already given as a Corollary to Proposition 2.[10] Taking the case of the hyperbola only, Fig. A.1.6 shows how this construction is added to the material previously assembled in Fig. A.1.5.

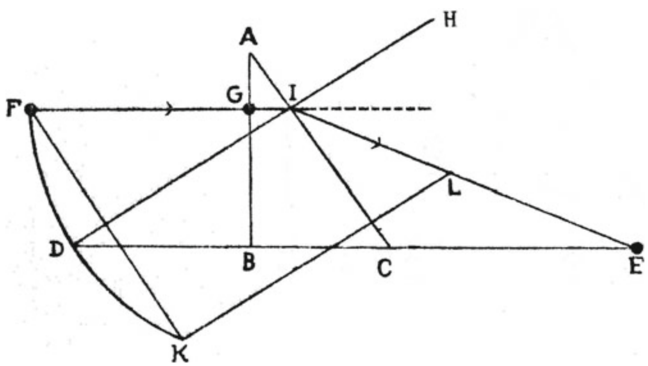

Fig. A.1.5 Mydorge letter, Mersenne (1938–1988) I, p.412, illustration of Proposition V, modified to show initial steps in Mydorge's demonstration

[10] Mersenne (1932–88) I. 406–7; In the *Dioptrique* (AT VI pp. 212–3) Descartes recapitulates the material in Mydorge's Proposition 2: He presents the same refraction device and then shows how to interpolate the transverse axis and foci of the anaclastic hyperbola into its geometry.

AC, representing the refracting surface of the prism, is considered to be tangent to the left branch of the hyperbola at I, the point of incidence. IE, the refracted ray, is taken to have passed through the distant focus at E. The near focus is then located by using the property of the hyperbola that a tangent to a point in the section bisects the angle between the lines drawn from that point to the two foci.[11] Thus AC bisects angle EIM and M is the left focus. IN (=IM) is marked off along IE. Then, according to the basic property of the hyperbola, the difference between IE and IN gives the length of the transverse axis, PQ, which can easily be inserted between the two foci. Finally, Mydorge must show that as IL:FI so PQ (or NE):ME. With the exception of its first step this proof is identical to that given in Proposition 3. The only difference is that Proposition 3 begins with a sine form of the law, whilst here we must start the proof with the radius form of the law, the product of Mydorge's introduction of the index measuring technique of Proposition 2.

The crucial step linking the radius form, IL:FI, to the sine form consists in a simple construction, which we add to Fig. A.1.6 in Fig. A.1.7.

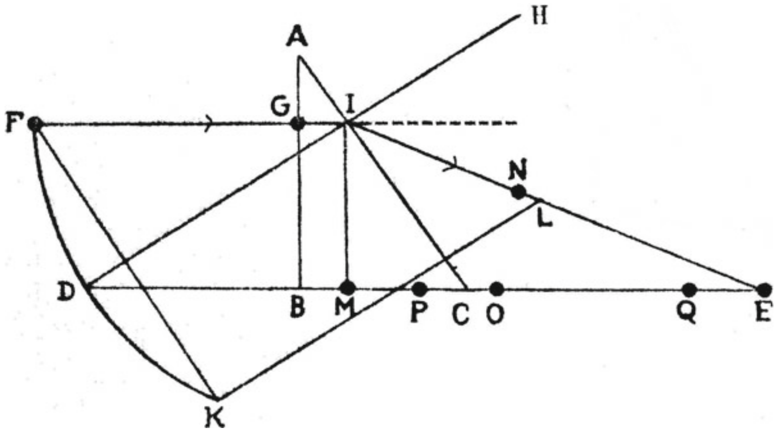

Fig. A.1.6 Mydorge letter, Mersenne (1938–1988) I, p.412, illustration of Proposition V, modified to show intermediate steps in Mydorge's demonstration

[11] Mydorge refers the reader to the relevant proposition in his own work on *Conics* first published in 1631; but he may of course be referring to draft material prior to that date. We can insist on dating the content of the Mydorge letter from 1631 or later, based on the publication history of his *Conics* and his use of routine material from it; or we can look closely at the absolutely novel aspects of the letter—the adducing of the cosecant form of the law and the working through from it to the 'antique' version of the sine law— placing all of that material in the context of whatever else we can reconstruct about Descartes' work on explaining the law of refraction and on lens theory. To reiterate, the latter considerations, argued here and in Schuster (2000), conduce to the hypothesis that the *material* in the letter dates from the very period of discovery of the law of refraction, 1626/1627, and not from the early 1630s when mature versions of all this material, well beyond the toing and froing of Mydorge's letter were known in the Descartes/Mersenne network, and Mydorge's presentation would have seen oddly out of date and out of touch.

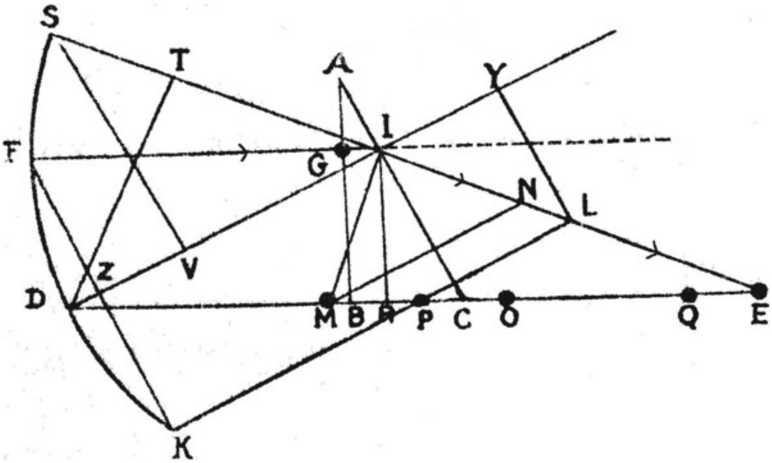

Fig. A.1.7 Mydorge letter, Mersenne (1938–1988) I, p.412, full illustration of Proposition V

Arc KDF is extended about point I, and ray IE is extended back to intersect the arc at S. SV and LY are dropped perpendicular to DI. Then FZ (=LY) and SV are the sines of the angles of incidence and refraction respectively, referred to the single reference circle KDFS. Triangle ISV is similar to triangle ILY, hence (SI:IL) = (SV:YL); but SI = FI and YL=KZ=FZ, hence (IL:FI) = (FZ:SV). After dropping DT normal to SIE and IR normal to ME, Mydorge can then continue the proof in the manner of Proposition 3 by making use of the equal triangles FIZ and IRD, and DTI and SVI; and the similar triangles DTE and IRE, and DIE and MNE.[12]

Proposition 5, as Mydorge presents it, marks the final step in his unfolding of his lens theory. First, using the radius form of the law only, he expounded the law of refraction (Proposition 1) and his index finding technique (Proposition 2). Then, using his 'antique' sine form of the law, he offered demonstrations linking the law of refraction to the ratio 'transverse axis:focal distance' of hyperbolas (Proposition 3) and ellipses (Proposition 4). In proposition 5 he turns to empirical cases of plano-hyperbolic and sphero-elliptical lenses, which means he must connect Proposition 2 to Propositions 3 and 4. This explains, in the context of the letter, Mydorge's construction of the 'antique' sine form of the law out of the radius form, and his redundant repetition of the proof structure of Propositions 3 and 4 in Proposition 5.

Of course, Mydorge's order of presentation of lens theory need not have corresponded to his and Descartes' order of research and discovery in this domain. If, as seems very likely, he initially had only the radius form of the law, he could not have pursued his synthetic lens theory through Propositions 3 and 4 without first having uncovered, by analysis, a way of linking the radius form to the fundamental properties of the conic sections. This way of linking was his construction of the 'antique'

[12] Cf above Note 5

sine form of the law out of the radius form. If one asks what Mydorge's (and Descartes') path of analysis might have looked like, a very plausible candidate springs to view—Proposition 5 itself. In the context of the letter, the bulk of the proof of Proposition 5 is redundant and repetitive; but, if Proposition 5 is read, as it were, backwards, as a remnant of an analysis, we obtain a story about Mydorge and Descartes' possible original analysis of the anaclastic problem which, given everything which has gone before, seems very plausible indeed.

A.6 Reconstruction of Descartes and Mydorge's First Analysis of the Anaclastic Problem, with the Cosecant Law of Refraction to Hand

Let us therefore try to reconstruct the analysis of which Proposition 5 seems to contain the remnants, and let us do this on the basis of the relevant facts we have already more or less established about Mydorge and Descartes' early optical work and intentions. First of all, as established in Chap. 4, we must imagine that Mydorge (and Descartes) obtained the radius form of the law by means of an image mapping technique similar to that used by Harriot. Next, we must hypothesize that with the radius form in hand, they moved to explore the possibility, hinted at by Kepler in his *Dioptrice* of 1611 (Proposition 59), that the hyperbola might be the anaclastic curve. Drawing upon their combined knowledge of the conic sections, they would have designed their elegant experimental device (if only on paper at first!) in such a way that they could easily interpolate an hyperbola whose defining property would be entailed by the geometry of the prism and the behavior of the empirically given ray, incident parallel to the intended transverse axis of the hyperbola. Then they would have had to attempt to prove the relation of the cosecant regularity to some expression of the defining property of the interpolated hyperbola, thus showing that the refraction to the distant focus holds for any parallel incident ray, and hence that the left branch of the hyperbola is an anaclastic surface. In the manner of classical geometry, and in accord with Descartes' explicit views on mathematical method, the analysis could then be reversed in so far as possible to guide the production of synthetic propositions such as Mydorge's Propositions 3 and 4.

With this background, the anaclastic problem would have taken the following form: Assume an incident ray parallel to the transverse axis of the hyperbola is refracted by the section to the distant focus. Can the law of refraction (reflecting the index of refraction) be related to the ratio 'transverse axis:focal distance' characterizing the hyperbola in question? So, let us imagine in Fig. A.1.8 what Mydorge and Descartes' analysis diagram might have looked like: Assume we are given hyperbola IPW, with foci M and E, and ray FI refracted at I to E. IC is tangent to the hyperbola at the point of incidence I. As accomplished geometers and experts on the conic sections, we also know that angle MIC=angle CIE; that IM=IN and NE=PQ = transverse axis. Next we construct the index of refraction in the only form we know, in radius form, as the ratio IL:FI.

Before proceeding any further, we need to make two observations. First, point D has not been located on the extension of ME, as in Proposition 5 (Figs. A.1.5, A.1.6 and A.1.7), because the exact location of D was probably discovered in the course of the analysis. Note also that Fig. A.1.8 has been constructed solely on the basis of (1) knowledge of the radius form of the law, (2) the suspicion that the hyperbola is the anaclastic curve, and (3) elementary knowledge of the properties of hyperbolas, which was second nature to Mydorge and Descartes.

Given Fig. A.1.8 the analytical problem is to show that (IL:FI) = (PQ:ME). Mydorge's synthetic demonstration of this relation in Proposition 5 relied on the establishment of the sine form of the law (FZ:SV) and on the relating of (FZ:SV) to (PQ:ME) via the set of equal and similar triangles pointed out earlier in Fig. A.1.7. Clearly, the successful analysis of the problem set in Fig. A.1.8, by Mydorge, Descartes or any one else, demands two crucial steps: One has to discover that point D must be located on the extension of EM, otherwise the series of interrelated equal or similar triangles does not materialize; and, one must also transform IL:FI into some ratio of lines relatable to the limbs of those triangles. *This is the very trick Mydorge accomplishes by constructing the 'one-sided' sine form of the law, FZ:SV in* Fig. A.1.7. Precisely how Mydorge and Descartes made those moves and in what order, we cannot know. That they performed an analysis of this general type is highly likely, given the close relation between Mydorge's Fig. A.1.7 and our Fig. A.1.8, constructed in light of what we can determine about the direction, background and tools of their early researches. This sort of analysis invites the construction of Mydorge's peculiar 'one-sided' version of the sine form, and such a route to

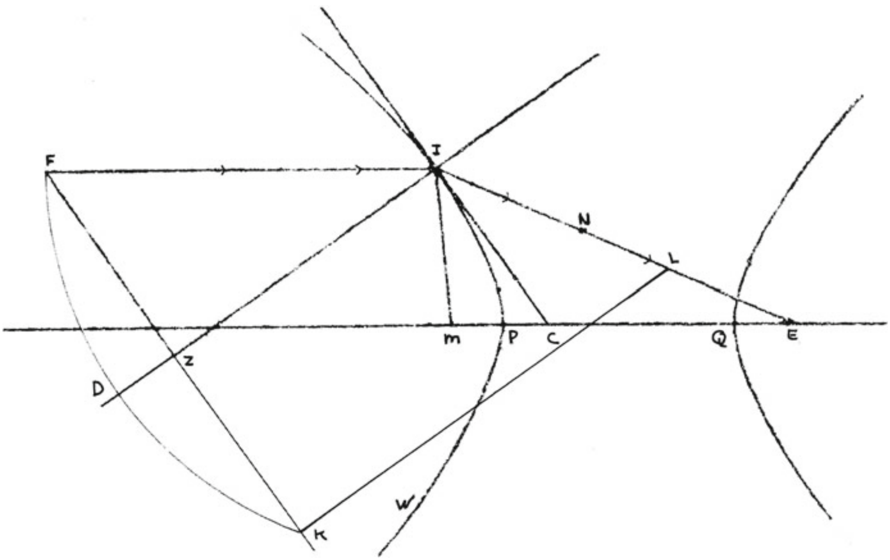

Fig. A.1.8 What Descartes and Mydorge's earliest analysis of the analclastic problem might have looked like, given initial possession of the 'radius' form of the law of refraction of light

Proposition 5 would explain Propositions 3 and 4 as later, synthetic versions of this material, launched, for simplicity's sake, on the basis of the 'antique' one–sided sine form. To put the matter quite generally, if you initially have only the radius form of the law of refraction and are analysing the anaclastic properties of hyperbolas, using the resources of classical and renaissance geometry, then you are very likely to construct the 'antique' sine form of the law in order to consummate the analysis. In this situation the 'one sided' form of the sine law is particularly useful and likely to turn up.

This necessarily technical section can be brought to a close by summarizing in 'synthetic' fashion the main conclusions we have reached through our complicated 'analysis' of the Mydorge letter, stage one of Descartes' lens theory. Below in drawing larger conclusions, we shall work in observations involving Stage two, the interaction with Beeckman in 1628, and Stage 3, the form of the theory published in the *Dioptrique* of 1637.

(1) The evolution of Mydorge and Descartes' lens theory shows that the content of the Mydorge letter dates from before 1628 and therefore approximates to the date of the discovery of the law of refraction in 1626/1627.
(2) Given (1), Mydorge's initial reliance upon the radius form of the law in his propositions 1 and 2 most likely indicates that this was the first form of the law with which he was acquainted, presumably because he and Descartes discovered the law through the 'Harriot-like' procedure of mapping image places using the traditional image placement rule, and deploying data which need have been no better than those supplied by Witelo.
(3) Given (1) and (2), Mydorge's Proposition 5 can be read as containing remnants of Mydorge and Descartes' initial analytical investigations of lens theory, using the radius form of the law as a tool. This path of analysis turned up the 'antique' sine form of the law, which was then used in devising the proofs of Propositions 3 and 4.

A.7 The Kramer-Milhaud Thesis: Discovering the Law of Refraction by Analysis of the Anaclastic Problem

Our results to this point permit us to evaluate a conjecture concerning the genesis of Descartes' law of refraction which has commanded a fair degree of credence for over a century. P. Kramer in 1882, followed by Gaston Milhaud in 1907, suggested that Descartes discovered the law of refraction as a result of posing and analysing the anaclastic problem in this form:

> Given an ellipse or hyperbola, on which a ray falls parallel to the focal axis, according to what geometrical condition will the ray be refracted to one of the foci?[13]

[13] Kramer (1882); Milhaud (1907).

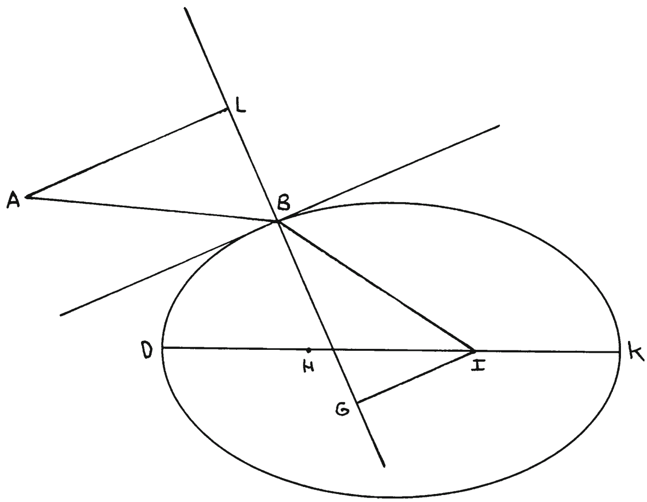

Fig. A.1.9 The Kramer–Milhaud conjecture: discovering the sine law of refraction by analysis of an ellipse assumed to behave as an anaclastic curve

Taking the case of the ellipse, Milhaud sketched an analysis beginning in the following fashion (Fig. A.1.9):

Ray AB enters the ellipse at B parallel to axis DK and is refracted to focus I. Lay off BA = BI and drop a normal LB to the tangent to the ellipse at point B. Then to this normal drop the sines of the angles of incidence and refraction, AL and IG respectively. Milhaud correctly stated that it can easily be shown that:

$$\frac{\mathrm{AL}}{\mathrm{IG}} = \frac{\mathrm{DK}}{\mathrm{HI}}$$

where DK is the transverse axis (t.a.) and HI the focal distance (f.a); hence that,

$$\frac{\sin \mathrm{i}}{\sin \mathrm{r}} = \frac{\mathrm{t.a.}}{\mathrm{f.d}}$$

and that the young Descartes could easily have done this.[14]

Milhaud did not give the rest of the analysis; but it would chiefly consist in a reversal of the steps found in Descartes' demonstration in the *Dioptrique* of the equivalent of Mydorge's Proposition 4. For Kramer and Milhaud this route of discovery had the virtue of exploiting Descartes' mathematical expertise whilst eliminating any appeal to experiment. (In addition one can point to the existence of a synthesis of the problem published by Descartes himself.)

Kramer and Milhaud were perfectly correct to believe the law could have been discovered in this fashion. Indeed, their conjecture could have been made even more

[14] Milhaud (1907) 226.

plausible had they defined less tendentiously the analytical problem they attribute to Descartes. There was no need to specify Descartes' (really Beeckman's) version of the sine form of the law as the fruit of the analysis. Mydorge's one sided version would have served equally well, as would any equivalent construction, for example, the neat construction used in Descartes' elegant but suppressed 1628 proof for the case of the ellipse, mentioned above. Strictly speaking, moreover, Kramer and Milhaud need not have specified any sine form of the law (or indeed any form of the law at all) in the data of the problem. It would have been more historically plausible simply to posit Descartes beginning with an ellipse or hyperbola and a parallel incident ray refracted to the distant focus. In such conditions, anyone with a knowledge of the conic sections could have easily discovered the relation between the ratio of the sines of incidence and refraction and the ratio of transverse axis to focal distance, by identifying the angles equal to the angles of incidence and refraction (or to their complements or supplements) and by applying the trigonometric law of sines.

Unfortunately, however, with or without such improvements, the Kramer-Milhaud conjecture suffers from one serious weakness: there is no positive evidence for it, and the evidence which can be teased out of the Mydorge letter runs directly counter to it. There is no evidence in Mydorge's letter of the law of refraction having been discovered by a straightforward analysis of the anaclastic problem. Mydorge's proofs are loaded with the one sided sine form and/or the radius form of the law; they are hardly the results of the elegant analysis envisioned in the Kramer-Milhaud thesis. Significantly, neither Harriot nor Snel give any evidence of having performed an analysis of that type. In addition, the evidence in Proposition 5 of Mydorge and Descartes' early analytical work in lens theory suggests that their analysis began with the radius form to hand. Their problem was to relate the radius form to the defining properties of an hyperbola or ellipse, operating on the not entirely wild suggestion of the authoritative Kepler that these conics could provide anaclastic surfaces. Descartes and Mydorge had the law, in cosecant form, already to hand, and needed to explore whether it could be related to the defining properties of the conics. Kramer and Milhaud require an initially entirely theoretical and mathematical procedure, producing a *candidate* law of refraction (in some trigonometric form or other as noted above), which then would have had to have been explored from an empirical point of view. However, it should be obvious from the total content of the Mydorge letter, properly interpreted, that the probability of the Kramer-Milhaud thesis being historically accurate is virtually nil.

A.8 Conclusions

[1] The reconstruction of the evolution of Descartes' lens theory confirms my claim in Chap. 4 that Mydorge and Descartes first stumbled on the law of refraction in cosecant rather than sine form, because it establishes that the sine form of the law only emerged during the course of analysis of the anaclastic problem, once the cosecant form was in hand.

[2] The reconstruction shows that the sine form provided more elegant lens theory propositions than the cosecant form, hence motivating its *overall* use in the *Dioptrique*, *even* in the problematical and confusing 'proof' of the law of refraction.

[3] This in turn further explains my finding in Chap. 4 that the 'natural' version of the sine law, when used in relation to the 'tennis ball model of light' proof of the law of refraction in the *Dioptrique,* created problems of exposition and understanding that would not have occurred had Descartes used the cosecant form, and a more explicit version of his 'dynamics of light (which on my reconstruction was derived from a physico-mathematical reading of the cosecant form).

[4] Similarly, the reconstruction shows that Beeckman introduced Descartes to the 'natural' form of the sine law for use in lens theory, whilst Descartes showed him an even more elegant representation for lens theory proof purposes. Because of its utility both in lens theory and in setting out the tennis ball model for the action of light in the attempted derivations of the optical laws, Descartes ultimately opted for the former over the latter. However, the original cosecant form of the law, which, as we have discovered, more accurately modeled the dynamical concepts underlying the optical proofs—having itself inspired their formulation—was never used by Descartes in geometrical representations of the law of refraction or in its supposed proof, although some of his verbal formulations in answers to critics of the *Dioptrique*, betray just that underlying conceptualization.[15]

[5] My argument shows that the Kramer–Milhaud reconstruction of Descartes' path of discovery of the law, which invoked a process of *de novo* and completely mathematically abstract analysis of the anaclastic problem, cannot be correct, given the documentary evidence available. However, it is fair to say that the Kramer/Milhaud conjecture was, as far as it goes, consistent with my claim that *the sine law did indeed emerge in the course of an analysis of the anaclastic problem*, provided, however, *Descartes and Mydorge already possessed and deployed in that analysis the cosecant form of the law*, itself having been obtained through other, quite different, and quite traditional mixed mathematical maneuvers in geometrical optics.

[6] In general, then, the sine form of the law emerged within, and became elegantly functional to, the development of lens theory, given the prior existence of the cosecant form of the law. By contrast, as we learned in Chap. 4, the original cosecant version of the law was intimately connected with Descartes' attempt to derive physico-mathematical capital from geometrical optics, first by reading out dynamical principles governing the behavior of light, and thence by promoting those principles to the level of a general dynamics of corpuscles.

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[15] See for example Descartes' remarks to Mydorge for Fermat in March 1638, Chap. 4 Note 25 and Schuster (2000) Note 24.

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Works of Descartes and Their Abbreviations

AT =*Oeuvres de Descartes* (revised edition, 12 vols.), edited by C. Adam and P. Tannery (Paris, 1964–76). References are by volume number (in roman) and page number (in Arabic).

SG = *The World and Other Writings*, edited and translated by Stephen Gaukroger (Cambridge,1998).

MM = *René Descartes, The Principles of Philosophy*, translated by V. R. Miller and R. P. Miller (Dordrecht, 1991).

MSM = *Rene Descartes, Le Monde, ou Traité de la lumière*, translated by Michael S. Mahoney (New York, 1979).

CSM(K) = *The Philosophical Writings Of Descartes*, 3 vols., translated by John Cottingham, Robert Stoothoff, and Dugald Murdoch, and (for vol. 3) Anthony Kenny, (Cambridge, 1988) References are by volume number (in roman) and page number (in arabic).

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Appendix 2: Decoding Descartes' Vortex Celestial Mechanics in the Text of *Le Monde*

This appendix unfolds some of the grounds for the synthetic reading of the technical details of the vortex celestial mechanics in *Le Monde* offered in Chap. 10, particularly Sects. 10.2.3, 10.2.4, and 10.4. It reflects some of the process by which I arrived at the reading given in the text. Additionally, in the spirit of the 'charitable hermeneutics of *Le Monde*, discussed in Sect. 10.2.1 of that same chapter, it provides some of the justification for the concepts, terms and diagrams I have used in the reconstruction of the theory of vortex celestial mechanics. It is assumed that the reader has examined the synthetic interpretation offered in Chap. 10, before assaying this appendix.

Descartes' discussion in *Le Monde*, of the motion and placement of the planets and comets, is quite extensive, covering 13 pages of text in the Adam-Tannery edition. Moreover, for Descartes the explanation is remarkably repetitive, vague and back-handed. It will, therefore, require a good deal of explication and interpretation of the text to bring out the underlying pattern of explanation. In addition, we shall have to use some passages from the *Principia Philosophiae* (1644) to confirm or clarify parts of the interpretation. This procedure presents obvious pitfalls. One surely does not want to attribute some later theory of 1644 to 1633. My approach will be to limit such appeal to the later work to passages in which it is extremely likely that nothing new has been added or that the sense has not been altered. It should be recognized, however, that a clarification of meaning is probably not strictly distinguishable for a change of meaning, and that any anachronistic appeal to later work is open to challenge. The value and validity of this procedure will have to rest with one's judgment of the overall interpretation which emerges.

Descartes first notes that planets made up of the third element must eventually move with the same agitation (*de même branle*) as the matter of the heavens surrounding them in which they float.

> For, if at first they were moving more quickly than that matter, then, not having been able to avoid pushing it upon colliding with it in their path; in a short time they had to transfer to it a part of their agitation. And if, on the contrary, they had in themselves no inclination to move, nevertheless, being surrounded on all sides by that matter of the heaven, they neces-

J. Schuster, *Descartes-Agonistes: Physico-mathematics, Method & Corpuscular-Mechanism 1618-33*, Studies in History and Philosophy of Science 27,
DOI 10.1007/978-94-007-4746-3, © Springer Science+Business Media Dordrecht 2013

> sarily had to follow its course, just as we see all the time that boats and diverse other bodies floating on water (both the largest and most massive and those that are less so) follow the course of the water they are in when there is nothing else to impede them from doing so.[1]

The sense of *branle* here seems to be speed of motion; for if the planet initially moved faster than the surrounding *boules*, it would be retarded; whereas, if it moved more slowly, it would gradually acquire speed. This entails that the force of motion of the planet and its surrounding *boules*, and consequently their respective centrifugal tendencies to motion, would be functions of their respective quantities of matter. Nevertheless, later in the discussion, Descartes will contend that although planets have the same agitation as the surrounding medium, they need not move as quickly as it does.[2] Thus, in the latter case, agitation conveys more the connotation of total force of motion. All that can initially be gathered from the two passages is that Descartes' terms have a conceptual looseness which seems to preclude a unique mechanical interpretation.

Some of the conceptual fuzziness is explained by the fact that, for the moment, Descartes is interested in pursuing a somewhat different point by exploiting his analogy of bodies floating in a river. As the subsequent passages make clear, the real thrust of his analogy is to establish that bodies pushed along in a current can be classified as belonging to one of two types: those so 'massive' and 'solid' that the centrifugal tendency arising from their inertial force of motion will induce a real centrifugal translation; and those 'less solid and composed of less massive parts',[3] which will not have sufficient centrifugal tendency to translate across the direction of flow;

> And note that, among the diverse bodies that thus float on water, those that are rather solid (*assez dur*) and rather massive (as, ordinarily, boats are, principally the largest and most heavily laden boats) always have much more force than the water to continue their motion, even though it is from the water alone that they have received their motion. By contrast, those floating bodies that are very light like those lumps of white scum that one sees floating long the shores during storms, have less force to continue moving. Thus, if you imagine two rivers that join with one another at some point and then separate again thereafter before their waters…have a chance to mix, then boats or other rather massive and heavy bodies that are borne by the course of the one river will be easily able to pass into the other river, while the lightest bodies will turn away from it and will be thrown back by the force of the water toward the places where it is least rapid.[4]

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[1] AT.XI. 57-58; SG 37-38; MSM 93-5.

[2] AT.XI. 68-9; SG 44; MSM 117.

[3] 3 AT.XI. 60; SG 39; MSM 97-99. *'…selon que chacune est plus ou moins solide, et composeé de parties plus ou moins grosse et massives.'* Recall that in Sect. 4.2, where Descartes' 'dynamics' was first discussed, we defined, in accordance with Descartes' laws of motion in *Le Monde*, the 'principal determination' of a body in motion or tending to motion as, *the directional quantity of force of motion directed along the tangent to the path of motion at a given instant*. Here we are using the term 'inertial force of motion' to denote the scalar quantity of the force of motion involved in the 'principal determination' at any instant of the motion—that is, the sheer amount of force of motion in play, which, in fact, is directed along the tangent to the trajectory at that point.

[4] 4 AT.XI. 58; SG 38; MSM 95. Mahoney renders 'assez durs' as 'rather solid'; Gaukroger translates it as 'rather big'. Larousse defines *dur* as *'ferme, solide, difficile à entamer…'*

Descartes can draw an analogy to the behavior of bodies floating in a celestial vortex. Comets are identified with the more solid and massive bodies, planets with those less so:

> By this example, it is easy to understand that, wherever the parts of matter that could not take the form of the second or of the first element may have been at the beginning, all the larger and more massive among them shortly had to take their course toward the outside circumference of the heavens that contained them and thereafter pass continually from one of these heavens into another without ever stopping for a very long period of time in the same heaven. By contrast, all the less massive had to be pushed, each toward the center of the heaven containing it, by the course of the matter of heaven. And (given the shapes that I have attributed to them) upon colliding with one another, they had to join together severally and compose large balls which, turning in the heavens, have there a motion tempered by all the motions their separate parts could have if they were in fact separate. Thus some tend to move toward the circumferences of those heavens, and others toward their centers.
>
> Know also that we should take those that thus tend to range toward the center of any heaven to be the planets, and we should take those that pass across different heavens to be comets.[5]

All this may very well be a suggestive, if mechanically vague, analogy; but it must be admitted that, to this point, Descartes has raised more questions than he has answered. The problem of whether a planet moves at the same speed as the medium still remains. In addition, Descartes has introduced the terms 'solid' and 'massive' without explaining what they mean, and whether, for example, they are conjointly reducible to density.[6] Finally, the analogy to rivers does not even begin to explain how it is that planets assume uniquely determined orbital distances from the center of a vortex. We must, therefore, pursue Descartes' exposition further. However, we can anticipate our conclusions for the sake of clarity: the problem of planetary placement will be resolved by a clearer understanding of the nature and role of 'solidity' and 'massiveness'; furthermore, as a result, the problem about planetary speed will lose its focal importance, although it will not be entirely resolved.

The next stage of Descartes' argument apparently represents a serious attempt to give a mechanical explication of his analogy. He advances a kind of *reductio ad absurdum* intended to show why a planet cannot but have the same 'force to continue in a straight line' as the second matter surrounding it. Considering first a planet ♄ (Saturn) following an orbit at radius K in Fig. A.2.1, he writes,

> But, in order to make you understand distinctly in what places the planets should stop, look for example at the one marked ♄, which I suppose to follow the course of the matter of the heaven toward the circle K, and consider that, if this planet had the slightest bit more force to continue its motion in a straight line than do the parts of the second element surrounding it, then, instead of always following that circle K, it would go toward Y and thus it would be more distant than it is from center S. Then, in as much as the parts of the second element that would surround it at Y move faster and even are a bit smaller (or at least are not larger) then those at K, they would give it still more force to pass beyond toward F, so that it would

[5] AT.XI. 60-1; SG 39-40; MSM 99-101.

[6] As we have seen, Descartes used '*solide'* at AT XI p.60; and '*dur'* at AT. XI. 58. It will be important to note where the notion of solidity appears, and does not appear, in the remainder of his presentation in *Le Monde.*

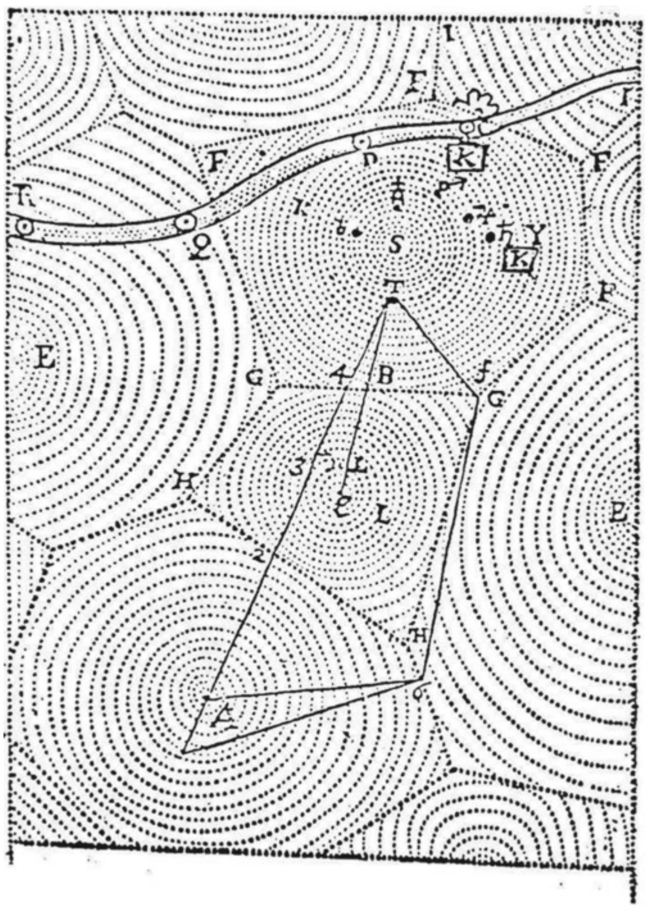

Fig. A.2.1 The vortex cosmos, Descartes, *Le Monde,* AT XI p.55

> go out to the circumference of that heaven, without being able to stop anywhere in between; then from there it would easily pass into another heaven, and thus, instead of being a planet, would become a comet.[7]

Thus, no planet can have more centrifugal tendency arising from its quantity of force of motion than does the second matter of the K layer; for, if it once started translating to the region beyond K, where the *boules* become swifter and smaller, more and more force of motion would be conveyed to it until it drifted right out of the vortex. Presumably, after penetrating the neighboring vortex to some distance, it would translate out again in the same manner, deriving increasing quantities of motion from each successive layer it passed. What is missing in this first branch of the reductio argument is an explicit statement about why such continuous centrifugal translation is caused beyond the K layer, but not below it.

[7] AT.XI. 64; SG 41-42,; MSM 109-111.

A similar lacuna appears in the second side of the reductio argument. Descartes takes up the case of the planet ♄ when it has 'less force than the parts of the second element surrounding it'. Here it happens that,

> those parts that follow it and that are placed a bit lower than it can divert it with the result that, instead of following circle K, it descends toward the planet marked 4 (Jupiter). The planet ♄ being there, it can happen that it is exactly as strong (*justement aussi forte*) as the parts of the second element that will then surround it. The reason for this is that, these parts of the second element being more agitated than those at K, they will also agitate the planet more; being in addition smaller, they will not be able to resist it as much. In this case, the planet will remain perfectly balanc ed in the middle of them and will there take its course in the same direction as they about the sun, without being at one time or another more or less distant from the sun, except insofar as they can also be more or less distant from it.[8]

In translating toward the sun, the plant meets *boules* which are increasingly more agitated and smaller than those farther out. By gaining more agitation, the planet will eventually attain sufficient centrifugal tendency to maintain a fixed orbit. Here we are helped by Fig. A.2.2 which we developed in Chap. 10 (where it was Fig. 10.4) to show the size, speed and hence force of motion distribution of the *boules* of the vortex. A glance at Fig. A.2.2 shows that this case is somewhat similar to the previous case of centrifugal translation, except for the fact that here the size of the *boules* must decrease in a greater proportion than their agitation increases, because their force of motion must decrease as one approaches the center.

The account is still rather vague. The planet drifts down to a level where the *boules* possess less force of motion but greater agitation than at the K-layer. The planet acquires some additional agitation from these lower layers of *boules* and eventually settles into a position, lower than its original one, in which a 'balance' has been struck. But in what does that balance consist? The passage cited implies that there is a balance between the agitation conveyed to the planet by the *boules* and the resistance they make to the motion of the planet. The agitation derived from the *boules* is so adjusted to the resistance to motion they offer that the planet now moves with exactly the same 'strength' (force, agitation, speed?) as the surrounding layer. This may be the case, but Descartes' very next remark only further clouds the issue, by mentioning an equality between the force of motion of the planet and that of the surrounding *boules*:

> But if this planet ♄ being at 4 still has less force to continue its motion in a straight line than has the matter of the heaven found there, it will again be pushed lower by the matter, toward the planet marked ♂ (Mars) and so on, until finally it is surrounded by a matter that has neither more nor less force than it.[9]

Thus, there appears to be a problem in reconciling the two interpretations of 'balance': The one between the planet's agitation and the medium's resistance; the other between their respective forces of motion. Furthermore, it is in this connection that we can note how this part of the reductio suffers from a lacuna analogous to the

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[8] Ibid. 65.

[9] AT.XI.65-66; SG 42; MSM 113.

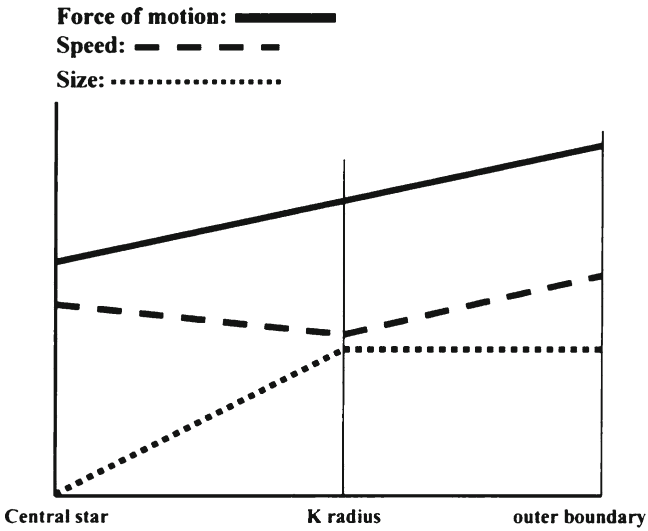

Fig. A.2.2 Size, speed and force of motion distribution of particles of second element, in a solar vortex

one in the first part; that is, 'Why is it that no continuous fall occurs here whilst in the first case continuous centrifugal rise occurs'?

So, let us ask in a pointed manner: Why is it the case that as soon as a planet well below K finds a layer in which its force of motion equals that of the surrounding matter, it does not start to translate outward again? After all, we know from Descartes' own discussion of the laws of nature that a centrifugal tendency arises from the constraint of what we called the 'principal' tendency to motion along a curved path.[10] Why does not that centrifugal tendency lead to a centrifugal translation out to layers of increasing force of motion, so that, as in the first stage of the argument from the K layer outward, the planet would continue right out of the vortex? In other words, the problem is why Descartes takes seriously the continuous centrifugal translation of planets once they pass the K layer, while he ignores this possibility for planets whose force is less than that of the *boules* of the K layer? For the region below K, all he does is invoke the notion of a balance of agitation and resistance, associated with the assertion that the planet will remain in a level of equal force. In summary, the reductio argument seems terribly inadequate; on the one hand, it suffers from a serious conceptual lacuna, while, on the other, it introduces a pair of unreconciled conditions for the placement of planets.

There is, however, a way out of this morass. Unfortunately it requires a brief mention of still one more peculiarity of the reductio argument. The discerning reader of *Le Monde* will probably notice that the reductio argument entirely omits any mention of the terms 'solidity' and 'massiveness'. This is odd, for we saw that

[10] See above Note 3.

in some way the analogy to bodies floating in a river involved differential placement according to their relative solidities or massivenesses. The key to Descartes' celestial mechanics lies in the correction of this omission. If we have been rather backhanded in arriving at it, that is because Descartes himself only falls into the solution in the passage following the reductio, and it is revealing to capture some of the confusion and disorder of the text of *Le Monde*.

As if taking note of the inadequacy of the reductio argument, Descartes writes,

> But, if I still have not made you understand well enough why it can happen that the parts of the heaven beyond the circle K, being incomparably smaller than the planets, do not cease to have more force than they to continue their motion in a straight line, consider that this force does not depend solely on the quantity of the matter that is in each body, but also on the extent of its surface. For even though when two bodies move equally fast it is correct to say that, if one contains twice as much matter as the other, it also has twice the agitation, that is not to say thereby that it has twice as much force to continue to move in a straight line; rather, it will have exactly twice as much if, in addition, its surface is exactly twice as extended, because it will always meet twice as many other bodies resisting it, and it will have much less force to continue if its surface is extended much more than twice.[11]

Thus, in considering the force of motion of bodies, or the resistance they encounter, one must take into account the role of the ratio of surface area to volume. In the *Principia* Descartes will explicitly call this quantity the 'solidity' of a body, a complex magnitude arising from the summed contributions of the volumes (quantities of matter) and surface areas of its constituent particles:

> What I understand here by the solidity of this star, is the quantity of the matter of the third element…in proportion to its volume and surface area.

and,

> That solidity does not depend on matter alone, but also on size and shape…[12]

Since shape contributes to surface area, we will subsequently express solidity as a ratio of volume to surface area (v/s), where solidity (and force of motion) increases as (v/s).

Given this passage from *Le Monde* and the definitions of solidity from the *Principia*, we can now clarify Descartes' entire line of argument. Let us recall the size and speed distribution of second element in the vortex (Fig. A.2.2). The resistance to being put in motion of the *boules* is a function of their solidity, their ratio of

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[11] AT.XI. 66-7; SG 43; MSM 113-115.

[12] *Principia Philosophiae* part III. para. CXXI, 'Per soliditatem hic intelligo, quantitatem materiae tertii elementi....cum eius mole et superificie comparatam.' And CXXII. 'Soliditatem non a sola materia, sed etiam a magnitudine ac figure pendere.' Miller and Miller pp.151–153 (The reader will also be interested in the claims made by Miller and Miller in their notes 120 and 121 thereto.) Descartes mentions a 'star' in the first passage because he is, here, in the process of describing how a dead star, its surface encrusted with third matter, becomes a planet and assumes an orbit in the vortex of another still viable star. A.J. Aiton, who wrote extensively on this and other aspects of Cartesian science, curiously came to the contrary judgment that Descartes conceived solidity to depend on the 'proportion of third matter contained in the body' See Aiton (1957) 261, Note 42, where he cites these very passages from the *Principia*.

surface to volume. The larger the spherical particle the larger (v/s) becomes and the more resistance it offers to being moved aside. Furthermore, as developed in Chap. 10, we may think of the surface layer of *boules* completely surrounding the parts of a planet as a kind of surface body or envelope (Fig. 10.5 above) whose total quantity of matter is proportional to the (v/s) of the *boules* making it up. That is, as long as the *boules* do not grow so large that just a few of them can cover the surfaces of a part of the planet, they will form an envelope around all the parts of the planet of greater or lesser total quantity of matter, depending on their volumes. We can now take Descartes' earlier statement about a balance between force of motion and resistance to motion to mean that planets settle into orbits where their centrifugal tendency to motion is just counter-balanced by the resistance to being put in motion of the surrounding surface layer of *boules*. A planet in the K layer with sufficient centrifugal tendency to start to move away from the center will encounter surface layers made up of progressively smaller *boules*. The resistance to centrifugal tendency offered by these layers will, therefore, progressively decrease, never being as great as it was at K, where the *boules* are largest. Thus, no resistance arising from surface envelopes will be able to prevent the planet from translating right out of the vortex. Hence, in Chap. 10 we were able to depict the resistance of the *boules* to being set in motion in Fig. 10.6 which plots (v/s) with distance. It is as if there exists a hump in the resistance curve at K. If a planet can surmount that hump, it will move as a comet between the levels Ka and Kb of these neighboring vortices.

We can now supply the rationale for the placement of the planets. A planet does not translate toward and beyond layer K because it is not sufficiently solid. As it moves out from near the sun toward K, it meets surface layers of *boules* of increasing resistance to being set in motion. A planet will be located at that layer where the *boules* (and hence the surface envelope) are of such magnitude that they just counteract the centrifugal tendency to motion generated by the quantity of matter, speed and (s/v) of the planet. Since planets themselves vary in their overall solidity they will be located at different distances from the sun. Descartes clearly states in the *Principia* that:

> Thus, when we now see the principal Planets, Mercury, Venus, the Earth, Mars, Jupiter and Saturn being transported around the Sun at different distances, we shall judge that this occurs because {they are not all equally solid, and that} those which are closer to the Sun are less solid than those further away. And we have no reason to think it strange that Mars, although smaller than the Earth, is further from the sun, because size is not the only factor which determines the solidity of bodies, so that Mars, {though smaller}, can be more solid than the Earth.[13]

In effect, contrary to the rhetorical thrust of Descartes' initial discussion in *Le Monde*, planets translate out until they meet countervailing resistances to their

[13] Miller and Miller 172, material in brackets appears first in the French edition of 1644. The original Latin passage at *Principia* Part III para. CXLVII reads as follows: 'Sicque iam videntes primarios Planetos, Mercurium, Venerem, Terram, Martem Novem et Saturnum, ad diversas distantias circa Solem deferri, judicabimus id ex eo contingere, quod eorum qui Soli viciniores sunt, soliditas sit minor quam remotiorum; Nec mirabimur Martem terra minorem, ipsa tamen magis a Sole distare, quia solidior nihilominus esse potest; cum soliditas a sola magnitudine non peneat'.

tendency to centrifugal motion. They are locked into orbits by a balance of their own centrifugal tendency and the resistance of the surface envelope to giving way to that tendency.

This interpretation can be confirmed by comparing the reductio argument of *Le Monde* with the one Descartes provides in the *Principia*. The latter argument is really more properly called a reductio, because it attempts to show that the planet will always return to its place, if it is posited to be out of position. It is presumed that the planet has already reached a layer below K in which the *boules* possess sufficient solidity to resist any further centrifugal translation.

> For if it descended closer to the Sun, it would there find itself surrounded by slightly smaller heavenly globules which it would exceed in force to recede from the center around which it revolves. These parts would also be more rapidly moved, which thus would increase its own agitation along with its force, causing it to ascend. If, on the other hand, it receded further from the Sun, it would encounter there heavenly globules which were somewhat less rapidly moved and would thus decrease its agitation, and which were slightly larger and would thus have the force to drive it back toward the Sun.[14]

By comparison, note that in *Le Monde* Descartes does not consider the second branch of this argument—the ascent of the planet from the sun and its return to its original place. In addition, the first branch of this argument also differs from that in *Le Monde*, for, in this case, the planet returns to its original level, not to some level below its starting point. Third and most importantly, in the *Principia* Descartes does not say that the planet descends or ascends because it has more or less force of motion than the surrounding medium, as he did in *Le Monde*. Rather, the argument proceeds by asking what follows, if we assume that a planet of given (v/s) (and therefore of determinate orbital distance) is not in its proper orbit, but higher or lower. How or why it ascended or descended is not important. The entire problem is to show that it must of necessity return to its proper place. If it is lower than it should be, it will move up, because it will circulate with *boules* which lack sufficient solidity to resist the centrifugal tendency which the planet acquires in moving. The planet will drift up, propelled by the centrifugal tendency, until it is locked in at its appropriate level; that is, the level which can offer countervailing resistance to its centrifugal drift. If the planet is higher than it should be, it will be slowed by impact with ever slower-moving *boules*, and, having less solidity than they, will be displaced or extruded downward by the centrifugal tendency of the *boules* lying just below it. Eventually it will reach its proper level, where the underlying boules will not be solid enough to extrude it, and where its own centrifugal tendency is just resisted by the surface envelope. A balance is achieved, on the one hand, between the centrifugal tendency of the planet and the resistance of the surface layer, and, on the other,

[14] *Principia* pt. III para. CXL; Miller and Miller p.169. 'Quippe si proprius accederet versus Solem, ibi versaretur inter globulos coelestes paulo minores, ac proinde quos superaret vi ad recedendum a centro circa quod gyrat; et celerius motos, ac proinde a quibus ista eius vis simul cum agitatione augeretur, sicque inde rursus regredi deberet. Si vero a Sole magis recederet ei occurrerent globuli coelestes aliquanto minus celeriter moti, ac proinde qui eius agitationem minuerent; et paulo majores, ac proinde qui vim haberent, ipsum versus Solem repellendi.'

between the centrifugal tendency of the immediately subjacent *boules* and the resistance to downward extrusion offered by the body's solidity.[15] This, by the way, clarifies the two kinds of balance discussed in *Le Monde*. 'Resistance' is determined by (v/s) and has to do with the locking mechanism provided by the surface envelopes. Consideration of 'force of motion' refers to the centrifugal tendency of the subjacent *boules* which can extrude a planet from an orbit of too large a radius; that is, an orbit in which the great solidity of the *boules* allows them to generate greater centrifugal tendency than the misplaced planet.

In general, the reductio argument of *Le Monde* is marred by Descartes' insufficient distinction between these two orders of consideration. In *Le Monde*, the planet descends *because it has less force of motion than the* boules *of its original layer*. Solidity does not enter the reductio argument.[16] In the *Principia,* the planet is posited lower than it should be (it has hypothetically descended) because for some reason it has *less force of motion than its solidity potentially allows it to assume.* Because it is more solid than the *boules* in its new surroundings, it will translate up to its proper level as determined by its solidity. There it is locked into an orbit, and, as a matter of fact, it will circulate with a certain force of motion. Thus it is clear that solidity is determinative of orbital distance, whereas, in fact, the force of motion is peripheral to the argument.[17] We do not have to specify what we mean by 'the same agitation' or 'the same force of motion' as the surrounding heaven; we need only grant that circulating in a heaven entails speed, force of motion and centrifugal tendency in the body. Wherever a planet is located, it will derive force of motion from the medium in which it floats. The real problem is whether the surrounding medium is sufficiently 'solid' to resist the centrifugal tendency to motion, which must be generated in the planet by the mere act of circulating.[18]

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[15] This is the source of the 'formula' for orbital equilibrium, the 'locking' of a planet into its orbital distance, given in Sect. 10.2.3.

[16] In *Le Monde*, the role of solidity does become slightly more clear later, when Descartes continues his discussion beyond the reductio argument, explicating his points about the distribution of surface to volume ratios amongst the *boules* at various distances from the central star, and extending his analogical remarks about types of bundles of third matter flowing in rivers. It is here that he comes closest to the interpretation we have developed. Continuing three paragraphs beyond the material cited above at Note 11, he writes, 'Whence you see how diverse planets can be suspended within circle K at diverse distances from the sun, and how it is not simply those that outwardly appear the largest, but those that are the most solid and massive in their interior, that should be the most distant.' [AT.X. 68; SG 44; MSM 117].

[17] As noted just above, Descartes finally achieves a clear statement of at least this point (solidity is determinative of orbital distance) somewhat late in his discussion, following the confused and confusing reductio passages we have been exploring.

[18] Recall Descartes' analogy of ships and flotsam in two confluent rivers to the behavior of comets and planets respectively. We can now see that when a planet gets too far out in the vortex for its solidity, it becomes 'flotsam' and is pushed down toward the central star. But, when a planet sinks closer to the star than its solidity, in principle, warrants, it becomes a 'ship' and drifts out. Since the planet's solidity is fixed, but the vortex corpuscles continuously vary in solidity and force of motion, as per the diagrams deployed above and in Chap. 10, the vortex is revealed as a locking and extruding mechanism.

In summarizing this tortuous and already overlong textual excursion, we can come to the following conclusions already embodied in our findings in Chap. 10. The Cartesian vortex is a kind of dual locking and extruding machine. There is a locking mechanism built into the (v/s) distribution of the *boules*. Planets of a solidity below a certain threshold value will be locked into the heavens below K at distances dependent on their solidities. Beyond K, the (v/s) distribution is such that planets cannot be locked in at any distance. Very solid objects which translate beyond K will become comets, which on Descartes' view oscillate between vortices above the K levels (as they orbit the centre of whichever vortex they presently occupy). All layers of the vortex are capable of extruding sun-ward a planet with too little centrifugal tendency. This is due to the fact that the force of motion of the *boules* constantly increases with distance from the sun. A planet of insufficient solidity, and consequently, too little centrifugal tendency will be displaced downward by the ascent of *boules* with greater centrifugal tendency.

References

Works of Descartes and Their Abbreviations

AT =*Oeuvres de Descartes* (revised edition, 12 vols.), edited by C. Adam and P. Tannery (Paris, 1964–76). References are by volume number (in roman) and page number (in Arabic).

SG = *The World and Other Writings*, edited and translated by Stephen Gaukroger (Cambridge, 1998).

MM = *René Descartes, The Principles of Philosophy*, translated by V. R. Miller and R. P. Miller (Dordrecht, 1991).

MSM = *Rene Descartes, Le Monde, ou Traité de la lumière*, translated by Michael S. Mahoney (New York, 1979).

CSM(K) = *The Philosophical Writings Of Descartes*, 3 vols., translated by John Cottingham, Robert Stoothoff, and Dugald Murdoch, and (for vol. 3) Anthony Kenny, (Cambridge, 1988) References are by volume number (in roman) and page number (in arabic).

HR = The Philosophical Works of Descartes, vol I translated by E.S. Haldane and G.R.T. Ross (Cambridge, 1968 [1st ed. 1911]).

Other

Aiton, E.J. 1957. The vortex theory of planetary motions 1. *Annals of Science* 13: 249–264.