# Appendix A
# Basic engineering aspects of the laminar–turbulent transition

In the previous chapters, the laminar–turbulent transition was discussed in detail as a continuous process starting from excitation of small-amplitude shear-layer disturbances up to establishment of a developed turbulent flow. Finally, we briefly comment applications of basic knowledge on instability and transition in shear flows to engineering problems of the laminar–turbulent transition prediction and control.

## A.1 Transition prediction

Basically, the goal of transition prediction is to clarify whether it takes place in a flow under consideration and to find the corresponding transition Reynolds number $\mathrm{Re}_\mathrm{T}$. In early studies, the laminar–turbulent transition in boundary layers was predicted through empirical correlations between the basic flow parameters and $\mathrm{Re}_\mathrm{T}$. One of the correlations was proposed by Michel (1951) and relates the Reynolds number based on the momentum thickness $\mathrm{Re}_\vartheta$ to the transition Reynolds number as

$$\mathrm{Re}_\vartheta = 1.174\left(1+\frac{22.400}{\mathrm{Re}_\mathrm{T}}\right)\mathrm{Re}_\mathrm{T}^{0.46}.$$

This formula is applicable for attached boundary layers on airfoils with Reynolds numbers based on the wing chord length $\mathrm{Re}_l \gtrsim 2\times 10^6$. There exist also some correlation techniques taking into account the level of free-stream turbulence.

If the transition is modeled without resorting to empirical correlations, the most justified are the methods based on the concept of hydrodynamic stability. In this case, the prediction procedure should include three basic elements:

1. Determination of the structure of initial boundary-layer disturbances excited by external perturbations.
2. Calculation of linear development of small disturbances in the boundary layer.
3. Determination and calculation of the dominant nonlinear processes that characterize the beginning of final laminar flow breakdown.

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To date certain techniques have been developed, allowing calculation of the initial amplitudes of Tollmien–Schlichting waves in some practical situations. The computations of the linear growth of boundary-layer disturbances are quite well advanced, and the adequacy of the description of Tollmien–Schlichting wave, cross-flow, and Görtler vortex amplification by the linear theory of hydrodynamic stability is confirmed experimentally. The calculation of the nonlinear stage of the transition is a more difficult problem; however, it can be sometimes bypassed in practice. Particularly, at a low free-stream turbulence level, nonlinear processes are usually very fast, and the development of small-amplitude disturbances described by the linear stability theory takes place in the major part of a transitional boundary layer. This enables the use of the linear theory for prediction of the transition 'point,' neglecting the details of nonlinear phenomena.

According to experimental data and estimations, the nonlinear processes usually begin to play a noticeable role at amplitudes of the Tollmien–Schlichting waves of about $u' \approx 1\%$ of external-flow velocity $U_e$. Since the nonlinear zone is relatively short, it is possible to admit the linear growth of disturbances in calculations a little above their real saturation amplitude and accept a transition 'point' at which the amplitude of a Tollmien–Schlichting wave of a certain frequency (defined during the calculations) reaches the value $u' = 2$–$4\%$ of $U_e$. The exponential growth of disturbances in the linear region makes the calculated Reynolds number insensitive to some arbitrariness in selection of the limit value $\mathrm{Re}_T$. In such a way, knowing the initial amplitude and frequency characteristics of the Tollmien–Schlichting waves (from an experiment or a solution of a relevant receptivity problem), it is possible to calculate the transition position with reasonable accuracy. This idea constitutes the essence of the so-called $e^N$-method originally suggested by van Ingen (1956), Smith and Gamberoni (1956), which can be applied to two-dimensional boundary layers as well as to three-dimensional and separated flows dominated by convective instabilities.

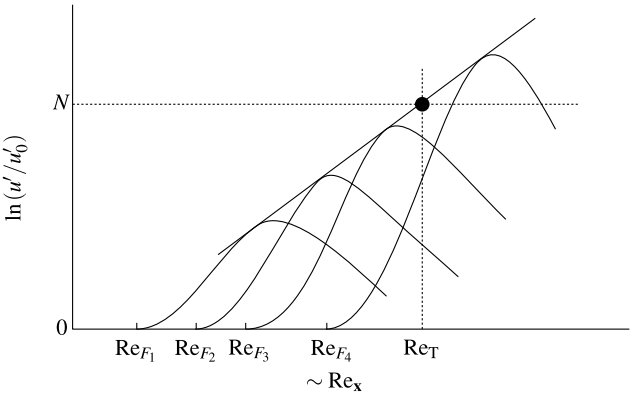


**Fig. A.1** To the $e^N$-method: amplification curves for instability waves with different frequency parameters $F$

According to the method, $\mathrm{Re_T}$ is determined by the minimum Reynolds number at which an envelope of the growth rate curves for instability waves of different frequencies (see, Fig. A.1)

$$\ln\left(u'/u'_0\right) = -\int_{\mathrm{Re}_F}^{\mathrm{Re}} \alpha_\mathrm{i} \mathrm{dRe}_\mathbf{x} \,,$$

where $\mathrm{Re}_F$ corresponds to branch I of the neutral stability curve for the wave with the frequency parameter $F$, reaches a predefined value $N$. Usually for plane and axisymmetric flows at a low level of free-stream turbulence, $N$ is about 8–10.

The value of $N$ varies depending on external-flow perturbations. For example, the results of experiments on the transition on gliders are well generalized by the $\mathrm{e}^N$-method, but with the exponent $N = 15$. Such a high value of the exponent, compared with other flight experiments, is stipulated by the absence of disturbing sources, e.g., a propulsion system. The influence of free-stream turbulence, assuming its isotropy $\overline{u'^2} \approx \overline{v'^2} \approx \overline{w'^2}$, so that $\mathrm{Tu} = \sqrt{\overline{u'^2}}/U_\mathrm{e}$, on the transition in a flat-plate boundary layer can be expressed by the interpolation formula

$$N = -8.43 - 2.4\ln(\mathrm{Tu}) \,,$$

which provides acceptable accuracy in the range $\mathrm{Tu} \approx 0.1$–$2\%$.

For transition prediction at a high free-stream turbulence level, methods based on consideration of spatial growth of optimal disturbances can be used. Let us assume that:

1. Initial kinetic energy of the optimal disturbances in the boundary layer is proportional to kinetic energy of isotropic disturbances of the external flow $E_\mathrm{e} = \mathrm{Tu}^2$ that models the receptivity of flow to external vortical perturbations;
2. Kinetic energy of the unstable disturbances is related to $E_\mathrm{e}$ as $E = \overline{G}\mathrm{Re}E_\mathrm{e}$, where $\overline{G}$ is independent of Re, i.e., the growth is in accordance with the growth of the optimal disturbances;
3. Transition to turbulence occurs when kinetic energy of the disturbances reaches certain value $E = E_\mathrm{T}$.

Combining these requirements yields

$$\mathrm{Tu}\sqrt{\mathrm{Re_T}} = K \,,$$

where $K$ is a constant. Experiments testify that $K$ is in the range of 1131–1506 at $\mathrm{Tu} = 0.9$–$6.0$.

The experimental observations of the transition in swept-wing boundary layers show the presence of quite extended regions of linear-waves development, which enables extension of the $\mathrm{e}^N$-method to three-dimensional flows. The transition to turbulence in a swept-wing boundary layer can occur in the region of the negative pressure gradient as a result of crossflow instability, or downstream where there is an amplification of the Tollmien–Schlichting waves. Then it is possible to apply the $\mathrm{e}^N$-method to both kinds of instability and to consider that the transition occurs if its criterion is satisfied for at least one of them. The main difficulty of application of the $\mathrm{e}^N$-method for three-dimensional flows is the necessity of determining the

direction of the disturbance growth, as outlined in Sect. 6.4.2. Nevertheless, Cebeci (1999), Cebeci et al (1991) applied the procedure and showed the efficiency of the $e^N$-method for the ONERA–D airfoil with the angles of sweep 49–60° and angles of attack 0–6° at $\mathrm{Re}_l = 1.2\text{–}1.6 \times 10^6$. Crouch and Ng (2000) proposed a further modification of the $e^N$-method to account for both the crossflow instability and the flow receptivity, which demonstrated promising results at experimental verification.

We also notice that streamwise surface curvature in the absence of centrifugal instabilities can be taken into account through local profiles of the basic flow. However, it is known that the Görtler vortices together with the Tollmien–Schlichting and crossflow waves promote an earlier transition to turbulence. Experiments on concave surfaces with domination of the Görtler vortices indicate that the transition occurs at the Görtler number based on boundary-layer momentum thickness depending on the free-stream turbulence level as

$$\mathrm{G\ddot{o}}_\vartheta = 9\mathrm{e}^{-17.3\mathrm{Tu}} .$$

Further advances in transition prediction are related to more accurate account of the nonparallel flow effects and curvature with the help of parabolized stability equations. This approach is is attractive because of the low cost of the calculations comparable with the standard $e^N$-method. The approach also admits account of the flow receptivity in a natural manner through modification of initial and boundary conditions.

## A.2 Outline of the linear control theory

The primary goal of the laminar–turbulent transition control is neutralizing disruptive shear-layer perturbations, thereby prolonging the laminar flow state. Research data on this subject are widely reported in original studies and reviewing papers while here we notice some fundamentals only.

The control strategies can be categorized based on the type of flow actuation as passive or active, depending on the absence or presence of energy consumption by the controlling device from external sources. Another classification scheme is based on the means by which the actuation changes in response to modifications in the flow. In such a way, the control strategy can be open-loop (without feedback) or closed-loop (with feedback). In open-loop control, actuator characteristics are designed in advance and remain fixed during its operation. Note that passive control is always the open-loop one.

In the control theory, the governing equations (the Navier–Stokes momentum equations and the continuity equation in our case) are called the descriptor system. In generalized form it can be written as

$$\mathscr{E}\frac{\partial \mathbf{q}}{\partial t} = \mathscr{A}\mathbf{q} + \mathscr{B}\mathbf{g} + \mathscr{D}\mathbf{w} ,$$

where $\mathbf{q}$ is called the state vector consisting of disturbance velocities and pressure, $\mathbf{w}$ is the vector of initial conditions, and $\mathbf{g}$ is the control vector of forcing by which

the system is manipulated with the actuator described by $\mathscr{B}$. The sensor gives information (usually limited) about the flow state as

$$\mathbf{y} = \mathscr{C}\mathbf{q}\,,$$

where $\mathbf{y}$ is the vector of measured values and $\mathscr{C}$ describes the sensor characteristics. To conduct the control in an open-loop manner, it is usually assumed that $\mathbf{y} = \mathbf{q}$ and forcing is changed through variations of actuator parameters, while the resulting measurements $\mathbf{y}$ are monitored to reach an objective.

A closed-loop control system, in addition to the actuator and the sensor, requires a controller. To close the loop, the control system must estimate the input given by $\mathbf{y}$ and try to tune the forcing in real time according to the measurements:

$$\mathbf{g} = \mathscr{K}\,\mathbf{y}$$

to reach an objective that is used to construct the feedback kernel $\mathscr{K}$.

It is frequently required to solve an optimization problem to design the actuator with account of given constraints, for example, to find a control technique that minimizes efforts to suppress the worst initial condition. The problem of optimization of a specified objective functional is called the optimal control problem. However, the presence of the control can make the worst initial condition different. This implies that both must be simultaneously optimized, to ensure control robustness. This approach is called the robust control.

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