

# Appendix A

## Microjunction Model

This appendix presents a pseudo-code implementation of the microjunction model discussed in Chap. 4, Sect. 4.5. The function `microjunction` provides the evolution rates of the area  $\partial A^*/\partial \tau$  and interference  $\partial \delta^*/\partial \tau$  for a given outer force loading history  $F^*(t)$ . The resulting interference and area evolution could be integrated by a standard ODE solver, e.g. a Runge Kutta scheme as indicated in the flowchart of Fig. 4.9.

The hat  $\hat{x}$  signifies the numerical approximations of some model value  $x$  within the function, e.g. the area evolution rate is written as  $\widehat{\frac{\partial A^*}{\partial \tau}}$ . For numerical and practical reasons, all variables are normalized to be of approximately unity magnitude. Interference  $\delta$ , area  $A$  and force  $F$  are normalized by their critical values  $\delta_c$ ,  $A_c$  and  $F_c$  and pressure  $p$  is normalized by the yield stress  $\sigma_Y$ . The normalized values are denoted by a star,  $x^*$ . Comments are denoted by a leading percentage sign %.

$$\mathbf{function} \left[ \widehat{\frac{\partial \delta^*}{\partial \tau}}, \widehat{\frac{\partial A^*}{\partial \tau}} \right] = \mathbf{microjunction} \left( \delta^*, A^*, F^*, \frac{\partial F^*}{\partial t} \right)$$

% Material parameters:

**global**  $\sigma_Y$   $E$   $\nu$   $R$   $C_1$   $C_2$

% Normalization constants:

$$C_\nu \leftarrow 1.295 \exp(0.736\nu)$$

$$E' \leftarrow \frac{E}{1-\nu^2}$$

$$\delta_c \leftarrow \left( \frac{C_\nu \pi \sigma_Y}{2E'} \right)^2 R$$

$$A_c \leftarrow \pi R \delta_c$$

$$F_c \leftarrow \frac{4}{3} \left( \frac{R}{E'} \right)^2 \left( \frac{1}{2} C_\nu \pi \sigma_Y \right)^3$$

% The low-delta augmentation is based on the *maximum*

% interference  $\delta_{\max}^*$  in hemisphere creep history

**global**  $\delta_{\max}^*$

$$\hat{\delta}_{\max}^* \leftarrow \max(\delta_{\max}^*, \delta^*)$$

% Hardness according to the JG model, Eq. (2.13)

$$A_{JG,\max}^* \leftarrow \begin{cases} \hat{\delta}_{\max}^* & \text{for } \hat{\delta}_{\max}^* \leq 1.9 \\ \hat{\delta}_{\max}^* \left( \frac{\hat{\delta}_{\max}^*}{1.9} \right)^{0.14 \exp(23 \frac{\sigma_Y}{E})} & \text{for } \hat{\delta}_{\max}^* > 1.9 \end{cases}$$

$$a \leftarrow \sqrt{\frac{A_{JG,\max}^* A_c}{\pi i}} \quad \% \text{ contact radius}$$

$$H_G \leftarrow \begin{cases} 2.84 \sigma_Y \left( 1 - \exp\left(-0.82 \left(\frac{a}{R}\right)^{-0.7}\right) \right) & \text{for } \frac{a}{R} \leq 0.41 \\ \sigma_Y \left( 7.32 \left(\frac{a}{R}\right)^3 - 14.1 \left(\frac{a}{R}\right)^2 + 6.28 \left(\frac{a}{R}\right) + 1.52 \right) & \text{for } 0.41 < \frac{a}{R} \leq 1 \\ \sigma_Y & \text{for } 1 < \frac{a}{R} \end{cases}$$

% Normalized time constant according to Eq. (2.33):

$$t_1 \leftarrow \frac{EC_1}{H_G}$$

% Creep law, according to Low  $\delta$ -adjustment in Sect. 4.4:

$$a_{A_1} \leftarrow 1.07727; \quad b_{A_1} \leftarrow -0.34304; \quad c_{A_1} \leftarrow -0.11265$$

$$a_{\alpha_1} \leftarrow 2.19198; \quad b_{\alpha_1} \leftarrow 0.50746; \quad c_{\alpha_1} \leftarrow -0.0874$$

$$a_{A_2} \leftarrow -1.85662; \quad b_{A_2} \leftarrow 1.52833; \quad c_{A_2} \leftarrow -0.05672$$

$$a_{\alpha_2} \leftarrow 4.29534; \quad b_{\alpha_2} \leftarrow 0.89977; \quad c_{\alpha_2} \leftarrow -0.06492$$

$$A_1 \leftarrow EC_1 \exp\left(a_{A_1} \left(1 + b_{A_1} \exp\left(\hat{\delta}_{\max}^* c_{A_1}\right)\right)\right)$$

$$\alpha_1 \leftarrow \frac{a_{\alpha_1}}{H_G} \left(1 + b_{\alpha_1} \exp\left(\hat{\delta}_{\max}^* c_{\alpha_1}\right)\right)$$

$$A_2 \leftarrow EC_1 \exp\left(a_{A_2} \left(1 + b_{A_2} \exp\left(\hat{\delta}_{\max}^* c_{A_2}\right)\right)\right)$$

$$\alpha_2 \leftarrow \frac{a_{\alpha_2}}{H_G} \left(1 + b_{\alpha_2} \exp\left(\hat{\delta}_{\max}^* c_{\alpha_2}\right)\right)$$

$$\hat{p} \leftarrow \frac{F^* F_c}{A^* A_c \sigma_Y} \frac{1}{\sigma_Y}$$

$$\widehat{\frac{\partial p}{\partial l}} \leftarrow -(A_1 \sinh(\alpha_1 \hat{p}) + A_2 \sinh(\alpha_2 \hat{p}))$$

$$\widehat{\frac{\partial p^*}{\partial \tau}} \leftarrow \widehat{\frac{\partial p}{\partial l}} \frac{1}{t_1 \sigma_Y}$$

% Stress reinsertion according to Eq. (3.36):

$$f_A \leftarrow 0.81$$

$$\widehat{\frac{\partial p^*}{\partial \delta^*}} \leftarrow f_A \frac{2E'}{\pi \sqrt{R \delta_c}} (A^*)^{-1/2} \frac{\delta_c}{\sigma_Y}$$

% Jackson--Green model for plastic behavior (see page 22):

$$A_{JG}^* \leftarrow \begin{cases} \delta^* & \text{for } \delta^* \leq 1.9 \\ \delta^* \left(\frac{\delta^*}{1.9}\right)^{0.14 \exp(23 \frac{\sigma_Y}{E})} & \text{for } \delta^* > 1.9 \end{cases}$$

$$\widehat{\frac{\partial A_{JG}^*}{\partial \delta^*}} \leftarrow \begin{cases} 1 & \text{for } \delta^* \leq 1.9 \\ \left(1 + 0.14 \exp\left(23 \frac{\sigma_Y}{E}\right)\right) \left(\frac{\delta^*}{1.9}\right)^{0.14 \exp\left(23 \frac{\sigma_Y}{E}\right)} & \text{for } \delta^* > 1.9 \end{cases}$$

$$\begin{aligned}
 a &\leftarrow \sqrt{\frac{A_c A_{JG}^*}{\pi}} \\
 H_G' &\leftarrow \begin{cases} 2.84\sigma_Y \left(1 - \exp\left(-0.81 \left(\frac{a}{R}\right)^{-0.7}\right)\right) & \text{for } \frac{a}{R} \leq 0.41 \\ \sigma_Y \left(7.32 \left(\frac{a}{R}\right)^3 - 14.1 \left(\frac{a}{R}\right)^2 + 6.28 \left(\frac{a}{R}\right) + 1.52\right) & \text{for } 0.41 < \frac{a}{R} \leq 1 \\ \sigma_Y & \text{for } 1 < \frac{a}{R} \end{cases} \\
 F_{JG}^* &\leftarrow \begin{cases} (\delta^*)^{3/2} & \text{for } \delta^* \leq 1.9 \\ \exp\left(-\frac{1}{4}(\delta^*)^{5/12}\right) (\delta^*)^{3/2} + \frac{4H_G'}{C_v\sigma_Y} \delta^* \left(1 - \exp\left(-\frac{1}{25}(\delta^*)^{5/9}\right)\right) & \text{for } \delta^* > 1.9 \end{cases} \\
 \widehat{\frac{\partial F_{JG}^*}{\partial \delta^*}} &\leftarrow \begin{cases} \frac{3}{2} (\delta^*)^{1/2} & \text{for } \delta^* \leq 1.9 \\ \exp\left(-\frac{1}{4}(\delta^*)^{5/12}\right) \frac{3}{2} (\delta^*)^{1/2} - \frac{5}{48} (\delta^*)^{11/12} \exp\left(-\frac{1}{4}(\delta^*)^{5/12}\right) + \frac{4H_G'}{C_v\sigma_Y} \left(1 - \exp\left(-\frac{1}{25}(\delta^*)^{5/9}\right)\right) + \frac{4H_G'}{C_v\sigma_Y} \frac{1}{45} \exp\left(-\frac{1}{25}(\delta^*)^{5/9}\right) (\delta^*)^{5/9} & \text{for } \delta^* > 1.9 \end{cases} \\
 &\% (H_G' \text{-dependence on } \delta^* \text{ neglected})
 \end{aligned}$$

% loading/unloading control

$$\frac{\partial F^*}{\partial \tau} \leftarrow \frac{\partial F^*}{\partial t} t_1$$

if  $\left(\frac{\partial F^*}{\partial \tau} < 0\right)$

% unloading, i.e. elastic behavior of force

$$\widehat{\frac{\partial F^*}{\partial \delta^*}} \leftarrow 2E' \sqrt{\frac{A^* A_c}{\pi}} \frac{\delta_c}{F_c} \quad \% \text{ Eqs. (3.30), (3.33)}$$

else

if  $(F^* < F_{JG}^*)$

% loading, below plastic curve,

% i.e. elastic behavior of force

$$\widehat{\frac{\partial F^*}{\partial \delta^*}} \leftarrow 2E' \sqrt{\frac{A^* A_c}{\pi}} \frac{\delta_c}{F_c} \quad \% \text{ Eqs. (3.30), (3.33)}$$

else

% plastic behavior of force

$$\widehat{\frac{\partial F^*}{\partial \delta^*}} \leftarrow \widehat{\frac{\partial F_{JG}^*}{\partial \delta^*}}$$

end if

end if

% combined interference creep rate (without load rate):

$$R_{res} \leftarrow \frac{4E'}{3F^* F_c} \left(\frac{A^* A_c}{\pi}\right)^{3/2} \quad \% \text{ Eq. (3.32)}$$

$$\widehat{\frac{\partial A^*}{\partial \delta^*}} \Big|_{el} \leftarrow \frac{R_{res}}{R} \quad \% \text{ Eq. (3.29) (normalized)}$$

$$\widehat{\frac{\partial \delta^*}{\partial \tau}} \Big|_{cr} \leftarrow -\widehat{\frac{\partial p^*}{\partial \tau}} \left( \widehat{\frac{\partial p^*}{\partial \delta^*}} + \frac{F^* F_c}{(A^*)^2 A_c} \widehat{\frac{\partial A_{JG}^*}{\partial \delta^*}} \right)^{-1} \quad \% \text{ Eq. (3.39)}$$

% Note: area contribution could be to good approx.  
omitted

% Interference rate (including load rate):

$$\frac{\widehat{\partial \delta^*}}{\partial \tau} \leftarrow \frac{\partial F^*}{\partial \tau} \left( \frac{\partial F^*}{\partial \delta^*} \right)^{-1} + \frac{\widehat{\partial \delta^*}}{\partial \tau} \Big|_{cr} \quad \% \text{ Eq. (4.12)}$$


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% Contact area evolution

**if**  $\left( \frac{\widehat{\partial \delta^*}}{\partial \tau} > 0 \right)$

    % decreasing interference, i.e. elastic behavior of  
area

$$\frac{\partial A^*}{\partial \delta^*} \leftarrow \frac{\partial A^*}{\partial \delta^*} \Big|_{el}$$

**else**

**if**  $(A^* < A_{JG}^*)$

        % below plastic curve, elastic area behavior

$$\frac{\partial A^*}{\partial \delta^*} \leftarrow \frac{\partial A^*}{\partial \delta^*} \Big|_{el}$$

**else**

        % plastic area behavior

$$\frac{\partial A^*}{\partial \delta^*} \leftarrow \frac{\partial A_{JG}^*}{\partial \delta^*}$$

**end if**

**end if**

% Area rate:

$$\frac{\widehat{\partial A^*}}{\partial \tau} \leftarrow \frac{\partial A^*}{\partial \delta^*} \frac{\widehat{\partial \delta^*}}{\partial \tau}$$

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