Appendix A
Elementary Particles

Many decades ago, when we the authors tried to learn physics, the field of elementary particles grew out of nuclear physics and was quite separate from what is presented in the earlier chapters of this book. In recent years, however, statistical physics and the theory of elementary particles haved moved closer together, e.g., by exchanging computer experience: the statistical physics of Sect. 4.3.9 learned from particle theoreticians, and statistical models as in Fig. 5.6 served as test cases for particle computer algorithms. Thus we try to summarize here the present status of this rapidly developing area, without asserting that this must be standard knowledge. Actually we learned it from Professors Rollnik and Heinloth of Bonn University, who in 1993 as missionaries preached quark wisdom to the heathen natives at Cologne, and from reviews of Jean Potvin at Cahokia. It may be taught directly after Chap. 4; if Chap. 5 is a lunch dessert, the present chapter is some sort of five o'clock tea, and as the Boston Tea Party made clear, not everybody likes such tea.

A.1 Basic Facts and Other Lies

A.1.1 Particles

The concept of "atoms" as undivisable units of matter goes back to more than 2000 years ago, and basically the fundamental particles such as quarks and leptons, which we will introduce here, are today's atoms. Historically, however, what was regarded as fundamental in one age turned out to be divisible later, and so the present use of the word "atom" no longer refers to undivisable units, and future centuries might teach us how to split quarks.

Macroscopic matter consists of molecules which are built out of atoms; these atoms define the elements. From the four elements of the ancient Greeks we moved to 92 natural and about 25 artificial elements, each of which may appear in several isotopes. Things became clearer when we learned that each atom consists of a small

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nucleus and a large electron cloud around it, with the nucleus again composed of Z charged protons and A–Z neutral neutrons (found by Chadwick in 1932). A neutral atom thus has Z electrons determining its chemical behaviour; thus Z fixes the element, whereas different atomic weights A for the same Z describe isotopes. For example, A is 3 and 4 for the two helium ($Z = 2$) isotopes ^{3}He and ^{4}He, whereas uranium ($Z = 92$) has the natural isotope with $A = 238$ and the splitable isotope with $A = 235$ to make bombs and reactors therefrom. (Uranium was cooked in a cosmic bomb, the supernova explosion.) So, in 1932 we had just three basic particles, the proton, the neutron, and the electron, to build all known tangible matter from.

"Unfortunately" the neutrino ν (=little neutron) was proposed in 1930 by Pauli to take away some of the energy, momentum and spin arising in the beta-decay of a neutron into a proton and an electron. These neutrinos show very little interest in any reactions and have very small mass; on average our bodies react a few times in their life with a neutrino (coming from the sun). Particles of intermediate masses (between that of an electron and that of a proton), called mesons, were found a few years later and called μ and π. Each particle seems to have an antiparticle of the same mass but with a different sign of the electric charge: the positron balances the electron, and the antiproton was found in a particle accelerator built particularly to produce such antiprotons according to $E = mc^2$. The electron and the myon (or heavy electron; originally mislabelled as the μ-meson), together with the quite heavy τ found only in 1975 at the Stanford two-mile accelerator, are called leptons or light-weights; each has its own type of neutrino. For the medium-weight mesons and heavy-weight baryons the proliferation of particle types due to new powerful accelerators was still more drastic; the Δ^{++} baryon carries two electric charges. These artificially produced particles decay after a short time and thus are called resonances: they show up as peaks of finite width$-$1/lifetime in scattering experiments, like in Fig. 3.8. In short, things became very messy in the 1960s, with an ever increasing number of elementary(?) particles.

Simplification came from a mathematical concept of group theory called SU(3) and from the idea of two or three quarks building up the mesons and baryons, as developed by Gell-Mann, Zweig and Ne'eman. Now the proton and neutron are no longer fundamental, but consist of the truly(?) fundamental quarks and the forces between them. Three quarks form a baryon such as the proton, neutron, or Δ, and two quarks form a meson such as π or ρ. (Their name is taken from James Joyce: "Three quarks for Muster Mark". SU(3) is the special unitary group of complex 3×3 matrices U with $\det(U) = 1$ and $U^\dagger = U^{-1}$.) Quarks have electric charges $\pm\frac{1}{3}$, $\pm\frac{2}{3}$ which explains the existence of the doubly charged Δ^{++} consisting of three quarks with 2/3 charge. Combinations of more than three quarks are also possible.

But again, things are not that simple: instead of two types of quarks they appear in six types (six "flavors"): u, d, c, s, t, b (=up, down, charm, strange, top, bottom). These six flavors are grouped into three generations, which correspond to the three

Table A.1 Fundamental particles of 1994, when evidence for the top quark was found

Leptons			Quarks		
ν_e	ν_μ	ν_τ	u	c	t
e	μ	τ	d	s	b

Table A.2 Masses in GeV and charges in e of fundamental particles

Name	Leptons mass	Charge	Name	Quarks mass	Charge
ν_e	0	0	u	0.005	+2/3
e	0.0005	−1	d	0.010	−1/3
ν_μ	0	0	c	1.5	+2/3
μ	0.105	−1	s	0.2	−1/3
ν_τ	0	0	t	174	+2/3
τ	1.784	−1	b	4.7	−1/3

leptons as shown in Table A.1; all these particles are Fermions obeying the Pauli principle. Thus the mesons with two quarks are Bosons, and the baryons with three quarks are Fermions.

To construct a Δ^{++} from three up quarks of the same charge $+2/3$ would violate the Pauli prohibition against more than one Fermion in the same state. Thus quarks, in contrast to leptons, appear in three "colors", from where the name quantum *chromo*dynamics comes for quark dynamics. And each of the quarks and leptons has its antiparticle; mesons are formed by one quark and one antiquark. Thus we have at present $6 \times 3 \times 2 = 36$ different quarks of various colors and flavors, and $6 \times 2 = 12$ different leptons, or 48 fundamental particles all together.

The proton consists of two up and one down quark, or in short: $p = (uud)$, the neutron is $n = (udd)$, the π^+ meson is formed by an up quark and a down antiquark, and their weights are 0.938, 0.940, and 0.140 GeV, respectively. (1 GeV $=$ 10^9 electron Volt is an energy unit and, via $E = mc^2$, corresponds to a mass of about $1.6\ 10^{-24}$ g, roughly the mass of a hydrogen atom.) So normal matter needs only e^-, u, d as constituents, a nice simplification. Table A.2 lists the masses (in GeV) and electric charges (in elementary charges) of the fundamental particles. The masses increase with increasing generation number 1, 2, and 3; the electric charges show a clear sense of order, and the neutrinos all have very little mass: These properties are not completely arbitrary. We also see that some of the masses of the three (up or down) quarks forming the proton or neutron are *much* smaller than the mass of that nucleon: most of the mass is hidden in the interaction energy due to the enormous color forces between quarks.

Experimental investigations of the possible types of reactions show that certain particle numbers are conserved in the sense that the number of incoming particles of this type must equal the number of particles of the same type after the reaction is over. Each of the three lepton generations has its own conserved number of particles, and so do the quarks for all generations together. Also the electric charge is always conserved, whereas the mass can be transformed into energy and back. However, antiparticles always count negative for the particle number and charge (not for mass). Thus radiation energy can form an electron-positron pair since then the number of e-leptons is still zero.

With these conservation laws we now understand a crucial difference between mesons (quark + antiquark) and baryons (three quarks): the meson number is not conserved since the quark and antiquark can annihilate each other; the baryon number is conserved since their quark number, whatever combination of antiquarks and quarks we try, is never zero. Indeed, the neutral pion π^0 decays into radiation within a microsecond whereas the proton is stable. The free neutron decays after 15 minutes into a proton, an electron, and an anti-electron-neutrino but this is allowed since both the proton and the neutron have the same baryon number of one. (A neutron star or pulsar does not decay into protons because of the strong forces between neutrons.)

Summary: baryon = three quarks; meson = quark and antiquark. Leptons do not consist of quarks. For quarks and for each of the three lepton generations the number is conserved, with antiparticles counting negative. Also electric charge is conserved, with antiparticles having the opposite charge.

So, why do we not observe in nature those fractional electric charges like 1/3 and 2/3? It seems the force between two quarks is quite strong and for large distances about 0.14 MegaNewton, independent of distance. So if we try to pull them apart, we need so much energy that we merely create new particles: somewhat like north and south poles of a magnetic dipole, we cannot observe quarks isolated. Only "white" combinations of quarks, where the color forces have cancelled each other (like quark-antiquark, or three quarks with the three fundamental colors), are observed as isolated particles. Now we look at the mechanism of these and other forces.

A.1.2 Forces

Gravitation and electric forces have already been introduced in the first two chapters; for two protons in an atomic nucleus, gravitation is about 10^{36} times weaker than their Coulomb force. (We still feel gravitation since there are no negative masses, in contrast to positive and negative electric charges which cancel each other in their force over long distances.) As Fig. 2.4 indicates, electric forces do not propagate with infinite velocity but only with the large but finite light velocity c. Light waves are called photons (γ) in quantum theory, that is Coulomb forces are transmitted via the quasi-particles called photons. Similarly, gravitational forces propagate with velocity c with the help of quantized gravity waves called gravitons (not yet detected as quantized quasi-particles). Photons (and gravitons) follow Bose statistics. Quite generally, forces are supposed to come from the exchange of intermediate Bosons. Decades ago we were taught that exchange of pions creates the "strong" or nuclear force which keeps the protons in a nucleus together against their ten times weaker

Coulomb repulsion. Now pions are no longer fundamental, this example is no longer valid, but the principle remains: forces arise from Boson exchange.

A simple analogy with this type of force are two grumpy old men playing tennis on a frozen lake: the small tennis balls give their momentum to each of the men when they are thrown back, and then the two players are slightly repelled from each other. If they used boomerangs, they would be attracted instead; see the movies for other types of intermediate particles.

The color forces between quarks are transmitted by gluons (i.e., by particles glueing the quarks together) of zero mass. They bind three quarks together as a nucleon (proton or neutron). At some distance from this nucleon some remnant of the color forces is felt, since they have not canceled each other exactly; this remnant is the strong force. Similarly, outside of an argon atom the positive and negative electronic charges cancel each other mostly but not fully; the remnants are the Lennard-Jones forces of Sect. 1.1.3 e. But neither these Lennard-Jones forces (or the exchange energy of homopolar binding, Sect. 3.3.6) nor the strong force between nucleons are fundamental. Nevertheless, these strong forces are 20 times stronger than the electromagnetic force for two protons in a nucleus.

The beta-decay of a proton (uud) into a neutron (udd) plus leptons does not involve color forces, since from the quark point of view an up is merely transformed into a down, a member of the same generation. Therefore this decay within a quarter of an hour is much slower than the color-force decay of a hadronic resonance within typically 10^{-22} s. The responsible force is called the "weak" force and is induced by the exchange of the weak $W^{\pm}$ and Z^0-Bosons, detected in 1982 at CERN, with masses of 81 and 91 GeV, respectively. For protons in a nucleus, their weak force is about ten million times weaker than their electric repulsion.

Summary: Forces are based on virtual Boson exchange: gluons of zero mass for the color force, photons of zero mass for the Coulomb force, and quite heavy weak Bosons for the weak force.

These intermediate Bosons (virtual particles) are packets of energy $\Delta E = mc^2$ with a short lifetime Δt, such that the energy-time uncertainty relation $\Delta E \Delta t \simeq \hbar$ allows their creation. If they move with a velocity close to c, within their lifetime they cover a distance of the order of the Compton wavelength:

$$\Delta r = c\Delta t \simeq c\hbar/\Delta E = \hbar/mc\,.$$

The heavier a particle is the less it can run and the shorter is the distance over which the interactions are felt. The force law then contains a factor $\exp(-r/\Delta r) = \exp(-rmc/\hbar)$ apart from pre-exponential factors. For a pion, the resulting Δr is of the order of a proton radius, 10^{-12} cm, as it should be if the "strong" proton-proton interaction came from pion exchange. Coulomb forces and gravitation are felt over infinite distances without exponential cut-off and thus have zero mass. Color forces also must have infinite range since otherwise we could isolate single quarks; thus also the gluons are massless. The weak interaction covers only very short distances because of the high mass of the corresponding intermediate Bosons.

Why are the $W^{\pm}$ and Z^0 so heavy? You can blame the US Congress. The so-called Higgs particles are connected with breaking a spontaneous symmetry. This is an effect already seen in Ising models: the interaction energy remains the same if all spins reverse their orientation; thus the Hamiltonian is up-down symmetric. The paramagnet at $T > T_c$ also has up-down symmetry. But if we cool it down to $T < T_c$ the actual state of the system loses this symmetry: either the majority of spins are up, or the majority of spins are down. This broken symmetry leads to domain walls or (in isotropic magnets) to spin waves or magnons. Similar broken symmetries are supposed to cause the mass of the (virtual) particles. Experimental evidence would be the detection of the Higgs particle, for which purpose construction of the superconducting super collider SSC was started in Texas. Then the project was stopped by parliament as being too expensive. Gell-Mann made a nice comment as to why fellow Nobel laureate Phil Anderson did not support SSC. The European Large Hadron Collider in 2012 filled the gap and found it at a mass of 126 GeV. Thus if the first author's Body Mass Index is too high, blame the Nobel laureates (2013) Higgs and Englert for this obesity.

What has been described here is the so-called *standard model* which includes color forces. The Grand Unified Theory (GUT) combines it with electromagnetic and weak forces, and the Theory of Everything would include gravity. If that theory could be established, it would be a good basis for starting anew the search for the true fundamentals, as has happened in the past: Max Planck was warned against taking up physics as a student, by being told that nearly all of physics is understood.

A.2 Quantum Field Theory of Elementary Particles

In magnetism, we understand on the basis of suitable assumptions, such as the Ising models, how a spontaneous magnetization can be formed. And we learned how to simulate this Ising model on a computer. Analogous computer simulations for quantum chromodynamics (QCD), i.e., for the forces building up the nucleons out of quarks, today constitute one of the most hungry consumers of computer time. Even special-purpose computers were built, which can simulate only QCD, but do this particularly efficient. These methods and their underlying theories are too complicated for our purpose; thus we only give here an impression of how it is done and what the results are.

A.2.1 Quantum and Thermal Fluctuations

What is the average energy of an harmonic oscillator? If we could have neither quantum nor thermal fluctuations, the oscillator would stay forever in its potential minimum of zero energy E. Both types of fluctuations are included in (4.48):

$$E = \hbar\omega(\langle n\rangle + 1/2)\,, \quad \langle n\rangle = 1/[\exp(\hbar\omega/kT) - 1]\,, \tag{A.1}$$

In the limits of very high and very low temperatures this energy becomes

$$E = \frac{\hbar\omega}{2}(\hbar\omega \gg kT)\,, \quad E = kT(\hbar\omega \ll kT) \tag{A.2}$$

corresponding to zero-point motion and equipartition theorem, respectively. Thus we see that $\hbar\omega$ plays, at low temperatures, the role which at high temperatures is played by kT. Similarly, we saw in the WKB approximation for tunnel effects that the wave function "propagates" as $\exp(-S/\hbar)$ through an energy barrier at zero temperature; at finite T without quantum effects an energy barrier would be overcome with probability $\exp(-E/kT)$: Again we found a correspondence of Boltzmann's constant (thermal fluctuation) with Planck's constant (quantum fluctuation). In this way, also the Metropolis simulations of the Ising model, Sect. 4.3.9, have a counterpart in *quantum field theory* at zero temperature.

Such theories generally start from Feynman's path integral formula

$$Z_{\mathrm{s}} = \sum e^{\mathrm{i}S/\hbar} \tag{A.3}$$

for the partition function, instead of the old thermal partition function

$$Z = \sum e^{-\mathrm{E_n/kT}} \tag{A.4}$$

of (4.2). Now the "action" S plays the role of Hamiltonian H or energy and is defined through the Lagrange function L:

$$S = \int L\,\mathrm{d}t\,. \tag{A.5}$$

The integral in (A.5) runs from one fixed point in time to another fixed point in time, just as for Hamilton's principle in (1.32). (We set the velocity of light equal to unity to have the same units for time and length.) And the sum in (A.3) runs over all possible paths $\boldsymbol{r}(t)$ from one fixed point to the other. To sum the thermal partition function (A.4), we learned to use a finite volume V and to replace summations over all wavevectors Q by integrals $(V/2\pi)^3 \int \mathrm{d}^3Q$. To distinguish between different paths for (A.3) we regard space and time as discrete variables, incremented by a minimum length a and a minimum time $\varDelta\tau$ instead of varied continuously. The paths in (A.3) are thus time-ordered connected chains of bonds in a four-dimensional hypercubic space-time lattice. We see this easier if we restrict ourselves to one space dimension x where the path is a function $x = x(t)$ of time t. In the discretized form, this path is a chain of bonds on a square lattice with time (in units of $\varDelta\tau$) plotted horizontally and space (in units of a) plotted vertically.

In this one-dimensional case, the Lagrange function L for a single particle in a potential $U(x)$ and with kinetic energy $mv^2/2$ is $L = \frac{m}{2}(\mathrm{d}x/\mathrm{d}t)^2 - U$. With the imaginary (so-called Euclidean) time $\tau = \mathrm{i}t$ and the Euclidean action $S_\mathrm{E} = -\mathrm{i}S$ we get

$$L = -\frac{m}{2}\,(\mathrm{d}x/\mathrm{d}\tau)^2 - U = -H\,,$$

with the usual Hamiltonian or energy; also

$$S_\mathrm{E} = -\mathrm{i}S = -\int L\,\mathrm{d}\tau = \int H\,\mathrm{d}\tau\,,$$

and thus

$$Z_\mathrm{s} = \sum e^{-S_\mathrm{E}/\hbar} = \sum \exp\left(-\int E(\tau)\,\mathrm{d}\tau/\hbar\right) = \sum \prod_\mathrm{n} \exp\left(-E_\mathrm{n}\varDelta\tau/\hbar\right) \quad (\mathrm{A.6})$$

looks very much like the thermal partition function (A.4) since we replaced the integral over τ by the discrete sum imposed by our lattice approximation. We just have $\hbar/t\,(N_\mathrm{t}\varDelta\tau)$ instead of kT. (The time integration over τ goes from 0 to $t = N_\mathrm{t}\varDelta\tau$, which is the analog of $\hbar/kT$: the larger T, the smaller is the length N_t in the time direction. Our discretization then transforms this time integral into a sum over N_t terms.)

A.2.2 Simulations at $T = 0$

Now the principle of a Monte Carlo simulation can be taken from the Metropolis program for Ising models:

1. choose a lattice site;
2. change slightly the variable belonging to this site;
3. calculate the resulting action change $\varDelta S_\mathrm{E}$;
4. calculate a random number z between 0 and 1;
5. accept the change if $z < \exp(-\varDelta S_\mathrm{E}/\hbar)$;
6. calculate the required quantities;
7. choose a new site and start again.

Our lattices are four-dimensional, of size $L \times L \times L \times N_\mathrm{t}$, for three space ($L$ sites) dimensions and one time (N_t sites) dimension. We obtain not the absolute masses of elementary particles but the ratios of their masses, just like in an Ising model we do not calculate the absolute value of the magnetization (in electromagnetic units per cubic centimeter) but its ratio to the saturation magnetization, as a function of the ratio J/kT. The only problem is now that the variable at each lattice site is no longer an Ising spin represented by $+1$ or -1 but a complex 3×3 matrix arising from the mysteries of SU(3) group theory (fundamental three-dimensional adjoint representation). And thus the simulated lattices of quantum chromodynamics are much smaller than the 336^4 sites of simulated Ising models.

Nevertheless, with lattices of sizes up to $30 \times 32 \times 32 \times 40$ and careful extrapolations to infinite lattices, the group of Weingarten at IBM obtained a quarter-century ago mass ratios for many hadrons which agreed within a few percent with the experimentally observed ratios. They built their own parallel computer, the GF11 (its name comes from 11 GigaFlops, i.e., 11×10^9 floating point operations per second) and used it for about one year for these calculations.

A.2.3 Simulations in the TeraKelvin Region

Just as with the harmonic oscillator, for QCD we can look at both quantum and thermal fluctuations together. At room temperature, the quarks will hardly feel this temperature, since one electron Volt (1 eV) corresponds to 10^4 K, and important quark energies are of the order of GeV. Thus only at temperatures of a TeraKelvin $= 10^{12}$ K are new effects found, such as the quark-gluon plasma.

For water we know that for temperatures below the boiling temperature (or the critical temperature), water droplets can be formed out of supersaturated vapor; if we increase the temperature beyond this limit, the droplets evaporate into vapor molecules which may form a nearly ideal gas. Similarly, at low temperatures the quarks form droplets of three quarks each, such as protons and neutrons which together with electrons, may form atoms, molecules, and life. For temperatures of one TeraKelvin, these nucleons “evaporate” into their three quarks; the gluons can no longer keep the quarks together. However, quarks at a long distance, in contrast to water molecules, still feel a strong force of the order of the weight of 16 tons, and thus they do not form an ideal gas. Instead, together with the gluons, they form a quark-gluon plasma, a soup where no long-lived bonds exist between these quarks. The transition temperature is about 2×10^{12} K corresponding to 0.2 GeV, of the order of the pion mass. It seems we have here a first-order transition (at least for quenched QCD) where the order parameter jumps to zero (like boiling water under atmospheric pressure, and not like the Ising Curie point). In one such simulation 20,000 Cray hours were invested.

Water has an unusually high surface tension at room temperature. Also the transition to a quark-gluon plasma seems to be associated with a surface tension of 5 MeV/(Fermi)^2, which is roughly 10^{19} times larger than that for water, but rather small by quark standards (1 Fermi $= 10^{-13}$ cm $=$ typical nucleon size). Shall we go swimming in it?

You may wonder how in a quenched approximation without moving quarks one can form nucleons at low temperatures out of the quark-gluon plasma. One cannot, but one sees something nevertheless: the gluons alone are thought to form glueballs since gluons, unlike the photons of electromagnetism, interact with each other. (QCD, in contrast to the Maxwell equations, is an intrinsically *nonlinear* theory.) On a more formal level, a phase transition may be connected with a broken symmetry: the interaction energy of the Ising model does not change if all down spins are flipped up and all up spins are flipped down. Thus the Ising Hamiltonian has up-down symmetry. However, for $T < T_c$ the Ising magnet has a spontaneous magnetization which favors one of the two orientations: The up-down symmetry is broken, the cooling of the quark-gluon plasma may similarly lead to a broken "chiral" symmetry.

All these theoretical studies would have been much easier had we watched out better shortly after the big bang of the present universe, some 10^{10} years ago. At that time, room temperature was indeed two Tera-Kelvin and more (if rooms were available at that time). Today we have to recreate such high temperatures with lots of effort in powerful particle accelerators. Perhaps future studies will show new phases of quark matter, or new and heavier quarks as part of an infinite spectrum of resonances, of which we have just seen the lower end with the six quark flavors of up, down, charm, strange, top, and bottom.

Questions

1. What did all natural matter consist of: (a) 2000 years ago; (b) 125 years ago; (c) 90 years ago; (d) for the last 50 years; (e) in 100 years time?
2. How many quarks form a meson, and how many form a baryon?
3. For which particle numbers do conservation laws hold?
4. What are baryons? What are μ-mesons?
5. Which fundamental forces are known, and what is their ratio?
6. What is the Theory of Everything supposed to do?
7. Which forces are transmitted by which particles?
8. What is the partition function of quantum field theory, analogous to $Z = \sum_n e^{-\beta E_n}$?
9. How do quantum fluctuations at zero temperature correspond to thermal fluctuations at high temperature, for the example of the harmonic oscillator?
10. How heavy are Cupid's arrows? ($\Delta r = 10\,\mathrm{m}$) (Cupid = Amor $\simeq$ Eros = ancient god of a dangerous disease; according to Einstein, gravity is not responsible for people falling in love.)

Appendix B
Answers to Questions

These answers should not be looked up in order to learn them, but to check if the reader has learned enough from the text. They are particularly suited to the preparation for an oral examination. Other answers may be also be reasonable; interpret questions such that an answer can be given.

B.1 Mechanics

Sect. 1.1

1. The orbital period is proportional to (major semi-axis)$^{3/2}$ for all solar planets.
2. If measured in an inertial system of coordinates.
3. $f = m\omega^2 r = mv^2/r$.
4. $mv^2/2 = GMm/R$, with the Earth's radius R and mass M; stone mass m cancels.
5. $GM/R = g = 10^3$ cm/s^2 with the Earth's radius R and mass $M = \rho(4\pi/3)R^3$.

Sect. 1.2

6. $\mu = m_1 m_2/(m_1 + m_2)$.
7. $\sum_\mathrm{i}(\boldsymbol{F}_\mathrm{i} - m_\mathrm{i}\mathrm{d}^2\boldsymbol{r}_\mathrm{i}/\mathrm{d}t^2)\delta\boldsymbol{r}_\mathrm{i} = 0$.
8. $\sum_\mathrm{i} \boldsymbol{F}_\mathrm{i}\delta\boldsymbol{r}_\mathrm{i} = 0$ or $\delta U = 0$.

Sect. 1.3

9. It would give nonsense if we moved the end point into the starting point.
10. Lagrange: coordinates and velocities. Hamilton: coordinates and momenta.
11. Atomic vibration; for zero wavevector, the acoustic (optic) phonon frequency is (non)zero.

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Sect. 1.4

12. $\boldsymbol{M} = \mathrm{d}\boldsymbol{L}/\mathrm{d}t$; $\boldsymbol{L} = \Theta\omega$; ω is only an axial vector.
13. If you put coordinate axes into the principal axes, the inertia tensor Θ is diagonal; Θ has as the diagonal elements the moments of inertia about each axis.
14. Zero, because a cube has three equal principal moments of inertia, like a sphere.
15. No, they are linear in ω_1 and ω_2: amplitude cancels out.
16. Because of the cross product: $\mathrm{d}\boldsymbol{L}/\mathrm{d}t = \boldsymbol{r} \times \boldsymbol{f}$.
17. $\omega \sim$ torque for symmetric gyroscope; used for NMR (magnetic resonance imaging).

Sect. 1.5

18. For ∂ the coordinate is fixed, for d it moves with the masses.
19. $\partial\rho/\partial t = \mathrm{div}\,\boldsymbol{j}$.
20. $\sigma = \boldsymbol{C}\varepsilon$; in isotropic fluids: $\sigma_{\mathrm{ik}} = -P\delta_{\mathrm{ik}}$.
21. Hurricane (over Atlantic Ocean) = typhoon (over Pacific Ocean); a tornado is much smaller (over some continents).
22. $\mathrm{d}\rho/\mathrm{d}t = 0$; $\mathrm{curl}\boldsymbol{v} = 0$; zero viscosity; $\mathrm{d}\boldsymbol{v}/\mathrm{d}t = 0$; $\boldsymbol{v} = 0$.
23. For small values only.
24. $D = k_{\mathrm{B}}T/6\pi\eta R$.

B.2 Electricity and Magnetism

Sect. 2.1

1. $U = eq/r$; $f = eq/r^2$.
2. $\mathrm{div}\boldsymbol{E} = 4\pi\rho$; $\mathrm{curl}\boldsymbol{E} = 0$; $\mathrm{div}\boldsymbol{B} = 0$; $\mathrm{curl}\boldsymbol{B} = 4\pi\boldsymbol{j}$.
3. Lorentz force $= q\boldsymbol{v} \times \boldsymbol{B}/c$.
4. $u = (E^2 + B^2)/8\pi$; $\boldsymbol{S} = (c/4\pi)\boldsymbol{E} \times \boldsymbol{B}$.
5. How to build up an arbitrary function out of superposition of sinus waves.
6. Potential = integral over density/distance, with ρ at time $t - R/c$. The electric forces need a time R/c to reach a distance R.
7. G is the effect (at time t) of a cause of unit strength at the origin (at time 0).
8. Put together a positive and a negative charge at a very small distance.
9. Zero; only the torque is nonzero. A field gradient would give a force.

Sect. 2.2

10. Potential = charge/$(r - \boldsymbol{r}\boldsymbol{v}/c)$.
11. Not really; we merely approximate the atomic structure through M and P.

Sect. 2.3

12. Look at the moon with a flashlight and move hand back and forth.
13. By a factor $1/(1 - v^2/c^2)^{1/2} \simeq 1 + v^2/2c^2$.
14. Not at all; but one can write them nicer in four dimensions.
15. The four-momentum $(\boldsymbol{p}, \mathrm{i}cm)$ transforms like $(\boldsymbol{r}, \mathrm{i}ct)$.

B.3 Quantum Mechanics

Sect. 3.1

1. Photo effect, stability of atoms, electron microscope, uncertainty of next exam?
2. No, only if the operators commute.
3. The unit operator. $|n\rangle\langle n|$ is an operator, $\langle n|n\rangle = 1$ a scalar product.
4. It must be unity since it is the space integral over the probability density.

Sect. 3.2

5. $-(\hbar^2/2m)\nabla^2\Psi + U(\boldsymbol{r})\Psi = E\Psi$ $(= \mathrm{i}\hbar\partial\Psi/\partial t)$.
6. If U is finite, then Ψ'' is finite and thus Ψ' is continuous.

Sect. 3.3

7. $Z = 9$ and 10: first and second level; $Z = 11$ also one electron in third level.
8. Photons coming from jumps to the first, second, third level, from higher levels.
9. Both have two protons, helium 3 has one neutron (Fermion), helium 4 has two (Boson).
10. Coulomb plus quantum mechanical indistinguishability gives exchange energy.

Sect. 3.4

11. To the square of the matrix element v_{kn}.
12. Elastic scattering gives Fourier transform in space, inelastic one gives it also in time.
13. Frequency dependence $\sim 1/(1 + \omega^2\tau^2)$.

B.4 Statistical Physics

Sect. 4.1

1. $\frac{1}{4}\,10^{20}$.
2. $2^{\mathrm{N}} \simeq 10^{0.3\mathrm{N}}$ configurations, $N = L^3 = 1, 8, 27, 64$; 1 µs per configuration.

Sect. 4.2

3. $\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V + \mu\mathrm{d}N + \ldots$ Extensive variables as differentials.
4. $E' = E-$ (intensive variable λ) $\times$ (extensive variable Λ) has λ as natural variable.
5. Helmholtz free energy F and $F + PV - BM$.
6. $C_{\mathrm{V}} = T(\partial S/\partial T)_{\mathrm{V}}$; $C_{\mathrm{P}} = T(\partial S/\partial T)_{\mathrm{P}}$; $\chi_{\mathrm{M}} = (\partial M/\partial B)_{\mathrm{M}} = 0$ (nonsense).
7. Vapor pressure and latent heat of vaporization.
8. $P_{\mathrm{osm}}V/kT =$ number of dissolved molecules.

Sect. 4.3

9. $\langle n\rangle = 1/(\mathrm{e}^{(\varepsilon-\mu)/kT} \pm 1)$ (Fermi: $+1$. Bose: -1. Maxwell: ± 1 negligible).
10. $S/kN = \log(\mathrm{const.}E^{3/2}V/N^{5/2})$; const. depends on $\hbar$!
11. Linear in T.
12. $\varepsilon_{\mathrm{F}} = kT_{\mathrm{F}} = p_{\mathrm{F}}^2/2m = \mu$ for Fermi energy ε_{F}, Fermi temperature T_{F}, Fermi momentum p_{F}.
13. Proportional to $T^{\mathrm{d/b}}$ in d dimensions, if $\omega(Q \to 0) \sim Q^{\mathrm{b}}$.
14. With third power for acoustic and infinite power for optic phonons. (Infinite power means $\exp(-\hbar\omega/kT)$; try to derive this: Debye versus Einstein.)
15. With $T^{3/2}$ for isotropic ferromagnets and T^3 for antiferromagnets.
16. A Taylor expansion in the density, to approximate real gases.
17. The line for liquid-vapor equilibrium cuts off equal areas in the P–V diagram.
18. P/P_{c} is a material-independent function of T/T_{c} and V/V_{c}.
19. $m = \tanh(\beta B + mT_{\mathrm{c}}/T)$; $T_{\mathrm{c}} = 0$ without interactions.
20. With the exponent $\delta = 3$ in old theories and nearly 5 in reality.

B.5 Appendix A: Elementary Particles

1. (a) fire, water, air and earth; (b) nearly 100 elements; (c) proton, neutron, electron; (d) quarks, leptons, ...; (e) heavy Staufferons, medium Stanleyons, and light Lesneons?
2. Quark and antiquark for mesons, three quarks for baryons.
3. For the quarks and for each of the three lepton generations.
4. Baryons are proton, neutron, and higher Fermi resonances. μ-mesons = wrong name for myons (leptons); like walfish.
5. Gravitational, weak, electromagnetic, and strong or color interactions. Ratio $10^{-36} : 10^{-7} : 1 : 10^1$ for two protons in a nucleus.
6. Should describe all four forces between all particles.
7. Electromagnetic forces by photons, strong forces by gluons, weak forces by $W^{\pm}$, Z^0.
8. $Z_{\mathrm{s}} = \sum \mathrm{e}^{\mathrm{i}S/\hbar}$ with the action S.
9. Energy $= \hbar\omega/2$ at $T = 0$ corresponds to Energy $= kT$ for $T \to \infty$.
10. $m = \hbar/c\Delta r = 10^{-27-10.5-3}$ g. Do you want to rely on *that*?

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