

Glossary

Acronyms

LCMP	Linearly constrained minimum power
LCMV	Linearly constrained minimum variance
MPDR	Minimum power distortionless response
MVDR	Minimum variance distortionless response
QCQP	Quadratically-constrained quadratic program
SNR	Signal-to-noise ratio
SOCP	Second-order cone programming
WNG	White noise gain

Mathematical operators

$\ \cdot \ $	2-norm
$(\cdot)^*$	Complex conjugate
$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian or complex transpose
$(\cdot)^\dagger$	Pseudo matrix inverse
$(\cdot)!$	Factorial
∇	Gradient
∇_x^2	Laplacian in Cartesian coordinates
∇_r^2	Laplacian in spherical coordinates
$E[\cdot]$	Expectation
$Im\{\cdot\}$	Imaginary part
$\kappa(\cdot)$	Condition number of a matrix
$Re\{\cdot\}$	Real part
$\Lambda(\cdot)$	Rotation operator

Greek symbols

α_q, α_q^{nm}	Sampling weights
α	Vector of sampling weights

δ_{nm}, δ_n	Kronecker delta function
$\delta(\cdot)$	Dirac delta function
θ	Elevation angle
ϕ	Azimuth angle
Ω	Solid angle

Symbols

$a(\cdot)$	Plane-wave decomposition in the space domain
a_{nm}	Plane-wave decomposition in the spherical-harmonics domain
$b_n(\cdot)$	Function relating pressure to plane-wave decomposition
DF	Directivity factor
DI	Directivity index
d_n	Axis-symmetric beamforming weighting function
$d_{mm'}^n(\cdot)$	Wigner-d function
$D_{mm'}^n(\cdot)$	Wigner-D function
\mathbf{d}_n	Axis-symmetric beamforming weighting vector
F	Front-back ratio
$h_n(\cdot)$	Spherical Hankel function of the first kind
$h_n^{(2)}(\cdot)$	Spherical Hankel function of the second kind
\mathbf{I}	Unit matrix
$j_n(\cdot)$	Spherical Bessel function of the first kind
k	Wave number
\mathbf{k}	Wave vector denoting propagation direction
$\tilde{\mathbf{k}}$	Wave vector denoting arrival direction
$L_2(\cdot)$	Space of square-integrable functions
N	Order of spherical harmonics
\mathbb{N}	Set of all natural numbers
\mathbf{n}	Noise vector in the space domain
\mathbf{n}_{nm}	Noise vector in the spherical harmonics domain
$P_n(\cdot)$	Legendre polynomial
$P_n^m(\cdot)$	Associated Legendre function
p	Sound pressure in the space domain
p_{nm}	Sound pressure in the spherical-harmonics domain
\mathbf{p}	Sound pressure vector
\mathbf{p}_{nm}	Sound pressure vector in the spherical-harmonics domain
Q	Number of samples or microphones
\mathbb{R}	One-dimensional space of real numbers
\mathbb{R}^3	Three-dimensional space of real numbers
\mathbf{r}	Vector of spherical coordinates
\mathbf{R}_y	Euler rotation matrix for rotations about the y axis
\mathbf{R}_z	Euler rotation matrix for rotations about the z axis
S^2	Unit sphere
\mathbf{S}	Spherical Fourier transform matrix
\mathbf{S}_{xx}	Cross-spectrum matrix in the space domain

$\mathbf{S}_{\mathbf{x}_{nm}\mathbf{x}_{nm}}$	Cross-spectrum matrix in the spherical-harmonics domain
\mathbf{S}_{nn}	Noise cross-spectrum matrix in the space domain
$\mathbf{S}_{\mathbf{n}_{nm}\mathbf{n}_{nm}}$	Noise cross-spectrum matrix in the spherical-harmonics domain
$T_M(\cdot)$	Chebyshev polynomial
\mathbf{v}	Steering vector in the space domain
\mathbf{v}_{nm}	Steering vector in the spherical-harmonics domain
WNG	White noise gain
$w(\cdot)$	Beamforming weighting function in the space domain
w_{nm}	Beamforming weighting function in the spherical-harmonics domain
\mathbf{w}	Beamforming weighting vector in the space domain
\mathbf{w}_{nm}	Beamforming weighting vector in the spherical-harmonics domain
$y_n(\cdot)$	Spherical Bessel function of the second kind
$Y_n^m(\cdot)$	Spherical harmonics
\mathbf{Y}	Matrix of spherical harmonics
\mathbb{Z}	Set of all integers

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