7. Conclusion

Kinematically redundant serial robots are increasingly gaining attention for industrial applications due to their extraordinary versatility and compliance especially in structured workspaces. As shorter cycle times yield improved cost-effectiveness of robotic operations, the requirement for time-optimimal trajectories along predefined task space paths arises. Because of topological differences to conventional non-redundant manipulators additional complexity is added to the task of trajectory planning.

While methods for point-to-point trajectory planning for kinematically redundant serial manipulators can be found in special literature, only a small number of approaches to finding trajectories along predfined geometric paths exists. Results obtained using methods that yield strict time-optimal trajectories are mostly not directly applicable to real manipulators as the continuity level of the resulting trajectories is low which can induce oscillations and may not be feasible due to the physical limits of the robot drives. Other approaches yield trajectories of arbitrary continuity but lack procedural efficiency.

This thesis presents two fundamentally different approaches to solve the problem of finding minimum-time trajectories for kinematically redundant manipulators along predefined task space paths. In the first method the trajectory of the parameter that parametrizes the task space path is described as a time-dependent B-spline curve. The trajectories of the individual robot joints are then computed by means of first and second-order inverse kinematics approaches. Additionally, those trajectories are improved by exploit-

A. Reiter, *Time-Optimal Trajectory Planning for Redundant Robots,*
BestMasters, DOI 10.1007/978-3-658-12701-5

ing a kinematically redundant manipulator's capability of performing null-space motion. Time-optimality is pursued by means of an optimization problem in which the trajectory end time is minimized and which is inequality constrained by physical and technological limitations of the robot. The second procedure presented in this thesis is based on a separation approach known from literature in which a kinematically redundant robot's joints are devided in two sets. For one selected robot joint a time-dependent B-spline curve is assumed as its trajectory. A separate B-spline trajectory with the same end time is assumed for the task space path parameter. From the task space path trajectory and the trajectory of the separated joint it is now possible to determine the trajectories of the remaining joints by means of analytical inverse kinematics. Trajectories with minimum end time are a result of an optimization process with the control points of the trajectories of the path parameter and the separated joint as parameters. Physical and technological restrictions of the manipulator are incorporated as inequality constraints to the optimization problem.

The presented methods for obtaining minimum-time trajectories were applied to two different examples of kniematically redundant serial manipulators, a planar three-link SCARA and a spatial industrial robot with seven degrees of freedom. For the approach based on numerical inverse kinematics it was found that the convergence of the optimization problem is slow and thus the computation times exceed feasible limits on current hardware due to the formulation. A possible direction of future work may be to investigate whether reformulating the optimzation problem leads to increased efficiency. Applying the separation-based method yielded time-optimal trajectories with respect to the given constraints. In special cases it even outperformed an approach from literature that generally yields trajectories with shorter end time due to lower continuity requirements. Encouraging results were obtained that can be further improved by modifying certain parameters of the methodology. Another opportunity is the investigation of continuation methods for analytical inverse kinematics solutions enabling the separation-based algorithm to be applied to task space paths of increased complexity.

Bibliography

[1] Mathias Brandstötter, Arthur Angerer, and Michael Hofbaur. An analytical solution of the inverse kinematics problem of industrial serial manipulators with an ortho-parallel basis and a spherical wrist. In *Proceedings of the Austrian Robotics Workshop 2014*, 2014.

[2] Hartmut Bremer. *Elastic Multibody Dynamics: A Direct Ritz Approach*. Springer Verlag, 2008.

[3] T. Chettibi, H.E. Lehtihet, M. Haddad, and S. Hanchi. Minimum cost trajectory planning for industrial robots. *European Journal of Mechanics A/Solids*, 23:703–715, 2004.

[4] International Organization for Standardization. EN ISO 9283:1998, February 1998.

[5] Hubert Gattringer. *Starr-elastische Robotersysteme: Theorie und Anwendungen*. Springer Verlag, 2011.

[6] Knut Graichen. *Vorlesungsskriptum Optimierung*. Institut für Automatisierungs- und Regelungstechnik, TU Wien, 2009.

[7] B. Jüttler. *Skriptum zur Vorlesung Geometrische Methoden*. Institut für angewandte Geometrie, Johannes Kepler Universität Linz, 2013.

[8] Johannes Kilian. Quasistatischer Folgeregler und Bahnoptimierung für einen Segway. Master's thesis, Johannes Kepler Universität Linz, Institut für Robotik, 2009.

[9] Shugen Ma and Mitsuru Watanabe. Time optimal path-tracking control of kinematically redundant manipulators. JSME International Journal, 2004.

[10] Wayne Piegl, Les; Tiller. *The NURBS Book*. Monographs in visual communication. Springer Verlag, second edition edition, 1997.

[11] Kurt Schlacher. *Vorlesungsskriptum Automatisierungstechnik I*. Johannes Kepler Universität Linz, 2006.

A. Reiter, *Time-Optimal Trajectory Planning for Redundant Robots,*
BestMasters, DOI 10.1007/978-3-658-12701-5

[12] Bruno Siciliano and Oussama Khatib, editors. *Handbook of Robotics*. Springer Verlag, 2008.

[13] Bruno Siciliano, Lorenzo Sciavicco, Luigi Villani, and Giuseppe Oriolo. *Robotics - Modelling, Planning and Control*. Advanced Textbooks in Control and Signal Processing series. Springer, 2009.

[14] The MathWorks Inc. Constrained nonlinear optimization algorithms, September 2014.