

Appendix

Bessel Function Generation by Chebyshev Polynomial Methods

One possible approach to generating the Bessel functions required in Sect. 3.6 to 3.10 is to evaluate the series expansions which serve as their definitions. From the point of view of numerical evaluation this is undesirable owing to the required fractional exponentiation for functions of fractional order. Furthermore it is difficult, with this method, to obtain an estimate of the error made by truncating the series at a particular term. A more efficient technique is to employ a Chebyshev polynomial expansion of the desired function in the form

$$f(x) = \sum'_{r=0}^{\infty} a_r T_r(x) \quad (\text{A1})^1$$

where the Chebyshev polynomial, $T_r(x)$, defined by

$$T_r(x) = \cos(r \cos^{-1} x), \quad -1 \leq x \leq 1$$

is subject to the recurrence relation

$$T_{r+1}(x) - 2xT_r(x) + T_{r-1}(x) = 0. \quad (\text{A2})$$

An approximation $g(x)$, to a function $f(x)$, can be produced by truncating the expansion such that

$$g(x) = \sum'_{r=0}^n a_r T_r(x) \quad (\text{A3})$$

where the error incurred by truncation has an upper bound given simply by

$$\varepsilon_n(x) = \sum_{r=n+1}^{\infty} |a_r|.$$

Owing to the rapid convergence of the series, a desired accuracy (and thus the value of n) can be chosen simply by inspection of the available range of expansion coefficients. As a result of this feature, and the fact that tables of Chebyshev polynomial expansion coefficients are readily available, this approach is an attractive practical one. Clenshaw and Picken [1] have produced tables of Chebyshev expansion coefficient for Bessel functions of fractional order. The coefficients are given to the 20th decimal place.

¹The prime on the summation indicates that the leading term in the series is halved.

The more usual method of carrying out a Chebyshev expansion is to make use of the recurrence relation in Eq. (A2). Since $T_0(x) = 1$ and $T_1(x) = x$ the higher polynomials are generated quite readily and may be incorporated into an algorithm for machine evaluation. A disadvantage with this approach lies in the number of arithmetic operations necessary to evaluate a given number of terms in the series. If computer time rationing is not a major consideration, the accompanying advantages of flexibility and accuracy offset slow computation. The technique is flexible in that a change in order of the approximation simply requires the addition of more terms to the expansion whilst accuracy is guaranteed by the properties of Chebyshev expansions.

The alternative to direct evaluation of the Chebyshev polynomials involves the rearrangement of a given Chebyshev series into a power series. This power series may then be evaluated by the well known technique of nested multiplication, leading to rapid computation. A requirement of this method is that the power series expansion coefficients must be generated from the Chebyshev coefficients. Whilst this in itself is particularly simple, it does represent a loss of flexibility since a change in the order of the approximation to a function requires a new set of power series coefficients. Another limitation is the possible ill-determination of the power series coefficients, thus leading to some uncertainty as to the accuracy of the technique. This is discussed in Clenshaw [2].

The accuracy of the Bessel function values generated by Chebyshev methods can be checked against the 10 decimal place tables produced by the National Bureau of Standards [3, 4].

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Scientific Fundamentals of Robotics 1

M. Vukobratović, V. Potkonjak

Dynamics of Manipulation Robots

Theory and Application

1982. 149 figures. XIII, 303 pages. ISBN 3-540-11628-1

In this monograph, complete mathematical models of the dynamics of open active mechanisms as applied to robotics are treated and presented for the first time.

It also presents the first parallel survey of computer-oriented methods for automatic construction of the dynamical equations of manipulation robots. These are based on a) general theorems of mechanics, b) second-order Lagrange's equations and c) Appel-Gibbs functions. Adjustment blocks are considered, this enabling the dynamics of spatial mechanisms to be applied to concrete manipulation tasks. The dynamical nominal states (functional movements) of manipulation systems are synthesized and the elastic properties of the manipulation systems are considered.

The basic idea of this book is to describe dynamic models and functional requirements of manipulator dynamics, to describe procedures for computer-aided design of the mechanisms of manipulation robots, and at the same time to introduce different criteria and corresponding constraints to practical relevance.

Scientific Fundamentals of Robotics 2

M. Vukobratović, D. Stokić

Control of Manipulation Robots

Theory and Application

1982. 111 figures. XIII, 363 pages. ISBN 3-540-11629-X

Control synthesis principles and control algorithms of manipulation robots based on their exact dynamic mathematical models are presented in this volume for the first time in monograph form. Chapter 1 gives a computer-aided method for the forming of mathematical models of dynamics of active, spatial, open-configuration mechanisms, as well as a computer-aided method for the forming of linearized mathematical models of dynamics of such mechanisms. Chapter 2 deals with the fundamentals of an original concept of two-stage control synthesis for large-scale mechanical systems which include robot and manipulation systems. Synthesis of control algorithms based on a decentralized structure is presented. Global control is introduced, taking into account the actual dynamic coupling among subsystems of manipulation systems. This results in complete dynamic control of manipulation robots using a relatively simple control structure.



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R.F. Curtain, A.J. Pritchard

Infinite Dimensional Linear Systems Theory

1978. VII, 297 pages

(Lecture Notes in Control and Information Sciences, Volume 8)

ISBN 3-540-08961-6

Contents: Semigroup Theory. – Controllability, Observability, and Stability. – Quadratic Cost Control Problem. – Stochastic Processes and Stochastic Differential Equations. – The State Estimation Problem. – The Separation Principle for Stochastic Optimal Control. – Unbounded Control and Sensing in Distributed Systems. – Time Dependent Systems.

Stochastic Differential Systems

Filtering and Control

Proceedings of the IFIP-WG 7/1 Working Conference

Vilnius, Lithuania, USSR, Aug. 28 – Sept. 2, 1978

Editor: B. Grigelionis

1980. IX, 363 pages

(Lecture Notes in Control and Information Sciences, Volume 25)

ISBN 3-540-10498-4

A wide range of problems connected with Itô stochastic differential equations and their applications to control and filtering of stochastic differential systems are covered in these contributions to an IFIP Working Conference. The conference was attended by 103 specialists from 11 countries, with the majority coming from the Soviet Union.

Stochastic Differential Systems

Proceedings of the 3rd IFIP-WG 7/1 Working Conference Visegrád, Hungary, September 15–20, 1980

Editors: M. Arató, D. Vermes, A.V. Balakrishnan

1981. VI, 250 pages

(Lecture Notes in Control and Information Sciences, Volume 36)

ISBN 3-540-11038-0

The subject of this conference was the modern theory of continuous-time stochastic processes and their applications in control theory, computer science, information processing and related fields. The objective was to bring together leading specialists from all parts of the world in order to report on advances of the past two years and to present the state of the art in further research.

E. Wong

Introduction to Random Processes

(A Dowden & Culver Book)

1982. Approx. 175 pages

ISBN 3-540-90757-2

With few exceptions currently available books on stochastic processes and systems analysis fall into two categories: either elementary or too advanced. There is a great need for an intermediate-level applications oriented book. *Introduction to Random Processes* fills that need.

This book is designed for a first course in random processes at the senior-graduate level. Although some background in probability theory is helpful, the book is largely self-contained. The first three chapters contain basic material on probability theory and random sequences while the last four treat random processes and their application to steady-state linear systems, to dynamical systems, and to signal estimation and detection.



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