

Bibliography

AM = Ann. of Math.; AMS = Ann. Math. Statist.; PAMS = Proc. Amer. Math. Soc.; PNAS = Proc. Nat. Acad. Sci. U.S.A.; TAMS = Trans. Amer. Math. Soc.; TV = Teor. Veroyatnost. i Primenen; ZW = Z. Wahrscheinlichkeitsrechnung.

- AUSTIN, D. G.: [1] Some differentiation properties of Markoff transition probability functions. PAMS **7**, 756—761 (1956).
— [2] Note on differentiating Markoff transition functions with stable terminal states. Duke Math. J. **25**, 625—629 (1958).
— [3] A new proof of the strong Markov theorem of CHUNG. PNAS **44**, 575—578 (1958).
- BLACKWELL, D.: [1] A renewal theorem. Duke Math. J. **15**, 145—150 (1948).
— [2] Extension of a renewal theorem. Pacific J. Math. **3**, 315—320 (1953).
— [3] On transient Markov processes with a countable number of states and stationary transition probabilities. AMS **26**, 654—658 (1955).
— [4] Another countable Markov process with only instantaneous states. AMS **29**, 313—316 (1958).
- BLACKWELL, D., and D. FREEDMAN: [1] The tail σ -field of a Markov chain and a theorem of Orey. AMS **35**, 1291—1295 (1964).
- BLUMENTHAL, R. M.: [1] An extended Markov property. TAMS **85**, 52—72 (1957).
- BREIMAN, L.: [1] On transient Markov chains with application to uniqueness problem for Markov processes. AMS **28**, 499—503 (1957).
— [2] Transient atomic Markov chains with a denumerable number of states. AMS **29**, 212—218 (1958).
- CHACON, R. V.: [1] Some theorems on continuous parameter Markov chains. Diss. Syracuse University 1956.
- CHUNG, K. L.: [1] An ergodic theorem for stationary Markov chains with a countable number of states. Proc. Intern. Congr. Math. Cambridge, Mass. 1950, Vol. I, p. 568.
— [2] Contributions to the theory of Markov chains. J. Res. Nat. Bur. Stand. **50**, 203—208 (1953).
— [3] Contributions to the theory of Markov chains. II. TAMS **76**, 397—419 (1954).
— [4] Foundations of the theory of continuous parameter Markov chains. Proc. Third Berkeley Symposium on Math. Statist. and Probability, Vol. II, pp. 29—40. University of California Press 1956.
— [5] Some new developments in Markov chains. TAMS **81**, 195—210 (1956).
— [6] On a basic property of Markov chains. AM **68**, 126—149 (1958).
— [7] Some aspects of continuous parameter Markov chains. Publ. Inst. Statist., Univ. Paris **6**, 271—287 (1957).
— [8] On last exit times. Illinois J. Math. **4**, 629—639 (1960).
— [9] Probability methods in Markov chains. Proc. Fourth Berkeley Symposium on Math. Statist. and Probability, Vol. II, pp. 35—56. University of California Press 1961.

- CHUNG, K. L.: [10] Some remarks on taboo probabilities. *Illinois J. Math.* **5**, 431—435 (1961).
- [11] On the Martin boundary for Markov chains. *PNAS* **48**, 963—968 (1962).
- [12] On the boundary theory for Markov chains. *Acta Math.* **110**, 19—77 (1963).
- [13] The general theory of Markov processes according to Doebelin. *ZW* **2**, 230—250 (1964).
- [14] On the boundary theory for Markov chains. II. *Acta Math.* **115**, 111—163 (1966).
- CHUNG, K. L., and J. L. DOOB: [1] Fields, optionality and measurability. *Amer. J. Math.* **87**, 397—424 (1964).
- CHUNG, K. L., and P. ERDÖS: [1] Probability limit theorems assuming only the first moment. *Mem. Amer. Math. Soc. No. 6*, 1—19 (1951).
- CHUNG, K. L., and W. H. J. FUCHS: [1] On the distribution of values of sums of random variables. *Mem. Amer. Math. Soc. No. 6*, 1—12 (1951).
- CHUNG, K. L., and D. ORNSTEIN: [1] On the recurrence of sums of random variables. *Bull. Amer. Math. Soc.* **68**, 30—32 (1962).
- CHUNG, K. L., and J. WOLFOWITZ: [1] On a limit theorem in renewal theory. *AM* **55**, 1—6 (1952).
- DERMAN, C.: [1] A solution to a set of fundamental equations in Markov chains. *PAMS* **5**, 332—334 (1954).
- [2] Some contributions to the theory of denumerable Markov chains. *TAMS* **79**, 541—555 (1955).
- DOBROUŠIN, R. L.: [1] Two limit theorems for the simplest random walk on a line. *Uspehi Mat. Nauk. (N.S.)* **10**, 139—146 (1955) [Russian].
- [2] On conditions of regularity of stationary Markov processes with a denumerable number of possible states. *Uspehi Mat. Nauk. (N.S.)* **7**, 185—191 (1952) [Russian].
- [3] An example of a countable homogeneous Markov process all states of which are instantaneous. *TV* **1**, 481—485 (1956) [Russian].
- [4] Some classes of homogeneous denumerable Markov processes. *TV* **2**, 377—380 (1957) [Russian].
- DOEBLIN, W.: [1] Sur les propriétés asymptotiques de mouvement régis par certains types de chaînes simples. *Bull. Math. Soc. Roum. Sci.* **39**, No. 1, 57—115; No. 2, 3—61 (1937).
- [2] Sur l'équation matricielle $A^{(t+s)} = A^{(t)}A^{(s)}$ et ses applications aux probabilités en chaîne. *Bull. Sci. Math. (2)*, **62**, 21—32 (1938); **64**, 35—37 (1940).
- [3] Sur certains mouvements aléatoires discontinus. *Skand. Aktuarietidskr.* **22**, 211—222 (1939).
- [4] Sur deux problèmes de M. KOLMOGOROFF concernant les chaînes dénombrables. *Bull. Soc. Math. France* **66**, 210—220 (1938).
- DOOB, J. L.: [1] Topics in the theory of Markoff chains. *TAMS* **52**, 455—473 (1942).
- [2] Markoff chains — denumerable case. *TAMS* **58**, 455—473 (1945).
- [3] Renewal theory from the point of view of probability. *TAMS* **63**, 422—438 (1948).
- [4] *Stochastic processes*. New York 1953.
- [5] Discrete potential theory and boundaries. *J. Math. Mech.* **8**, 433—458 (1959).
- DYNKIN, E. B.: [1] Some limit theorems for sums of independent random variables with infinite mathematical expectation. *Izv. Akad. Nauk. SSSR., Ser. Mat.* **19**, 247—266 (1955) [Russian].

- DYNKIN, E. B.: [2] Foundations of the theory of Markov processes. Moscow 1959 [Russian].
- DYNKIN, E. B., and A. A. YUŠKEVIČ: [1] Strong Markov processes. *TV* **1**, 149—155 (1956) [Russian].
- ERDŐS, P., W. FELLER and H. POLLARD: [1] A theorem on power series. *Bull. Amer. Math. Soc.* **55**, 201—204 (1949).
- ERDŐS, P., and M. KAC: [1] On certain limit theorems of the theory of probability. *Bull. Amer. Math. Soc.* **52**, 292—302 (1946).
- FELLER, W.: [1] On the integro-differential equations of purely discontinuous Markoff processes. *TAMS* **48**, 488—575 (1940); *Errata* **58**, 474 (1954).
 — [2] Fluctuation theory of recurrent events. *TAMS* **67**, 98—119 (1949).
 — [3] An introduction to probability theory and its applications. New York, Vol. 1, 1950 (first ed.), 1957 (second ed.); Vol. 2, 1966.
 — [4] Boundaries induced by stochastic matrices. *TAMS* **83**, 19—54 (1956).
 — [5] On boundaries and lateral conditions for the Kolmogoroff differential equations. *AM* **65**, 527—570 (1957).
 — [6] On the Fourier representation for Markov chains and the strong ratio theorem. *J. Math. Mech.* **15**, 274—283 (1966).
- FELLER, W., and H. P. MCKEAN, Jr.: [1] A diffusion equivalent to a countable Markov chain. *PNAS* **42**, 351—354 (1956).
- FOSTER, F. G.: [1] On Markoff chains with an enumerable infinity of states. *Proc. Cambridge Phil. Soc.* **47**, 587—591 (1952).
- FRANK, P.: [1] Taboo generating functions and other topics in Markov chains. *Diss. Columbia University* 1959.
- FRÉCHET, M.: [1] *Recherches théoriques modernes sur le calcul des probabilités, Vol. II, Methode des fonctions arbitraires. Théorie des événements en chaîne dans le cas d'un nombre fini d'états possibles.* Paris 1938.
- GNEDENKO, B. V., and A. N. KOLMOGOROFF: [1] Limit distributions for sums of independent random variables. Moscow-Leningrad 1949 (English translation by K. L. CHUNG. Cambridge, Mass. 1954).
- HARDY, G. H.: [1] *Divergent series.* Oxford 1949.
- HARRIS, T. E.: [1] First passage and recurrence distributions. *TAMS* **73**, 471—486 (1952).
 — [2] Transient Markov chains with stationary measures. *PAMS* **8**, 937—942 (1957).
- HARRIS, T. E., and H. ROBBINS: [1]: Ergodic theory of Markov chains admitting an infinite invariant measure. *PNAS* **39**, 860—864 (1953).
- HARTMAN, P., and A. WINTNER: [1] On the law of the iterated logarithm. *Amer. J. Math.* **63**, 169—176 (1941).
- HILLE, E., and R. S. PHILLIPS: [1] *Functional analysis and semi-groups.* Amer. Math. Soc. Colloq. Publ. 1957.
- HODGES, J. L., and M. ROSENBLATT: [1] Recurrence-time moments in random walks. *Pacific J. Math.* **3**, 127—136 (1953).
- HSU, P. L.: [1] The differentiability of the probability transition function of a purely discontinuous stationary Markoff process on the Euclidean space. *Peking Univ. J.* **4**, No. 2, 257—270 (1958).
- HUNT, G. A.: [1] Markoff chains and Martin boundaries. *Illinois J. Math.* **4**, 313—340 (1960).
- JAIN, N.: [1] Some limit theorems for a general Markov process. *Diss. Stanford University* 1965 (to appear in *ZW*).

- JURKAT, W. B.: [1] On semi-groups of positive matrices, I. *Scripta Math.* **24**, 123—131 (1959).
 — [2] On semi-groups of positive matrices, II. *Scripta Math.* **24**, 207—218 (1959).
 — [3] On the analytic structure of semi-groups of positive matrices. *Math. Z.* **73**, 346—365 (1960).
- KARLIN, S., and J. MCGREGOR: [1] Representation of a class of stochastic processes. *PNAS* **41**, 387—391 (1955).
- KATO, T.: [1] On the semi-groups generated by KOLMOGOROFF's differential equations. *J. Math. Soc. Japan* **6**, 1—15 (1954).
- KELLEY, J. L.: [1] *General topology*. New York 1955.
- KEMENY, J., and L. J. SNELL: [1] *Denumerable Markov chains*. New York 1966.
- KENDALL, D. G.: [1] Some analytical properties of continuous stationary Markov transition functions. *TAMS* **78**, 529—540 (1953).
 — [2] Some further pathological examples in the theory of denumerable Markov processes. *Quart. J. Math.* **7**, 39—56 (1956).
 — [3] A note on DOEBLIN's central limit theorem. *PAMS* **8**, 1037—1039 (1957).
 — [4] Unitary dilations of one-parameter semigroups of Markov transition operators, and the corresponding integral representations for Markov process with a countable infinity of states. *Proc. London Math. Soc.* (3) **9**, 417—431 (1959).
 — [5] Some recent developments in the theory of denumerable Markov processes (to appear).
- KENDALL, D. G., and G. E. H. REUTER: [1] Some pathological Markov processes with a denumerable infinity of states and the associated semigroups of operators on l . *Proc. Intern. Congr. Math. Amsterdam 1954*, Vol. III, pp. 377—415.
 — [2] The calculation of the ergodic projection for Markov chains and processes with a countable number of states. *Acta Math.* **97**, 103—144 (1957).
- KINGMAN, J. F. C.: [1] An approach to the study of Markov processes. *J. Roy. Statist. Soc.* (13) (to appear).
- KINGMAN, J. F. C., and S. OREY: [1] Ratio limit theorem for Markov chains. *PAMS* **15**, 907—910 (1964).
- KOLMOGOROV, A. N.: [1] Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung. *Math. Ann.* **104**, 415—458 (1931).
 — [2] Grundbegriffe der Wahrscheinlichkeitsrechnung. *Ergebn. Math.* **2**, No. 3, Berlin 1933.
 — [3] Anfangsgründe der Theorie der Markoffschen Ketten mit unendlichen vielen möglichen Zuständen. *Mat. Sbornik N.S. Ser.* 607—610 (1936); *Bull. Univ. Moscou* **1** (1937) [Russian].
 — [4] A local limit theorem for classical Markov chains. *Izv. Akad. Nauk. SSSR., Ser. Mat.* **13**, 281—300 (1949) [Russian].
 — [5] On the differentiability of the transition probabilities in homogeneous Markov processes with a denumerable number of states. *Učenyje Zapiski MGU* **148**, *Mat.* **4**, 53—59 (1951) [Russian].
- KRENGEL, U.: [1] On the global limit behavior of Markov chains and of general nonsingular Markov processes (to appear in *ZW*).
- LAMPERTI, J.: [1] An occupation time theorem for a class of stochastic processes. *TAMS* **88**, 380—387 (1958).
- LÉVY, P.: [1] Systèmes markoviens et stationnaires. *Cas dénombrable. Ann. Sci. Ecole Norm. Sup.* (3) **68**, 327—381 (1951).
 — [2] Complément à l'étude des processus de MARKOFF. *Ann. Sci. Ecole Norm. Sup.* (3) **69**, 203—212 (1952).

- LÉVY, P.: [3] Processus markoviens et stationnaires du cinquième type (infinité dénombrable des états possibles, paramètre continu). *C. R. Acad. Sci. Paris* **236**, 1630—1632 (1953).
- [4] Processus semi-markovien. *Proc. Intern. Congr. Math. Amsterdam 1954*, pp. 416—426.
- [5] Processus markoviens et stationnaires. Cas dénombrable. *Ann. Inst. H. Poincaré* **16**, 7—25 (1958).
- LOÈVE, M.: [1] *Probability theory*. 3rd ed. New York 1963.
- MAULDON, J. G.: [1] On non-dissipative Markov chains. *Proc. Cambridge Phil. Soc.* **53**, 825—835 (1957).
- NEVEU, J.: [1] Une généralisation des processus à accroissements croissants indépendants. *Abh. Math. Sem. Univ. Hamburg* **25**, 36—61 (1961).
- [2] Lattice methods and submarkovian processes. *Proc. Fourth Berkeley Symposium of Math. Statist. and Probability*, Vol. II, pp. 347—391. University of California Press 1961.
- [3] Sur les états d'entrée et les états fictifs d'un processus de Markov. *Ann. Inst. H. Poincaré*, **17**, 324—337 (1962).
- [4] Chaines de Markov et théorie du potentiel. *Ann. Fac. Sci. Claremont*, No. 24, 37—89 (1964).
- OREY, S.: [1] Sums arising in the theory of Markov chains. *PAMS* **12**, 847—856 (1961).
- [2] Strong ratio limit property. *Bull. Amer. Math. Soc.* **67**, 571—574 (1961).
- [3] An ergodic theorem for Markov chains. *ZW* **1**, 174—176 (1962).
- ORNSTEIN, D.: [1] The differentiability of transition functions. *Bull. Amer. Math. Soc.* **66**, 36—39 (1960).
- PORT, S. C.: [1] Ratio limit theorems for Markov chains. *Pacific J. Math.* **15**, 989—1017 (1965).
- RAY, D.: [1] Resolvents, transition functions and strongly Markovian processes. *AM* **70**, 43—72 (1959).
- REUTER, G. E. H.: [1] Über eine Volterrasche Integralgleichung mit totalmonotonem Kern. *Arch. Math.* **7**, 59—66 (1956).
- [2] Denumerable Markov processes and the associated contraction semi-groups on l . *Acta Math.* **97**, 1—46 (1957).
- [3] Denumerable Markov processes. II. *J. London Math. Soc.* **34**, 81—91 (1959).
- [4] Denumerable Markov processes. III. *J. London Math. Soc.* **37**, 63—73 (1962).
- REUTER, G. E. H., and W. LEDERMANN: [1] On the differential equations for the transition probabilities of Markov processes with enumerably many states. *Proc. Cambridge Phil. Soc.* **49**, 247—262 (1953).
- ROMANOVSKII, V. I.: [1] *Discrete Markov chains*. Moscow-Leningrad 1949 [Russian].
- SAKS, S.: [1] *Theory of the integral*. Warszawa-Lwow 1937.
- SARYMSAKOV, T. A.: [1] *Elements of the theory of Markov processes*. Moscow 1954 [Russian].
- SMITH, G. J.: [1] Instantaneous states of Markov processes. *TAMS* **110**, 185—195 (1964).
- SMITH, W. L.: [1] Asymptotic renewal theorems. *Proc. Roy. Soc. Edinburgh (A)* **64**, 9—48 (1954).
- SPITZER, F.: [1] *Principles of random walk*. New York 1964.
- TITCHMARSH, E. C.: [1] *The theory of functions*. Oxford 1939 (second ed.).

- URBANIK, K.: [1] Limit properties of homogeneous Markoff processes with a denumerable set of states. *Bull. Acad. Polon. Sci. Cl. III* **2**, 374—373 (1954).
- USPENSKY, U. V.: [1] Introduction to mathematical probability. New York 1937.
- VEECH, W.: [1] The necessity of HARRIS' condition for the existence of a stationary measure. *PAMS* **14**, 856—860 (1963).
- VERE-JONES, D.: [1] Geometric ergodicity in denumerable Markov chains. *Quart. J. Math.* **13**, 17—28 (1962).
- WIDDER, D. V.: [1] The Laplace transform. Princeton 1946.
- WILLIAMS, D.: [1] A new method of approximation in Markov chain theory and its application to some problems in the theory of random time substitution. *Proc. London Math. Soc. (3)*, **16**, 213—240 (1966).
- [2] The process extended to the boundary. *ZW* **2**, 332—339 (1963).
- [3] On the construction problem for Markov chains. *ZW* **3**, 227—246 (1964).
- YUŠKEVIČ, A. A.: [1] On strong Markov processes. *TV* **2**, 187—213 (1957) [Russian].
- [2] On differentiability of transition probabilities of homogeneous Markov processes with a countable number of states. *Yčeny Zapiski MGY* **186**, Mat. **9** [Russian].

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 If $\sum_n \mathbf{P}(A_n) < \infty$, then $\mathbf{P}(\limsup_n A_n) = 0$. If the A_n 's are independent and $\sum_n \mathbf{P}(A_n) = \infty$, then $\mathbf{P}(\limsup_n A_n) = 1$.
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 Each of the four derivates and the difference quotients of a continuous function in an (open) interval have the same bounds.
DINI'S theorem on uniform convergence:
 If a nondecreasing sequence of continuous functions converges to a continuous limit in a compact interval, then the convergence is uniform there.

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