Appendix A
The WLP of Ternary Monomial Complete Intersections in Positive Characteristic

So far in this monograph we have considered the Lefschetz properties mostly in characteristic zero. In this appendix we indicate some amazing results on the WLP in positive characteristic recently discovered by J. Li and F. Zanello [80] and also by C. Chen et al. [15].

The following is due to Migliore et al. [102]. (Here we state it in a weaker form than the original.)

Proposition A.1. *Suppose that* $A = \oplus A_{i=0}^{e} A_i,\ \ A_e \neq 0$ *is a standard Artinian algebra over* A_0*, an arbitrary field. Suppose that* $l \in A$ *is a linear form and* $\phi_d : A_d \to A_{d+1}$ *is the homomorphism defined by the multiplication by* l *for* $d \geq 0$.

1. *If* ϕ_{d_0} *is surjective, then* ϕ_d *is surjective for all* $d \geq d_0$.
2. *If* ϕ_{d_0} *is injective, then* ϕ_d *is injective for all* $d \leq d_0$.
3. *If* A *is Gorenstein and if* e *is odd, then* A *has the WLP if and only if* $\phi_{(e-1)/2}$ *is bijective.*

The numerical function $M(a,b,c)$ defined below plays a crucial role in determining the WLP for a monomial complete intersection in three variables over a field of characteristic $p > 0$.

Definition A.2. For any integers a, b, c, define the numerical function $M(a,b,c)$ by

$$M(a,b,c) = \begin{cases} \prod_{i=1}^{a} \prod_{j=1}^{b} \prod_{k=1}^{c} \dfrac{i+j+k-1}{i+j+k-2} & \text{if } a,b,c \text{ are positive,} \\ 1 & \text{otherwise.} \end{cases}$$

Theorem A.3. *1. Let* α, β, γ *be positive integers such that* $\alpha + \beta + \gamma$ *is even, and let*

$$A = \mathbb{Q}[x,y,z]/(x^{\alpha}, y^{\beta}, z^{\gamma}).$$

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Let U be the square matrix representing the multiplication map,

$$\times(x+y+z) : R_m \to R_{m+1}, \text{ where } m = (\alpha+\beta+\gamma-4)/2,$$

in the monomial bases of R_m and R_{m+1}. Then

$$|\det U| = M\left(\frac{-\alpha+\beta+\gamma}{2}, \frac{\alpha-\beta+\gamma}{2}, \frac{\alpha+\beta-\gamma}{2}\right).$$

2. *Let K be a field of characteristic $p > 0$, α, β, γ positive integers such that $\alpha + \beta + \gamma$ is even, and let $A = K[x, y, z]/(x^\alpha, y^\beta, z^\gamma)$. Then A fails to have the WLP if and only if*

$$p \left| M\left(\frac{-\alpha+\beta+\gamma}{2}, \frac{\alpha-\beta+\gamma}{2}, \frac{\alpha+\beta-\gamma}{2}\right)\right.$$

(If $a + b + c$ is odd, there is an analogous criterion but it is a little more complicated. We omit the description.)

Remark A.4. In the above Theorem we may assume that $\alpha \le \beta \le \gamma$. We note that the socle degree e of A is odd; $e = \alpha + \beta + \gamma - 3$. So U is a square matrix. Thanks to Theorem A.1, the WLP of A is determined by the injectivity of U. Li and Zanello proved that $|\det U| = \det\left(\binom{c}{\beta-c+i-j}\right)_{1\le i,j\le c}$, where $c = \frac{\alpha+\beta-\gamma}{2}$. It turns out that this determinant is equal to $M(\frac{-\alpha+\beta+\gamma}{2}, \frac{\alpha-\beta+\gamma}{2}, \frac{\alpha+\beta-\gamma}{2})$. This was proved in Krattenthaler [74, 2.17]. Visibly (1) implies (2).

Definition A.5. A plane partition of a positive integer n is a two-dimensional array $N = (n_{i,j})$ such that $n_{i,j} \ge n_{i,j+1} \ge 1$, $n_{i,j} \ge n_{i+1,j} \ge 1$ for all i, j and $\sum_{i,j} n_{i,j} = n$. We say that a plane partition $A = (a_{i,j})$ is contained in a box of size $a \times b \times c$ if $1 \le i \le a$, $1 \le j \le b$, $1 \le n_{i,j} \le c$.

The set of Young diagrams contained in a rectangle of size $a \times b$ may be considered as a lattice of ideals of two chains. In Chap. 1 this was denoted by $\mathscr{I}(a^b)$. It was shown that it is isomorphic to the lattice of ideals of two chains $\mathscr{I}(a^b) \cong \mathscr{I}(b^a) \cong J(C(a-1) \times C(b-1))$. Likewise, with a little contemplation, one sees that the set of plane partitions contained in a box of size $a \times b \times c$ is, with the containment order, isomorphic to the lattice of ideals of three chains

$$J(C(a-1) \times C(b-1) \times C(c-1)).$$

Theorem A.6. *The number of plane partitions contained in a box of size $a \times b \times c$ is equal to $M(a, b, c)$.*

This was discovered by MacMahon [86]. For a combinatorial proof see Krattenthaler [75].

C. Chen et al. [15] gave a bijective proof for this fact: $|\det U|$ counts the number of plane partitions in a box of size $a \times b \times c$. This is stated in the next theorem.

Theorem A.7 ([15]). *Let $R = \mathbb{Z}[x, y, z]/(x^{a+b}, y^{b+c}, z^{c+a})$ with a, b, c positive integers and set $m = a + b + c - 2$. Let U be the square matrix defining the multiplication map $\times(x + y + z) : R_m \to R_{m+1}$ in the monomial $\mathbb{Z}$-bases. Then the determinant of U is equal, up to sign, to the permanent of U and each nonzero term in its permanent corresponds to a plane partition in a box of size $a \times b \times c$. Therefore $|\det U| = M(a, b, c)$.*

As a corollary we have

Corollary A.8. *Let K be a field of characteristic $p > 0$, a, b, c positive integers and $A = K[x, y, z]/(x^{a+b}, y^{b+c}, z^{c+a})$. Then A fails to have the WLP if and only if $p|M(a, b, c)$.*

If $a = 1$, it gives us the following

Corollary A.9. *With the same K as above, let b, c be positive integers. Then p divides $M(1, b, c) = \binom{b+c}{b}$ if and only if $K[x, y, z]/(x^{b+1}, y^{b+c}, z^{c+1})$ fails to have the WLP.*

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Edited by J.-M. Morel, B. Teissier; P.K. Maini

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