List of Symbols

The items below are listed by order of appearance. The list is not meant to be exhaustive, but symbols that occur repeatedly and which are potentially nonstandard have been enclosed in the list.

$\overline{k}$	The algebraic closure of the field k, see (2.7)
Gal_k	The absolute Galois group of the field k, see (2.8)
$\overline{X}$	The base change $\overline{X} = X \times_k \bar{k}$ of X to $\bar{k}$, see (2.9)
$\pi_1(X, \bar{x})$	The étale fundamental group of X with base point $\bar{x}$, see Definition 16
$\pi_1(\overline{X}, \bar{x})$	The geometric fundamental group of X, see page xiii
$\pi_1(X/k)$	The fundamental exact sequence associated to the geometrically connected X/k, see Definition 19
s_a	The Diophantine section associated to the rational point a, see Definition 4
$[s]$	The conjugacy class of the section s, see Definition 7
$\mathscr{S}_{\pi_1(X/k)}$	The space of conjugacy classes of sections of $\pi_1(X/k)$; also denoted by $\mathscr{S}_{\pi_1(X/k,\bar{a})}$ when the base point $\bar{a} \in \overline{X}$ is emphasized, see page xiv and Definition 23
κ	The (profinite) Kummer map, see Definition 1
Gal_k^0	The kernel of the cyclotomic character $\mathrm{Gal}_k \to \hat{\mathbb{Z}}^*$, see Conjecture 6
$\mathscr{S}_{\pi_1(X/k)}(k')$	The space of conjugacy classes of sections of $\pi_1(X/k)$ defined over $\mathrm{Gal}_{k'}$ for an extension k'/k, see Definition 27
$\mathscr{S}_{\pi \to \Gamma}$	The space of sections of $1 \to N \to \pi \to \Gamma \to 1$ up to N-conjugacy, see Definition 7
$\mathscr{S}_{N \rtimes \Gamma}$	The pointed space of sections of $N \rtimes \Gamma \twoheadrightarrow \Gamma$ up to conjugation by N, with the canonical splitting as the special element, see Definition 7
$\mathrm{Tors}_\Gamma(N)$	The pointed set of Γ-equivariant right N-torsors, see page 4
$\delta(t, s)$	The difference cocycle of two sections s and t, see (1.2)

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$\mathrm{Ext}(\Gamma, G')$	The set of extensions $1 \to G' \to E \to \Gamma \to 1$ up to isomorphy, see Definition 9
$[\mathrm{Ext}(\Gamma, G')]_G$	The orbit space of $\mathrm{Ext}(\Gamma, G')$ by the G-action by pushout with conjugation where $G' \trianglelefteq G$ is a normal subgroup, see Definition 9
$Z_{\overline{\pi}}(s_0)$	The centraliser of (the image of) a section s_0 in $\overline{\pi}$, see Definition 11
$\mathsf{Rev}(X)$	The Galois category of finite étale covers of X, see (2.1)
$Y[\bar{a}]$	The fibre in $\bar{a}$ of the object Y, see (2.2)
$P_{\bar{a}}$	The path space pro-representing the fibre functor $\bar{a}$, see (2.3)
$\mathsf{Pro{-}Rev}(X)$	The pro-category associated to $\mathsf{Rev}(X)$, see (2.3)
G^{opp}	The opposite group to G with the same elements but composition reversed, see page 14
$\Pi_1(X)$	The fundamental groupoid of fibre functors of $\mathsf{Rev}(X)$, see Definition 16
$\pi_1(X; \bar{a}, \bar{b})$	The set of étale paths from $\bar{a}$ to $\bar{b}$, see Definition 16
k^{sep}	The separable closure of the field k, see (2.7)
$\pi_1(X/k, \bar{a})$	The fundamental exact sequence associated to the geometrically connected X/k with base point $\bar{a} \in \overline{X}$ emphasized, see Definition 19
Γ^{ab}	The abelianization of the profinite group Γ, see (3.1)
$\pi_1^{\mathrm{ab}}(X/k)$	The maximal geometrically abelian quotient extension of the fundamental group extension $\pi_1(X/k)$, see Definition 26
s^{ab}	The abelianization of the section s, see Definition 26
Alb_X	The Albanese variety of X/k, see (3.2) and (7.1)
$\alpha_X : X \to \mathrm{Alb}^1_X$	The Albanese torsor map of X/k, see (3.3) and (7.1)
$\mathrm{res}_{K/k}$	The restriction map $\mathrm{Gal}_K \to \mathrm{Gal}_k$ for a field extension K/k, see (3.4)
$s_K = s \otimes K$	The base change of the section s with respect to the field extension K/k, see Definition 27
π_s	The anabelian fibre in a section s with respect to a fibration, see Definition 30
$h^{-1}(s)$	The anabelian fibre in a sections s along a finite étale map $h : Y \to X$, see Definition 32
$\underline{M}$	The étale sheaf of sets on $\mathrm{Spec}(k)_{\text{ét}}$ associated to a finite Gal_k-set M, see Definition 32
$\mathrm{Ext}[G]$	The category of (continuous) extensions $1 \to N \to E \to G \to 1$ of a pro-finite group G, see Definition 37
$\mathrm{Ind}_H^G(-)$	The non-abelian induction, see page 34
$\mathrm{R}_{L\vert K}\, X$	The Weil restriction of scalars of the variety X/L, see (3.7)
$\mathrm{Sub}(\pi)$	The space of closed subgroups of π, see Section 4.1
$(\mathbb{N}, <)$	The set $\mathbb{N}$ as an ordered set with respect to $<$, see page 38
$Q_n(\overline{\pi})$	The characteristic quotient of $\overline{\pi}$ related to finite quotients of order $\leq n$, see (4.1)

$Q_n(\pi_1(X/k))$	The characteristic quotient extension of $\pi_1(X/k)$ related to finite quotients of $\pi_1(\overline{X})$ of order $\leq n$, see (4.2)
X_s	The decomposition tower of a section s of $\pi_1(X/k)$, see Definition 51
κ_f	The Kummer torsor associated to a unit f, see (5.1)
ev_s	The evaluation map of units in sections, see Definition 56 and Definition 57
$\mathbb{N}'$	The partially ordered set of all $n \in \mathbb{N}$ prime to the characteristic of the base field, ordered by divisibility, see (5.2)
$\widehat{k^*}$	The pro-$\mathbb{N}$ completion $\varprojlim_{n\in\mathbb{N}'} k^*/(k^*)^n$ of k^*, see (5.3)
$\widehat{\mathbb{G}}_{\mathrm{m},k}$	The pro-$\mathbb{N}$ completed sheaf $\mathbb{G}_{\mathrm{m}}$, see (5.4)
$\mathsf{Ab}_{\mathrm{ML}}^{\mathbb{N}'}$	The category of $\mathbb{N}'$-systems of abelian groups localised at Mittag–Leffler-zero objects, see page 48
$\mathrm{T}'(M)$	The prime to p Tate module of an abelian group M, see (5.5)
$\mathrm{cl}_s^{\mathrm{graph}}$	The cycle class of a section as defined via graphs, see (6.1)
cl_Z	The cohomological cycle class associated to a cycle Z, see Section 6.1
$\mathrm{cl}_s^{\mathrm{norm}}$	The cycle class of a section as defined by norm compatibility, see (6.3)
$\mathrm{cl}_s^{\mathrm{group}}$	The cycle class of a section as defined by a group extension, see (6.4)
$\mathrm{cl}_s^{\mathrm{dual}}$	The cycle class of a section as defined by duality, see (6.6)
cl_s	The cycle class of a section s, see Definition 62
$\mathrm{W}_{-1}\,\pi_1^{\mathrm{ab}}(\overline{U})$	The weight -1 quotient of $\pi_1^{\mathrm{ab}}(\overline{U})$, see Definition 68
$\mathrm{W}_{-1}\,\pi_1^{\mathrm{ab}}(U/k)$	The weight -1 quotient extension of $\pi_1(X/k)$, see Definition 68
NS_X	The Néron–Severi group scheme of X/k, see (7.2)
G^D	The Cartier dual of a finite flat group scheme G, see page 70
$G^{\text{ét}}$	The maximal étale quotient group scheme of a finite flat group scheme G, see page 70
δ_{kum}	The boundary map in the Galois cohomology of the Kummer sequence, see Corollary 71
$\pi_1^{\text{pro-}p}(X/k)$	The maximal geometrically pro-p quotient extension of the fundamental group extension $\pi_1(X/k)$, see (7.7)
$\pi_1^{\text{pro-}p}(\overline{X})$	The maximal pro-p quotient of $\pi_1(\overline{X})$, see (7.7)
$\pi_1^{(\text{pro-}p)}(X)$	The maximal geometrically pro-p quotient of the fundamental group $\pi_1(X)$, see (7.7)
$\rho_{X/k}$	The outer (pro-p) Galois representation associated to X/k, see (7.8)
sp	The specialisation map of fundamental groups, see (8.1)
$R_{\mathscr{X}/S}$	The cokernel of the geometric specialisation map, see (8.3)
$\mathrm{ram}(s)$	The ramification of a section, see Definition 82
$\mathscr{S}_{\pi_1(X/k)}^{\mathrm{nr}}$	The space of unramified sections, see Definition 83
sp_σ	The σ-specialisation map, see (8.6)

$\mathrm{ram}_s^{\mathrm{ab},\ell}$	The pro-ℓ abelianized ramification of a section, see (8.7)
k_v	The completion of an algebraic number field at the place v, see page 107
$\mathrm{index}(X)$	The index of the variety X, see Definition 113
$\mathrm{period}(X)$	The period of the variety X, see Definition 113
$\mathrm{Br}(X/k)$	The relative Brauer group, Definition 113
BS_A	The Brauer–Severi variety associated to the Azumaya algebra A, see (10.1)
$\psi_p(s)$	a $\mathbb{Q}_p$-linear form on the Lie algebra $\mathrm{Lie}(\mathrm{Pic}_X^0)$ induced by a section s, see (10.6)
$\mathrm{Br}^0(X/k)$	Pic^0-part of relative Brauer group, Definition 121
$\mathbb{A}_k$	The adele ring of a number field k, see page 120
$X(\mathbb{A}_k)_\bullet$	The space of modified adelic points of X, see (11.1)
$X(\mathbb{A}_k)_\bullet^{\mathrm{Br}}$	The Brauer kernel, see (11.3)
$\mathscr{S}_{\pi_1(X/k)}(\mathbb{A}_k)$	The space of adelic sections, see Definition 123
$\mathscr{S}_{\pi_1(X/k)}(\mathbb{A}_k)^{\mathrm{Br}}$	The Brauer kernel, see Theorem 127
$X(\mathbb{A}_k)_\bullet^h$	The descent obstruction set induced by a torsor h, see (11.6)
$X(\mathbb{A}_k)_\bullet^{\mathrm{descent}}$	The descent obstruction set, see (11.7)
$\mathscr{S}_{\pi_1(X/k)}(\mathbb{A}_k)^{\varphi}$	The descent obstruction posed by the torsor φ on adelic sections, see Definition 137
$\mathscr{S}_{\pi_1(X/k)}(\mathbb{A}_k)^{\text{f-descent}}$	The finite descent obstruction set on adelic sections, see Theorem 138
$\mathscr{S}_{\pi_1(X/k)}(\mathbb{A}_k)^{\text{cf-descent}}$	The constant finite descent obstruction set on adelic sections, see Definition 143
$\mathscr{S}_{\pi_1(X/k)}(\mathbb{A}_k)^{\text{ét-Br}}$	The étale Brauer–Manin obstruction to adelic sections, see Definition 149
$\mathrm{H}^2(k,(\overline{G},\rho))$	The non-abelian second cohomology set with coefficients in the k-kernel, see page 147
$\mathrm{H}^2_{\mathrm{nt}}(k,(\overline{G},\rho))$	The subset of $\mathrm{H}^2(k,(\overline{G},\rho))$ of neutral classes, see page 148
$\mathrm{H}^1(k,\mathscr{G})$	The non-abelian first cohomology set with coefficients in a k-gerbe $\mathscr{G}$, see (12.1)
$\mathrm{H}^2_{\mathrm{c}}\left(k,(\overline{G},\rho)\right)$	The non-abelian compactly supported second cohomology with coefficients in a k-kernel, see Definition 164
$\mathrm{H}^1_{\mathrm{c}}\left(k,\mathscr{G}\right)$	The non-abelian compactly supported second cohomology with coefficients in a k-gerbe, see Definition 167
$\mathrm{Div}(M)$	The maximal divisible subgroup of M, see (13.4)
$\mathrm{div}(M)$	The subgroup of divisible elements of an abelian group M, see (13.5)
G^{sc}	The universal finite étale (isogeny) cover of the semisimple algebraic group G by a simply connected semisimple algebraic group, see (13.7)
G_{lin}	The maximal connected linear algebraic subgroup of a connected algebraic group, see (13.8)
$R(G)$	The radical $R(G)$ of G_{lin}, see (13.8)
$R_{\mathrm{u}}(G)$	The unipotent radical $R_{\mathrm{u}}(G)$ of G_{lin}, see (13.8)

$\mathrm{Div}^0_{\overline{X},\overline{Y}}$	The group of divisors of degree 0 on $\overline{X}$ supported on $\overline{Y}$, see page 170
$X^{(d)}$	The dth symmetric product of X, see page 171
$\pi_1^{\mathrm{nilp}}(\overline{X})$	The maximal pro-nilpotent quotient of $\pi_1(\overline{X})$, see (14.1)
$\pi_1^{\mathrm{nilp}}(X/k)$	The maximal geometrically pro-nilpotent quotient extension of $\pi_1(X/k)$, see (14.2)
κ_{nilp}	The nilpotent Kummer map, see (14.3) and Definition 196
κ_p	The pro-p Kummer map, see (14.3)
$C_\bullet\Gamma$	The descending central filtration on the profinite group Γ, see Definition 194
$C_{\geq -n}\big(\pi_1(X/k)\big)$	The geometrically n-step nilpotent quotient extension of the fundamental group extension $\pi_1(X/k)$, see Definition 195
$C_{\geq -n}\big(\pi_1^{\text{pro-}\ell}(X/k)\big)$	The geometrically n-step pro-ℓ nilpotent quotient extension of $\pi_1(X/k)$, see Definition 195
κ^{ab}	The abelian Kummer map, see Definition 196
κ_n	The n-step nilpotent Kummer map, see Definition 196
κ_ℓ	The pro-ℓ Kummer map, see Definition 196
$\kappa_{\ell,n}$	The n-step nilpotent pro-ℓ Kummer map, see Definition 196
$\overline{\pi}$	an abbreviation for $\pi_1(\overline{X})$, see (14.4)
$\overline{\pi}^\ell$	an abbreviation for $\pi_1^{\text{pro-}\ell}(\overline{X})$, see (14.4)
$\mathrm{Lie}(\Gamma)$	The graded $\mathbb{Z}_\ell$-Lie algebra associated to the descending central filtration on a pro-ℓ group Γ, see (14.6)
$\mathfrak{p}$	The graded $\mathbb{Z}_\ell$-Lie algebra associated to $\pi_1^{\text{pro-}\ell}(\overline{X})$ for a smooth projective curve X/k, see (14.7)
$\mathfrak{p}_n$	The $-n$th graded piece of $\mathfrak{p}$, see page 180
$\mathfrak{p}_K$	The scalar extension to a K-Lie algebra, see (14.8)
$[\mathfrak{p}]$	The Poincaré series of $\mathfrak{p}_K$, see (14.9)
h	The size of the Picard group over a finite field, see 15.2
$\kappa_{\mathbb{R}}$	The version of the Kummer map over $\mathbb{R}$, see (16.1)
k^{cyc}	The maximal cyclotomic extensions $k(\mu_\infty)$ of an algebraic number field k, see page 217
X_y	The étale local scheme $\mathrm{Spec}(\mathcal{O}^{\mathrm{h}}_{X,y})$ of the henselisation of the local ring for a point $y \in X$, see (18.1)
U_y	The scheme of étale nearby points $X_y \times_X U$, see (18.2)
$\mathscr{S}^{\mathrm{cusp}}_{\pi_1(U/k)}$	The space of cuspidal sections, see Definition 248
s_v	The tangential section associated to a tangent vector v, see Proposition 249
$\mathrm{D}_{\tilde{y}\|y}$	The decomposition subgroup, see (18.6)
$\mathrm{I}_{\tilde{y}\|y}$	The inertia subgroup, see (18.7)
$W_{-2}(U)$	The anabelian weight filtration of U, see Definition 252
$\mathscr{S}_{\pi_1(k(X)/k)}$	The space of birational sections for X/k, see Definition 258
κ_{bir}	The birational Kummer map, see (18.10)
$\mathscr{S}^{\mathrm{bir}}_{\pi_1(X/k)}$	The space of birationally liftable sections, see Definition 266

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