Appendix A
Encoding Amplitude Information on Phase-Only Diffractive Optical Elements

When light fields are shaped holographically with DOEs, usually only amplitude or phase can be specified rather than the full complex wavefront because typical modulators cannot encode the complex amplitude $\hat{U} = |\hat{U}|e^{i\Psi}$ directly. In particular, it is often preferred to utilise phase-only modulators because they offer superior diffraction efficiency and hence utilisation of the available light. For many applications, the (real) amplitude information can be neglected. This, for example, is successfully done in HOT when only the argument of the complex superposition of multiple gratings and holographic lenses [cf. Eq. (7.2)] is taken into account while the amplitude is discarded. For the generation of advanced light fields with high fidelity, however, complex modulation is essential. For example, the mode purity of holographically generated LG beams [cf. Eq. (2.17)] is limited to approximately 0.85 when generated with phase-only DOEs (Ando et al. 2009). There are a couple of methods available that enable additional encoding of (real) amplitude information on phase-only DOEs (Kirk and Jones 1971; Davis et al. 1999; Kettunen et al. 1997; Davis et al. 2003; Arrizón et al. 2009), yielding—in this example—a mode purity very close to one (Ando et al. 2009).

The basic idea of encoding amplitude information on a phase-only modulator is simple. In general, a high frequency carrier grating is utilised that redistributes light between different orders of diffraction, resulting in locally varying intensities corresponding to $|\hat{U}|$ in one order. For example, we assume a desired amplitude and phase distribution as depicted in Fig. A.1a and b, respectively. The carrier signal in this example is a blazed diffraction grating (Fig. A.1c) that efficiently diffracts light into the +1st order of diffraction. When the contrast of the diffraction grating is reduced, i.e. it has a modulation of less than 2π, diffraction efficiency is also reduced and the light which is not diffracted into the $+1$st order remains in the 0th order. The concept thus is to reduce the diffraction efficiency in those areas of the DOE where the desired amplitude is low while maintaining the diffraction efficiency where high amplitude levels are desired as illustrated in Fig. A.1d. This modulated diffraction grating yields the desired amplitude distribution in the $+1$st order of diffraction. The desired phase distribution can be simply added to the

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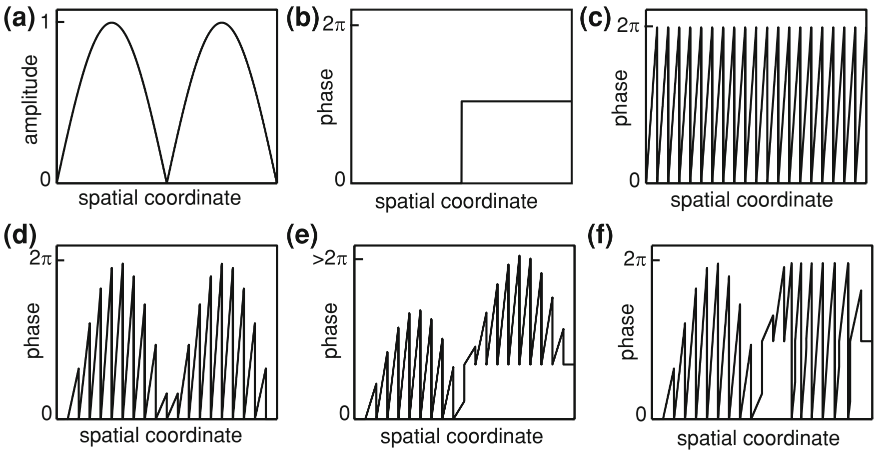

Fig. A.1 Basic principle of encoding complex amplitudes on phase-only DOEs. **a** and **b** show a sample (real) amplitude and a sample phase, respectively. The blazed grating in (**c**) reflects all light into the +1st order of diffraction. When the contrast of the blazed grating is locally modulated, so is the diffraction efficiency. By this means, the amplitude information is encoded on the blazed grating (**d**). The desired phase distribution can be be added (**e**) and the result, if necessary modulo 2π (**f**) will produce a wave front with the desired complex amplitude distribution in the +1st order

grating (cf. Fig. A.1e), resulting in a diffracting element that effectively modulates both, the amplitude and the phase of a light wave. Since typical phase modulators are only capable of introducing a phase retardation of one wavelength, i.e. $(0\ldots2\pi)$ radians, the phase shift to be applied can be wrapped accordingly if necessary (cf. Fig. A.1f).

The illustrated encoding method for the realisation of complex valued transmission or reflection functions can be understood in terms of a Fourier series representation, assuming that the complex function $U(u)\exp(\mathrm{i}\Psi(u))$ is supposed to be encoded for the spatial coordinate u. One concrete implementation for the encoding of this complex function on phase-only modulators is the choice of a phase distribution (Davis et al. 1999)

$$\Phi(u) = \exp(\mathrm{i}U(u)\Psi(u)), \tag{A.1}$$

i.e. a multiplication of the amplitude and the phase. This phase distribution can be represented by a Fourier series as (Davis et al. 1999)

$$\Phi(u) = \sum_{n=-\infty}^{\infty} \Phi_n(u)\exp(\mathrm{i}n\Psi(u)), \tag{A.2}$$

with the coefficients

$$\Phi_n(u) = \exp(\mathrm{i}(n - U(u))\pi)\frac{\sin(\pi(n - U(u)))}{\pi(n - U(u))}. \tag{A.3}$$

For the first diffraction order, $m = 1$, the phase term in $\Phi(u)$ reproduces the desired phase $\Psi(u)$ and the coefficient $\Phi_n(u)$ approximates the desired amplitude $U(u)$ (Davis et al. 1999). The phase term in Eq. (A.3) and the sinc function introduce errors in the reproduced amplitude. These can be compensated by using a distorted amplitude $U'(u)$ that can be calculated from the desired $U(u)$ for a given order of, say $n = 1$ here, such that it annuls the distortions introduced by the nonlinear relation in Eq. (A.3) (Davis et al. 1999). In order to spatially separate the different orders of diffraction, a linear phase term is added to the phase distribution, i.e. $\Psi(u) \to \Psi(u) + \frac{2\pi}{\Lambda} u$. This introduces, after wrapping the phases, an underlying blazed grating with the period Λ which spatially separates the desired image from the 0th order of diffraction. Alternatively, a quadratic phase could be added that separates the different diffraction orders along the beam axis (cf. Sect. 7.1.1).

There are various different approaches to encode complex modulation on a phase-only modulator. They differ in the choice of the carrier signal (Kirk and Jones 1971; Arrizón et al. 2009), in the choice of the encoding function (Eq. (A.1)) (Ando et al. 2009; Arrizón et al. 2007), and in the employed order of diffraction (Arrizón et al. 2009). Furthermore, the quantisation of the phase levels needs to be taken into account. While complex amplitude encoding can be an advantage even with two available phase levels (Davis et al. 2003), all modulators employed for the experiments described in this thesis were capable of applying $N = 256$ phase levels. Since the intensity of resulting noise is proportional to $1/N^2$, we can safely neglect additional errors introduced by the phase quantisation.

Appendix B
Mathematical Functions

B.1 Details on the Calculation of Mathieu Functions

The Mathieu Eq. (5.10) has even $(\mathrm{ce}_m(u,q))$ and odd $(\mathrm{se}_m(u,q))$ solutions (Whittaker 1912; Arscott 1964). The solutions are dependent on the parity of the order m so that we expect four equations:

$$\mathrm{ce}_{2n}(\eta,q)=\sum_{r=0}^{\infty}A_{2r}(q)\cos(2r\eta) \tag{B.1}$$

$$\mathrm{ce}_{2n+1}(\eta,q)=\sum_{r=0}^{\infty}A_{2r+1}(q)\cos((2r+1)\eta) \tag{B.2}$$

$$\mathrm{se}_{2n+1}(\eta,q)=\sum_{r=0}^{\infty}B_{2r+1}(q)\sin((2r+1)\eta) \tag{B.3}$$

$$\mathrm{se}_{2n+2}(\eta,q)=\sum_{r=0}^{\infty}B_{2r+2}(q)\sin((2r+2)\eta), \tag{B.4}$$

with $n=0,1,2,\ldots$ The expansion coefficients $A_i,\ B_i$ of this Fourier series can be obtained as elements of the eigenvectors $A,\ B$ of the Mathieu equation (Gutierrez-Vega et al. 2003).

While the ordinary Mathieu functions $\mathrm{ce}_m(u,q)$ and $\mathrm{se}_m(u,q)$ can be directly identified as the solutions of the angular Mathieu equation, solutions of the radial Mathieu equation are obtained with the substitution $\eta=\mathrm{i}\xi$. Hence, the modified Mathieu functions are given as:

$$\mathrm{Je}_{2n}(\xi,q)=\mathrm{ce}_{2n}(\mathrm{i}\xi,q)=\sum_{r=0}^{\infty}A_{2r}(q)\cosh(2r\xi) \tag{B.5}$$

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$$\mathrm{Je}_{2n+1}(\xi,q) = \mathrm{ce}_{2n+1}(\mathrm{i}\xi,q) = \sum_{r=0}^{\infty} A_{2r+1}(q)\cosh((2r+1)\xi) \tag{B.6}$$

$$\mathrm{Jo}_{2n+1}(\xi,q) = \mathrm{se}_{2n+1}(\mathrm{i}\xi,q) = \sum_{r=0}^{\infty} B_{2r+1}(q)\sinh((2r+1)\xi) \tag{B.7}$$

$$\mathrm{Jo}_{2n+2}(\xi,q) = \mathrm{se}_{2n+2}(\mathrm{i}\xi,q) = \sum_{r=0}^{\infty} B_{2r+2}(q)\sinh((2r+2)\xi), \tag{B.8}$$

where $\mathrm{Je}_m(\xi,q)$ and $\mathrm{Jo}_m(\xi,q)$ are the even and odd modified Mathieu functions of order m.

The functions are calculated numerically in MATLAB,[1] following References (Stamnes and Spjelkavik 1995) and (Cojocaru 2008). For a given value of q there are four infinite sequences of eigenvalues a, corresponding to the respective four solutions. Substituting Eqs. (B.1–B.4) into the Mathieu Eq. (5.10) yields recurrence relations for the expansion coefficients A_i, B_i. These relations can be written in matrix form (Stamnes and Spjelkavik 1995) and the eigenvalue problem is solved in MATLAB (Cojocaru 2008). In order to limit the computational expense to a reasonable value, the number of expansion coefficients in Eqs. (B.1–B.4) and (B.5–B.8) is set to $r_{\max} = 25$, following convergence considerations (Cojocaru 2008).

B.2 Details on the Calculation of Ince Polynomials

The Ince Eq. (6.6) has even $(C_p^m(\eta,\epsilon))$ and odd $(S_p^m(\eta,\epsilon))$ solutions which additionally depend on the parity of degree m and order p.[2] In contrast to the Mathieu functions, the four solutions are finite sums and called Ince polynomials (Arscott 1964; Bandres and Gutierrez-Vega 2004):

$$C_{2n}^{2k}(\eta,\epsilon) = \sum_{r=0}^{n} A_r(\epsilon)\cos(2r\eta) \qquad k = 0,\ldots n \tag{B.9}$$

$$C_{2n+1}^{2k+1}(\eta,\epsilon) = \sum_{r=0}^{n} A_r(\epsilon)\cos((2r+1)\eta) \qquad k = 0,\ldots n \tag{B.10}$$

$$S_{2n}^{2k}(\eta,\epsilon) = \sum_{r=1}^{n} B_r(\epsilon)\sin(2r\eta) \qquad k = 1,\ldots n \tag{B.11}$$

[1] MathWorks MATLAB website–http://www.mathworks.de, Sept 2011.

[2] Recall that degree m and order p have the same parity.

$$S_{2n+1}^{2k+1}(\eta, \epsilon) = \sum_{r=0}^{n} B_r(\epsilon) \sin((2r+1)\eta) \qquad k = 0, \ldots n, \tag{B.12}$$

with $n = 0, 1, 2, \ldots$ Analogous to the Mathieu solutions, the expansion coefficients A_r, B_r are obtained from an analysis of the eigenvalues a. Therefore, Eqs. (B.9–B.12) are substituted into the Ince Eq. (6.6), yielding recurrence relations for the coefficients. From these relations, finite tridiagonal matrices are constructed (Bandres and Gutierrez-Vega 2004) and the eigenvalue problem is solved numerically with MATLAB. For a given order p, a finite ensemble of eigenvalues is obtained. The elements of the eigenvector A or B corresponding to the mth eigenvalue in an ordered list can be identified as the expansion coefficients A_r or B_r, respectively (Bandres and Gutierrez-Vega 2004).

While these Ince polynomial $C_p^m(\eta, \epsilon)$ and $S_p^m(\eta, \epsilon)$ can be directly identified as the solutions of the ("angular") Ince Eq. (6.6), solutions of the "radial" Ince Eq. (6.5) are obtained with the substitution $\eta = \mathrm{i}\xi$ (Bandres and Gutierrez-Vega 2004):

$$C_{2n}^{2k}(\mathrm{i}\xi, \epsilon) = \sum_{r=0}^{n} A_r(\epsilon) \cosh(2r\xi) \qquad k = 0, \ldots n \tag{B.13}$$

$$C_{2n+1}^{2k+1}(\mathrm{i}\xi, \epsilon) = \sum_{r=0}^{n} A_r(\epsilon) \cosh((2r+1)\xi) \qquad k = 0, \ldots n \tag{B.14}$$

$$S_{2n}^{2k}(\mathrm{i}\xi, \epsilon) = \sum_{r=1}^{n} B_r(\epsilon) \sinh(2r\xi) \qquad k = 1, \ldots n \tag{B.15}$$

$$S_{2n+1}^{2k+1}(\mathrm{i}\xi, \epsilon) = \sum_{r=0}^{n} B_r(\epsilon) \sinh((2r+1)\xi) \qquad k = 0, \ldots n. \tag{B.16}$$

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