Epilogue: The Anthropic Principle

> *This World, he said, is your commandment and in your bowels is written.*
>
> O. Elytis

Abstract Do the Laws of physics and of the values of the Universal physical constants fit the phenomenon of life?

Summary The biological fitness of the laws of Nature and the biological fitness of the values of the universal physical constants constitute a part of one version of the anthropic principle. A stronger version claims that the statement ‘Life exists’ is not only the shortest but the most comprehensive statement about the World. Most versions of the anthropic principle include also an ‘explanation’ of either the fitness or the assumed necessity of the physical laws and the values of the physical constants for the phenomenon of life as we know it on Earth.

In our short journey over the structures of the World we bypassed the most interesting ones, namely the living structures. The reason was that living structures do not seem to fit with what was the purpose of this booklet: *To derive some select but important quantitative (or semi-quantitative) results regarding structures of matter in a simple way starting from a few established general laws of Nature and the values of the universal physical constants.* The out of thermodynamic equilibrium living matter seems to be too complex and diverse to fit within this limited purpose and thus allow a simple derivation of quantitative results from a few fundamental laws of physics.

However, the opposite direction, from the fact that “life exists” to the “laws of nature and to the values of the physical constants” is not without some merit [1.1, E1]. It must be admitted that this subject known under the generic name **anthropic principle** is rather controversial, because, on the one hand it is ideologically loaded, and on the other, it does not usually provide concrete quantitative conclusions. Nevertheless, in this short epilogue we shall mention a few points showing that one can deduce some fundamental laws of Nature or at least demonstrate their necessity from the fact that life exists. In what follows we shall refer mostly to the system Earth/Sun where carbon based life exists. However,

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most of our arguments are applicable to any other planet/star or satellite/star system sustaining earth-like life.

The admirably precise reproducibility from generation to generation of living structures strongly suggests that matter is “digital” not “analog”, meaning that matter is of discrete not of continuous character. It is very difficult to imagine a continuous complex medium in contact with an ever changing environment to emerge again and again with exactly the same form and the same features. For this impressive phenomenon to occur, the huge amount of information stored in the “central bank” of the DNA molecule has to be kept intact. This in turn requires the double discretization shown in Fig. 3.1 (d or d′) because, otherwise, the DNA molecule would undergo a continuous change and it would never be the same. In other words, from the precise reproducibility of the living structures one is led to the discretization along both the vertical and the horizontal axis of Fig. 3.1(d or d′) The discretization along the vertical axis is consistent with the atomic idea. Similarly, the discretization along the horizontal axis is consistent with the wave-particle duality [E1]. Whether one can reverse the argument and deduce from the double discretization the atomic structure of matter (and forces) as well as the particle/wave duality is not clear.

The existence of an extremely weak, attractive, long range force, such as gravity, seems to be necessary in order to keep the atmosphere, the lakes, the oceans, and the living matter on the surface of the Earth; as well as to keep the Earth going around the Sun and drawing from it both the energy and the information required for the emergence and maintenance of life. This leads to the necessity of another force, the electromagnetic one, which has a dual responsibility: To transmit the energy and the information from the Sun to the Earth; and to bring and keep together all matter including the one from which the living structures consist. The strength of the E/M force has to be enormous in comparison with gravitational one. This becomes apparent by considering what the size of a single-cell organism would be, if it would be held together by gravitational forces instead of the electromagnetic ones: its diameter would be an absurd size of 10^{21} m, instead of 10^{-5} m which is its actual size! Moreover, the E/M force by being both attractive and repulsive manages to be both long range in transmitting energy and information from the Sun and short range in bringing and keeping together the constituents of living matter.

Life on Earth requires the continuous flow of energy from the Sun over a long period (of the order of a few billion years). Such a supply of energy cannot be chemical, because it would be exhausted in a period of thousands of years. There must be another source of energy about a million times larger than the chemical one. Assuming that the available energy is of the same order of magnitude as the kinetic energy and taking into account the Heisenberg principle, we conclude that the required supply of energy E per particle must come from a region of size r and from particles of mass m such that $E/E_{chem} \approx \left(m_e a_B^2/m\, r^2\right) \approx 10^6$. The nuclear size of $r = 1$ fm and the proton mass of 1836 m_e produces the necessary boost of 1.5×10^6. Hence, it is clear that the nucleus is the source of the energy supply in the Sun (and in any other active star). This implies that nuclei are composite particles held together by a third type of force- the strong one capable to overcome the kinetic energy of the nucleons and the Coulomb repulsion among the protons. Finally, a fourth interaction, the weak one, is necessary in order to allow the transformation from proton to neutron and vice versa. Without this, the fusion of two protons to a deuteron (with the emission of a positron and a neutrino) would not be possible, neither would be possible the eventual fusion of four protons to a helium nuclei, the main reaction providing the energy of the stars.

Let us consider now the values of some universal physical constants. The mass of the electron is less than the mass difference between a neutron and a proton. This allows the neutron, if free, to decay to a proton, an electron, and an antineutrino. If this were not the case, electrons and protons would combine to free neutrons in the early Universe leaving very few protons to act as fuel in the stars. Thus, a neutron has to be heavier than the sum of an electron and a proton but not much heavier, because in such a case very few neutrons would be produced during the baryogenesis or within the nuclei [see (7.2)]. As a result of the greater percentage of protons heavier elements necessary for life would not be stable. Elements heavier than He and up to Fe are made by fusion within active stars. This stellar nucleosynthesis passes through a bottle-neck to reach the carbon nucleus. The reason is that the Be^8 nucleus consisting of four protons and four neutrons is extremely metastable with a life-time of the order of 10^{-15} s, because it is energetically favorable to break into two He nuclei. Thus it is extremely improbable for a third He atom to be incorporated in the Be^8 to produce the crucial for life carbon atom, unless a resonance effect is present. Indeed, such an effect occurs in the sense that the energy of Be^8 plus the energy of the third He nucleus coincides with an excited energy level of carbon at 7.65 MeV above its ground state. This coincidence greatly enhances the probability of the incorporation of the third He nucleus and makes the carbon nucleus possible. It is worth mentioning that Fred Hoyle *predicted* the occurrence of such an excited level in the carbon nucleus before it was found experimentally.

The existence of life implies that the dimensionless gravitational constant must have a value close to the measured one. Indeed, it has been estimated that about 10 billion years are needed for life to emerge in the Universe, since life is the final stage and the crowning of the matter dominated epoch. On the other hand, life ought to appear and evolve before the dark energy dominates, since the latter tends to disolve the structures of matter. These two requirements imply that $\sqrt{g}\,t \approx 1$ for $t \approx 10^{10}$ ys [see (13.13)]. This in turn means that $a_G \approx 4.7 \times 10^{-39}$ i.e. about 20% smaller than its measured value. (The estimated value of $\rho_{de} = \varepsilon_{de}/c^2 \approx 0.721\times 1.034 \times 10^{-26}\mathrm{kg/m}^3$ was used in obtaining this value of a_G).

As it was mentioned before, supernova explosions, in particular core-collapse type II supernova explosions (see below), are important for the creation and the recycling of the elements in general and for the emergence of life in particular. The heavier than iron elements are created during such explosions. Moreover, all elements are dispersed as a result of supernovae very efficiently in the interstellar medium to be recycled for the production of the next generation of stars, of planetary systems and, eventually, of humans. In rough terms, a Type II supernova explosion occurs as follows: In a big star upon the exhaustion of its nuclear fuel, its central part collapses to a core so dense that even neutrinos are trapped in it. This superdense core attracts and accelerates the outer part of the star, which hits the core at high speed. This internal collision forces the outer part to rebound and at the same time releases the neutrinos which push the outer part with explosive force leading to the supernova II. For the neutrino flux to be capable of catapulting the outer part of the star against the gravitational attraction, the following relation connecting the weak dimensionless coupling constant a_W with the gravitational one a_G, must be satisfied:

$$a_W^4 \approx a_G \left(m_p / m_e \right)^6 \quad \Rightarrow \quad a_W \approx 2.18 \times 10^{-5}$$

The resulting value of a_W is in reasonable agreement with its theoretical and measured value.

We shall conclude this epilogue by drawing attention to the mystery of the cosmological constant Λ, or equivalently the density of the dark energy, ε_{de}. If we accept the established value of the gravitational constant, G, the measured value of Λ has the right order of magnitude for the transition from matter domination to dark energy domination to occur at $t \approx 10$ billion years, i.e., when the time was ripe for life to emerge. This looks like an extremely fine tuning of the value of Λ, in view of theoretical considerations producing values of Λ 10^{120} larger than the actual one!

Appendix I: Oscillations and Waves

Oscillations occur locally in systems where their energy can change back and forth from the form of potential energy to the form of kinetic energy. A typical mechanical example is the pendulum shown in Fig. I (a). A typical electrical example is the LC circuit shown in Fig. I (b); in this case the potential energy is equal to $\frac{1}{2}Q^2/C$, while the kinetic-type energy is $\frac{1}{2}LI^2/c^2$ (which includes both the systematic kinetic energy $\frac{1}{2}N_e m_e \upsilon^2$ of the electrons and the magnetic energy associated with the current $I = (N_e/\ell)\, e\, \upsilon$). C is the capacitance, L is the self-inductance $I = dQ/dt$, and ℓ is the overall length of the wire making up the circuit. In cases of macroscopic oscillations, as those in Fig. I(a) and (b) macroscopic kinetic energy is transferred through collisions with microscopic particles to the internal energy of both the system and the environment. This process is described macroscopically as the action of frictional “forces”, which eventually lead to the “loss” of the macroscopic oscillating energy and the eventual termination of the macroscopic oscillation. If there is no transfer of energy from the oscillating system, then conservation of energy, $E_t = E_K + E_P$ implies that $E_{KM} - E_{Km} = E_{PM} - E_{Pm}$, where the subscripts M and m indicate maximum and minimum, respectively. This observation allows us to obtain the angular frequency $\omega = 2\pi f$ (f is the frequency) of the oscillation. E.g., in the case of Fig. I(b) by setting $LI_M^2/c^2 = Q_M^2/C$, and taking into account that $I^2 = \omega^2 Q^2$, we obtain $\omega^2 = c^2/LC$. (See also Sect. 5.4, p. 39.)

Waves are oscillations which migrate from local subsystems to neighboring subsystems within an extended medium (and, consequently, spread and are delocalized). The medium can be discrete, as the coupled pendulums shown in Fig. I(c), or continuous, as in the case of sea-waves shown schematically in Fig. I(d). A wave in addition to the angular frequency ω of the propagating oscillation is usually characterized by its wavelength λ (or equivalently by its wavenumber $k \equiv 2\pi/\lambda$) and its velocity υ of propagation. The three quantities ω, k, υ are related as follows:

$$\omega = \upsilon k \tag{I.1}$$

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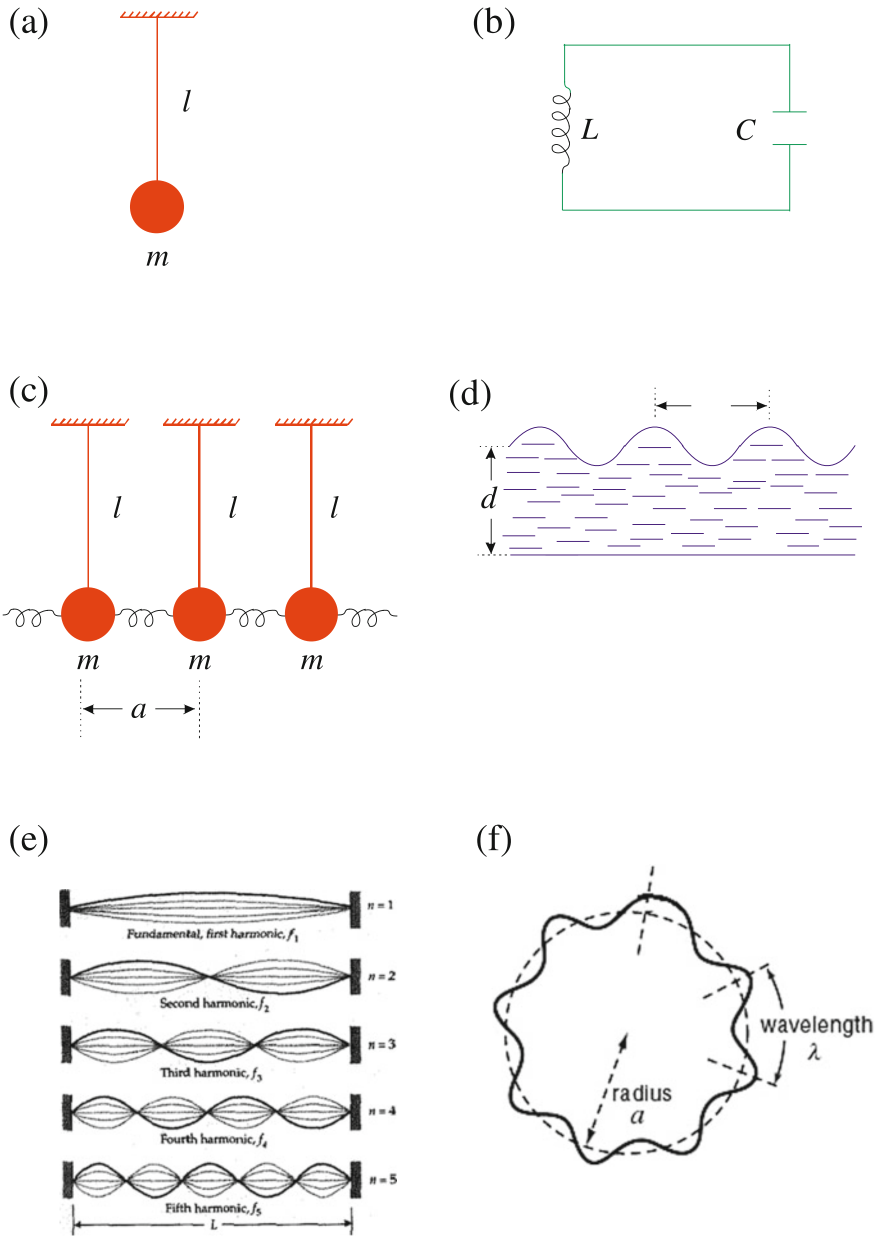

Fig. I **a** A pendulum. **b** An LC circuit. **c** A medium of coupled pendulums. **d** Sea waves. **e** Standing or stationary waves in a string of fixed ends. **f** Standing or stationary wave around a circle

The velocity υ depends on the properties of the medium and possibly of the wavenumber k. (See also Sect. 5.4, p. 39.)

Waves confined in a finite region of space have to satisfy certain boundary conditions, which restrict the allowed values of the angular frequency to a discrete set, ω_o, ω_1, ω_2, … (see Fig. 3.1, p. 23). To each value ω_i correspond one or more waves, called stationary (or standing) waves which, having spread over the available space, are not traveling anymore. Examples of standing waves are the ones appearing in a string of fixed ends [see Fig. I(e)] on the ones going around in a circle [see Fig. I(f)].

Selected Problems

4.1 Taking into account (4.12), plot schematically the Gibbs free energy as a function of temperature under constant pressure for the three phases of matter. Then, obtain the phase diagram in the plane T, P.

4.2 Prove that $\Omega = -PV$.

5.1 Estimate the drag force on an object moving with velocity υ within a fluid.

5.2 Obtain a formula for the viscosity of a liquid. Dimensions of viscosity are pressure× time. Estimate the viscosity of water at $T \approx 295\,K$.

5.3 Estimate the surface-tension coefficient of water. See Sect 7.1.

5.4 Obtain the so-called Poiseuille's formula giving the volume of running fluid passing per second through any cross-section of a pipe.

5.5 High frequency current density flowing in a metallic wire of conductivity σ is confined in a region near the surface of depth δ. Obtain δ, called skin-depth.

5.6 Estimate the value λ_m at which the maximum of the black body radiation plotted as a function of the wavelength appears. Similarly, for ω_m in the I_ω versus ω curve. Is $\omega_m = 2\pi/\lambda_m$? If not, why?

5.7 Estimate the life-time of a classical model of a hydrogen atom. If there were no radiation, the electron will follow a circular orbit of radius a_B.

5.8 Estimate the lifting force on the wings of a plane. Assume that the wings are rectangular flat rigid metallic sheets.

6.1 Consider the baryons $\Delta^{++},\ \Sigma^{o},\ \Xi^{-}$. What is their composition? What is the Feynman diagram describing their decay? Can you estimate their life-time?

6.2 The same for the mesons $\pi^{\mathrm{o}},\ \pi^{+},\ \pi^{-},\ \mathrm{K}^{\mathrm{o}},\ \bar{\mathrm{K}}^{\mathrm{o}},\ \mathrm{K}^{+},\ \mathrm{J}/\psi$.

7.1 The composition of natural uranium is 99.3% U-238 and 0.7% U-235. Their half-lifes are 4.51×10^{9}s and 7.1×10^{8} s respectively. Obtain limits for the age of our planetary system and the age of the Universe.

7.2 The distribution of the mass number of the fragments of the neutron induced fission of U-235 in a nuclear reactor exhibits a double peak at $A = 92$ and $A = 140$. (Eisberg-Resnick, *Quantum physics*.) On the contrary, the fragments of a fission bomb exhibit a single peak at $A \approx 116$. Any explanation?

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8.1 Estimate the first, the second, and the third ionization potential for the atom of Li. (See Fig. 8.2).

9.1 The energy E vs. distance d between two argon atoms is $E = 4\varepsilon\left[(\sigma/d)^{12} - (\sigma/d)^{6}\right]$, where $\varepsilon = 10.4\,\text{meV}$ and $\sigma = 3.4\,\text{A}$. Calculate (a) the bond length (b) the oscillation frequency (c) the dissociation energy (d) the fluctuation of the bond length. Will the molecule survive at room temperature?

9.2 Identify the four eigenoscillations of CO_2. Classify them according to the size of their frequency. Which one is responsible for the greenhouse effect?

9.3 Determine the stereochemistry of C_2H_2, C_2H_4, C_2H_6, C_4H_6, C_4H_{10}.

9.4 For the NaCl molecule the relevant atomic levels and the bond length are, $\varepsilon_{\text{Na}} \approx -4.95\,\text{eV}$, $\varepsilon_{\text{Cl}} \approx -13.78\,\text{eV}$, $d = 2.36\,\text{A}$. Determine the bonding and antibonding molecular levels, the polarity index and the charge transfer from Na to Cl. The ionization potential of Na is 5.14 eV and the chemical affinity of Cl is 3.61 eV. What is the dissociation energy of NaCl?

9.5 In analogy with (9.8) the equations satisfied by the 6 p_z orbitals in benzene are $(\varepsilon - \varepsilon_v)c_n^{(v)} + V_2 c_{n-1}^{(v)} + V_2 c_{n+1}^{(v)} = 0$, $n = 1, \ldots, 6$; $c_7^{(v)} \equiv c_1^{(v)}$. Show that Eqs. 9.15 satisfy these equations with $\varepsilon_v = \varepsilon_p + 2V_{2,zz}\cos\varphi_v$. Draw the levels ε_v for the 6 values of v.

10.1 The low-temperature specific heat of a metal is of the form $C = a_1 T + a_2 T^3$, where the first term is due to electronic excitations and the second to vibrational ones. Using the principles of Pauli and Schrödinger explain this behavior.

10.2 Make a rough estimate of the melting temperature of a solid.

10.3 In a diagram show the relevant atomic levels, the hybrid level, the bonding and antibonding molecular levels of Si, as well as its valence and conduction bands. What is the value of the energy gap in Si?

10.4 The same for the III–V semiconductor GaAs. (see Table 4; $d = 2.45$ A).

10.5 What is the value of $\bar{r}$ for liquid water? What is the velocity of sound in water (keep in mind that the hydrogen bond is 3 to 4 times weaker than a typical metallic bond)? What is the value of the bulk modulus of water?

10.6 Consider pure crystalline Si. Replace one Si atom by a P atom. Four of the valence electrons of P will participate in the four tetrahedral bonds. The fifth one will be detached from the immediate vicinity of the parent atom, but will be trapped around it at a distance r much larger than a_B. Taking into account that the dielectric constant of Si is 13.1 and that the effective electron mass is $0.32m_e$, find the distance r and the corresponding binding energy. If one atom of Si in every million is replaced by a P atom, what would be the effect on the resistivity at room temperature?

11.6 Mars's year is longer than Earth's by a factor of 1.88. What is the temperature at the surface of Mars?

11.7 The period of Moon around the Earth is 27.32 days, while the period of full moon is 29.5 days. Provide a quantitative explanation for this difference.

11.8 The period of tides is about 12 hours. Why? When is the phenomenon of tide stronger? At full-moon? At new-moon? At half-moon? Explain.

12.1 What would be the radius and the entropy of Earth, if it would become a black hole?

12.2 For an active star in the main sequence (see [11.2] or [12.1]) we have the following approximate formulae: Pressure at their center, $P_c \approx 2\rho GM/R$. Temperature at their center, $T_c \approx P_c m/k_B \rho$. Radius, $R \propto M^a;\ a \approx \frac{1}{2}$. Surface temperature, $T_S \propto M^{\beta};\ \beta \approx \frac{1}{2}$. Luminosity, $L \propto R^2 T_S^4 \propto M^{2a+4\beta};\ 2a+4\beta \approx 4$ for $0.5M_S \leq M \leq 2M_S$ and 3.5 for $2M_S \leq M \leq 20M_S$. Life-time $t \approx M/L$. Can you justify these formulae. Apply them to our Sun. Do they work?.

12.1 What was the pressure of baryons, of photons, and of neutrinos at $t \approx 380\,\text{kyr}$?

13.2 Consider a photon gas in thermodynamic equilibrium with electron/positron pairs. The temperature is much higher than $m_e c^2/k_B$ and the lepton number is negligible. The energy of each species is: $E_{ph} = 2\sum_k \varepsilon_k b_k$, $E_e \approx E_{e^+} = 2\sum_k \varepsilon_k$, $f_k \varepsilon_k = \hbar c\, k$, $b_k = [\exp(\varepsilon_k/k_B T) - 1]^{-1}$, $f_k = [\exp(\varepsilon_k/k_B T) + 1]^{-1}$. Justify the above formulae. Taking into account that $\sum_k \rightarrow (V/2\pi)^3 \int d^3k$ show that $E_e = \frac{7}{8} E_{ph}$.

Bibliography

P1 S.W. Hawking, L. Mlodinow, *The grand Design* (Bantam books, NY, 2010)
1.1 P. Morrison, P. Morrison, *Powers of Ten* (Scientific American Books, NY, 1982)
2.1 M. Veltman, *Facts and Mysteries in Elementary particle physics* (World Scientific, New Jersey, 2003)
3.1 R. Feynman, R. Leighton, M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, 1964)
3.2 P. Ghose, *Testing Quantum Mechanics on New Ground* (Gambridge University Press, Cambridge, 1999)
3.3 R. Griffiths, *Consistent Quantum Theory* (Gambridge University Press, Cambridge, 2002)
3.4 H. Paul, *Introduction to Quantum Theory* (Gambridge University Press, Cambridge, 2008)
3.5 A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic Publishers, Dordrecht, 1995)
3.6 A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47**, 777 (1935)
3.7 A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. **49**, (1982)
3.8 R. Shankar, *Principles of Quantum Mechanics*, 2nd edn. (Springer, NY, 1994)
4.1 E. Fermi, *Thermodynamics* (Dover Publications, NY, 1956)
4.2 L. Landau, E. Lifshitz, *Statistical Physics, Part 1* (Pergamon Press, Oxford, 1980)
6.1 D. Perkins, *Introduction to High Energy Physics* (Addison-Wesley, Reading, 1987)
6.2 W. Cottingham, D. Greenwood, *An Introduction to the Standard Model*, 2nd edn. (Cambridge University Press, Cambridge, 2007)
6.3 F. Wilczek, *The Lightness of Being: Mass, Ether, and the Unification of Forces* (Basic Books, New York, 2008)
7.1 R. Leighton, *Principles of Modern Physics* (McGraw-Hill, NY, 1959)
7.2 A. Das, T. Ferbel, Introduction to nuclear and particle physics
9.1 J. McMurry, *Organic Chemistry* (Brooks/Cole, Belmont, MA, 1996)
9.2 W. Harrison, *Electronic structure and the poperties of solids* (Dover Publications, NY, 1989)
10.1 E. Economou, *The physics of solids, essentials and beyond* (Springer-Verlag, Heidelberg, 2010)
11.1 E. Cohen, D. Lide, G. Trigg (eds.), *AIP physics desk reference* (Springer-Verlag, NY, 2003)
11.2 F. Shu, *The physical universe: an introduction to astronomy* (University Science Books, Mill Valley, CA, 1982)
12.1 B. Caroll, D. Ostlie, *An introduction to modern astrophysics* (Addison-Wesley, Reading, 2007)
12.2 S. Lloyd, Y.J. Ng, Scientific American, Nov. 2004, p. 53; J. D. Bekenstein, Scientific American, **289**, 58, Aug. 2003

E. N. Economou, *A Short Journey from Quarks to the Universe*,
SpringerBriefs in Physics, 1, DOI: 10.1007/978-3-642-20089-2,

13.1 S. Weinberg, *The first three minutes* (Fontana Paperbacks, London, 1983)
E1. E. Schrödinger, *What is life?* (Gambridge University Press, Cambridge, 1967)

The Castoriadis quote is from the Castoriadis and Evangelopoulos book, *Philosophy and Science* (Editions Eurasia, Athens, 2010)

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Table 1 Physical constants. Numbers in parentheses give the standard deviation in the last two digits

Quantity	Symbol	Value (year 2008)	Units
Planck constant over 2π	$\hbar = h/2\pi$	$1.054\,571\,628\,(53) \times 10^{-34}$	$\mathrm{J \cdot s}$
	$\hbar$	$6.582\ 118\ 99(16) \times 10^{-16}$	$\mathrm{eV \cdot s}$
Velocity of light	c	299 792 458	$\mathrm{m \cdot s^{-1}}$
Gravitational constant	G	$6.674\ 28(67) \times 10^{-11}$	$\mathrm{m^3 kg^{-1} s^{-2}}$
Proton charge	e	$1.602\ 176\ 487(40) \times 10^{-19}$	C
Electron mass	m_e or m	$9.109\ 382\ 15(45) \times 10^{-31}$	kg
Proton mass	m_p	$1.672\ 621\ 637(83) \times 10^{-27}$	kg
Neutron mass	m_n	$1.674\ 927\ 211(84) \times 10^{-27}$	kg
Atomic mass constant $\frac{1}{12}m(C^{12})$	m_u(or u)	$1.660\ 538\ 782(83) \times 10^{-27}$	kg
Vacuum permittivity	e_0	$8.854\ 187\ 817\ldots \times 10^{-12}$	$\mathrm{F \cdot m^{-1}}$
Vacuum permeability	μ_0	$4\pi \times 10^{-7}$	$\mathrm{N \cdot A^{-2}}$
Boltzmann constant	k_B	$1.380\ 6504(24) \times 10^{-23}$	$\mathrm{J \cdot K^{-1}}$
Avogadro constant	N_A	$6.022\ 141\ 79(30) \times 10^{23}$	$\mathrm{mol^{-1}}$
Fine-structure constant	α^*	$(137.035\ 999\ 679(94))^{-1}$	
Magnetic flux quantum	$\Phi_0{}^*$	$2.067\ 833\ 667(52) \times 10^{-15}$	Wb
Quantum Hall resistance	$R_H{}^*$	25 812.8075(80)	Ω
Bohr magneton	$\mu_B{}^*$	$927.400\ 915(23) \times 10^{-26}$	$\mathrm{J \cdot T^{-1}}$
Nuclear magneton	$\mu_N{}^*$	$5.050\,783\,24\,(13) \times 10^{-27}$	$\mathrm{J \cdot T^{-1}}$
Electron magnetic moment	μ_e	$-1.001\ 159\ 652\ 181\ 11(74)$	μ_B
Proton magnetic moment	μ_p	2.792 847 356(23)	μ_N
Neutron magnetic moment	μ_n	$-1.913\ 042\ 73\ (45)$	μ_N
Gas constant	$R \equiv N_A k_B$	8.314 472(15)	$\mathrm{J \cdot mol^{-1} \cdot K^{-1}}$
Bohr radius	$a_B \equiv 4\pi e_0 \hbar^2/m_e e^2$	$0.529\ 177\ 208\ 59(36) \times 10^{-10}$	m

$\alpha = e^2/4\pi e_0 \hbar c$, $\Phi_0 = h/2e$, $R_H \equiv h/e^2$, $\mu_B \equiv e\hbar/2m_e$, $\mu_N \equiv (m_e/m_p)\mu_B$ (all in SI)

$\alpha = e^2/\hbar c$, $\Phi_0 = hc/2e$, $a_B = \hbar^2/m_e e^2$, $\mu_B = e\hbar/2m_e c$, $\mu_N \equiv (m_e/m_p)\mu_B$ (all in G – CGS)

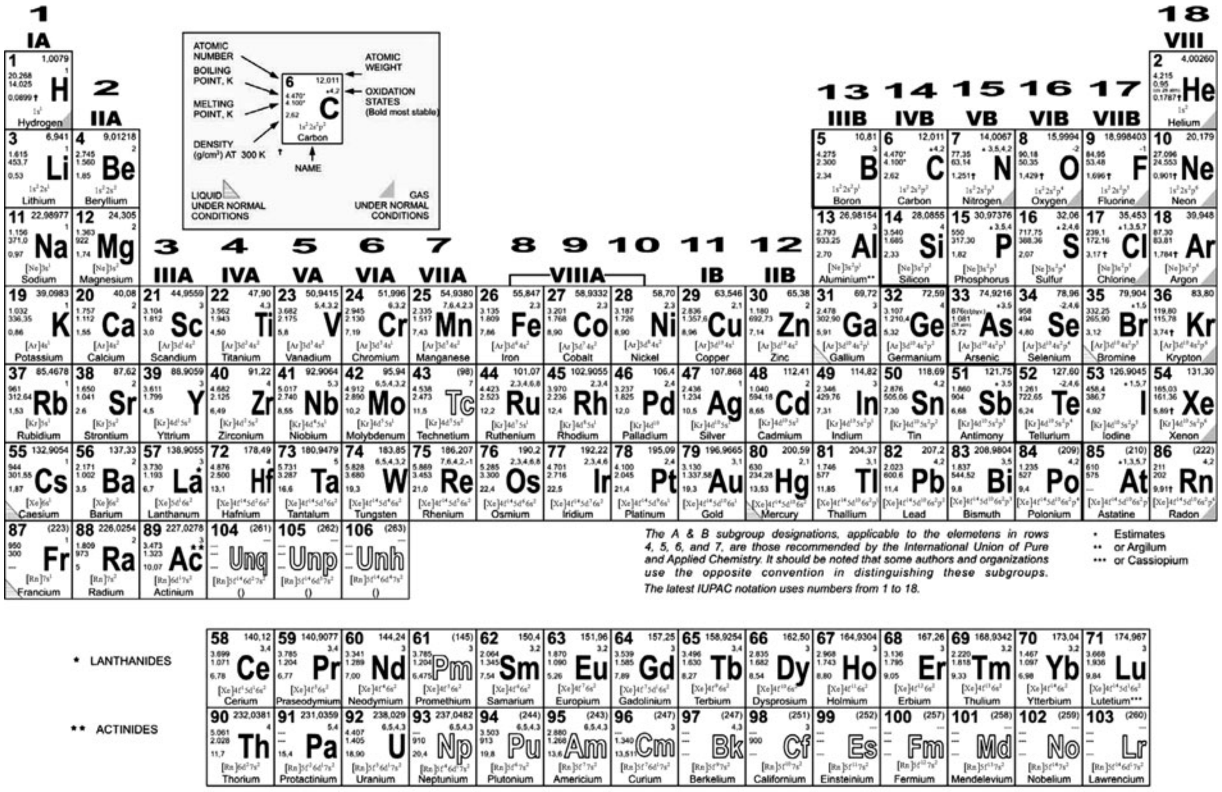
Table 2 Atomic system of units ($m_e = 1$; $e = 1$; $a_B = 1$; $k_B = 1$) $c = 1/\alpha$ in G—CGS; $\varepsilon_0 = 1/4\pi$, $\mu_0 = 4\pi\alpha^2$ in SI

Length	$l_0 = a_B$
Mass	$m_0 \equiv m_e$
Charge	$q_0 \equiv e$
Time	$t_0 \equiv m_e a_B^2/\hbar = 2.418\,884 \times 10^{-17}\ \mathrm{s}$
Energy	$E_0 \equiv \hbar^2/m_e a_B^2 = 4.359\,744 \times 10^{-18}\ \mathrm{J} = 27.211\,384\ \mathrm{eV}$
Angular frequency	$\omega_0 \equiv \hbar/m_e a_B^2 = 4.134\,137 \times 10^{16}\ \mathrm{rad/s}$
Velocity	$\upsilon_0 \equiv a_B/t_0 = \hbar/m_e a_B = \alpha c = 2\,187.\,691\,\mathrm{km/s}$
Mass density	$\rho_0 = m_e/a_B^3 = 6.147\,315\ \mathrm{kg/m^3}$
Temperature	$T_0 \equiv E_0/k_B = \hbar^2/m_e a_B^2 k_B = 315\,775\ \mathrm{K}$
Pressure	$P_0 \equiv E_0/a_B^2 = \hbar^2/m_e a_B^5 = 2.942\,101 \times 10^{13}$ $\mathrm{N/m^2} = 2.942\,101 \times 10^8\ \mathrm{bar}$
Electrical resistance	$R_0 \equiv \hbar/e^2 = R_H/2\pi = 4\,108.236\ \Omega$
Resistivity	$\rho_{\rho 0} = R_0 a_B = \hbar a_B/e^2 = 21.739\,848\ \mu\Omega \times \mathrm{cm}$
Conductivity	$\sigma_0 = 1/\rho_{c0} = e^2/\hbar a_B = 4.599\,848 \times 10^7\ \Omega^{-1}\mathrm{m}^{-1}$
Electric current	$i_0 \equiv e/t_0 = 6.623\,618\ \times 10^{-3}\mathrm{A}$
Voltage	$V_0 \equiv E_0/e = 27.211\,384\,\mathrm{V}$
Electric field	$\mathrm{E}_0 \equiv V_0/a_B = 5.142\,206 \times 10^{11}\ \mathrm{V/m}$
Magnetic field[1]	$B_0 \equiv c\hbar/e a_B^2 = 2.350\,517 \times 10^5\,\mathrm{T}$
Electric polarizability [2]	$a_{e0} = 4\pi e_0 a_B^3 = 1.648\,777 \times 10^{-41}\ \mathrm{Fm^2}$
Electric induction	$D_0 \equiv e/a_B^2 = 57.214\,762\ \mathrm{C/m^2}$
Magnetic moment	$\mu_0 \equiv 2\mu_B = 1.854\,802 \times 10^{-23}\ \mathrm{JT^{-1}}$
Magnetization	$M_0 \equiv \mu_0/a_B^3 = 1.251\,682 \times 10^8\,\mathrm{A/m}$
Magnetic field	$H_0 \equiv M_0 = 1.251\,682 \times 10^8\ \mathrm{A/m}$

For any quantity X we define $\bar{X} = X/X_0$ [see (2.16)]

[1] In SI set $c = 1$. [2] In G–CGS set $4\pi e_0 = 1$

Table 3 Periodic table of the elements



1 IA	2 IIA	3 IIIA	4 IVA	5 VA	6 VIA	7 VIIA	8 VIIIA	9 VIIIA	10 VIIIA	11 IB	12 IIB	13 IIIB	14 IVB	15 VB	16 VIB	17 VIIB	18 VIII
1 1,0079 1 20.268 14.025 0,0899† **H** $1s^1$ Hydrogen																	2 4,00260 4.215 0.95 (at 26 atm) 0,1787† **He** $1s^2$ Helium
3 6,941 1 1.615 453.7 0,53 **Li** $1s^2 2s^1$ Lithium	4 9,01218 2 2.745 1.560 1,85 **Be** $1s^2 2s^2$ Beryllium											5 10,81 3 4.275 2.300 2,34 **B** $1s^2 2s^2 p^1$ Boron	6 12,011 ±4,2 4.470* 4.100* 2,62 **C** $1s^2 2s^2 p^2$ Carbon	7 14,0067 ±3,5,4,2 77,35 63,14 1,251† **N** $1s^2 2s^2 p^3$ Nitrogen	8 15,9994 -2 90,18 50,35 1,429† **O** $1s^2 2s^2 p^4$ Oxygen	9 18,998403 -1 84,95 53,48 1,696† **F** $1s^2 2s^2 p^5$ Fluorine	10 20,179 27,096 24,553 0,901† **Ne** $1s^2 2s^2 p^6$ Neon
11 22,98977 1 1.156 371,0 0,97 **Na** $[Ne]3s^1$ Sodium	12 24,305 2 1.363 922 1,74 **Mg** $[Ne]3s^2$ Magnesium											13 26,98154 3 2.793 933,25 2,70 **Al** $[Ne]3s^2p^1$ Aluminium**	14 28,0855 4 3.540 1.685 2,33 **Si** $[Ne]3s^2p^2$ Silicon	15 30,97376 ±3,5,4 550 317,30 1,82 **P** $[Ne]3s^2p^3$ Phosphorus	16 32,06 ±2,4,6 717,75 388,36 2,07 **S** $[Ne]3s^2p^4$ Sulfur	17 35,453 ±1,3,5,7 239,1 172,16 3,17† **Cl** $[Ne]3s^2p^5$ Chlorine	18 39,948 87,30 83,81 1,784† **Ar** $[Ne]3s^2p^6$ Argon
19 39,0983 1 1.032 336,35 0,86 **K** $[Ar]4s^1$ Potassium	20 40,08 2 1.757 1.112 1,55 **Ca** $[Ar]4s^2$ Calcium	21 44,9559 3 3.104 1.812 3,0 **Sc** $[Ar]3d^1 4s^2$ Scandium	22 47,90 4,3 3.562 1.943 4,50 **Ti** $[Ar]3d^2 4s^2$ Titanium	23 50,9415 5,4,3,2 3.682 2.175 5,8 **V** $[Ar]3d^3 4s^2$ Vanadium	24 51,996 6,3,2 2.945 2.130 7,19 **Cr** $[Ar]3d^5 4s^1$ Chromium	25 54,9380 7,6,4,2,3 2.335 1.517 7,43 **Mn** $[Ar]3d^5 4s^2$ Manganese	26 55,847 2,3 3.135 1.809 7,86 **Fe** $[Ar]3d^6 4s^2$ Iron	27 58,9332 2,3 3.201 1.768 8,90 **Co** $[Ar]3d^7 4s^2$ Cobalt	28 58,70 2,3 3.187 1.726 8,90 **Ni** $[Ar]3d^8 4s^2$ Nickel	29 63,546 2,1 2.836 1.357,6 8,96 **Cu** $[Ar]3d^{10} 4s^1$ Copper	30 65,38 2 1.180 692,73 7,14 **Zn** $[Ar]3d^{10} 4s^2$ Zinc	31 69,72 3 2.478 302,90 5,91 **Ga** $[Ar]3d^{10} 4s^2p^1$ Gallium	32 72,59 4 3.107 1.210,4 5,32 **Ge** $[Ar]3d^{10} 4s^2p^2$ Germanium	33 74,9216 ±3,5 876(subl.) 1.081 (28 atm) 5,72 **As** $[Ar]3d^{10} 4s^2p^3$ Arsenic	34 78,96 -2,4,6 958 494 4,80 **Se** $[Ar]3d^{10} 4s^2p^4$ Selenium	35 79,904 ±1,5 332,25 265,90 3,12 **Br** $[Ar]3d^{10} 4s^2p^5$ Bromine	36 83,80 119,80 115,78 3,74† **Kr** $[Ar]3d^{10} 4s^2p^6$ Krypton
37 85,4678 1 961 312,64 1,53 **Rb** $[Kr]5s^1$ Rubidium	38 87,62 2 1.650 1.041 2,6 **Sr** $[Kr]5s^2$ Strontium	39 88,9059 3 3.611 1.799 4,5 **Y** $[Kr]4d^1 5s^2$ Yttrium	40 91,22 4 4.682 2.125 6,49 **Zr** $[Kr]4d^2 5s^2$ Zirconium	41 92,9064 5,3 5.017 2.740 8,55 **Nb** $[Kr]4d^4 5s^1$ Niobium	42 95,94 6,5,4,3,2 4.912 2.890 10,2 **Mo** $[Kr]4d^5 5s^1$ Molybdenum	43 (98) 7 4.538 2.473 11,5 **Tc** $[Kr]4d^5 5s^2$ Technetium	44 101,07 2,3,4,6,8 4.423 2.523 12,2 **Ru** $[Kr]4d^7 5s^1$ Ruthenium	45 102,9055 2,3,4 3.970 2.236 12,4 **Rh** $[Kr]4d^8 5s^1$ Rhodium	46 106,4 2,4 3.237 1.825 12,0 **Pd** $[Kr]4d^{10}$ Palladium	47 107,868 1 2.436 1.234 10,5 **Ag** $[Kr]4d^{10} 5s^1$ Silver	48 112,41 2 1.040 594,18 8,65 **Cd** $[Kr]4d^{10} 5s^2$ Cadmium	49 114,82 3 2.346 429,76 7,31 **In** $[Kr]4d^{10} 5s^2p^1$ Indium	50 118,69 4,2 2.876 505,06 7,30 **Sn** $[Kr]4d^{10} 5s^2p^2$ Tin	51 121,75 ±3,5 1.860 904 6,68 **Sb** $[Kr]4d^{10} 5s^2p^3$ Antimony	52 127,60 -2,4,6 1.261 722,65 6,24 **Te** $[Kr]4d^{10} 5s^2p^4$ Tellurium	53 126,9045 ±1,5,7 458,4 386,7 4,92 **I** $[Kr]4d^{10} 5s^2p^5$ Iodine	54 131,30 165,03 161,36 5,89† **Xe** $[Kr]4d^{10} 5s^2p^6$ Xenon
55 132,9054 1 944 301,55 1,87 **Cs** $[Xe]6s^1$ Caesium	56 137,33 2 2.171 1.002 3,5 **Ba** $[Xe]6s^2$ Barium	57 138,9055 3 3.730 1.193 6,7 **La*** $[Xe]5d^1 6s^2$ Lanthanum	72 178,49 4 4.876 2.500 13,1 **Hf** $[Xe]4f^{14} 5d^2 6s^2$ Hafnium	73 180,9479 5 5.731 3.287 16,6 **Ta** $[Xe]4f^{14} 5d^3 6s^2$ Tantalum	74 183,85 6,5,4,3,2 5.828 3.680 19,3 **W** $[Xe]4f^{14} 5d^4 6s^2$ Tungsten	75 186,207 7,6,4,2,-1 5.869 3.453 21,0 **Re** $[Xe]4f^{14} 5d^5 6s^2$ Rhenium	76 190,2 2,3,4,6,8 5.285 3.300 22,4 **Os** $[Xe]4f^{14} 5d^6 6s^2$ Osmium	77 192,22 2,3,4,6 4.701 2.716 22,5 **Ir** $[Xe]4f^{14} 5d^7 6s^2$ Iridium	78 195,09 2,4 4.100 2.045 21,4 **Pt** $[Xe]4f^{14} 5d^9 6s^1$ Platinum	79 196,9665 3,1 3.130 1.337,58 19,3 **Au** $[Xe]4f^{14} 5d^{10} 6s^1$ Gold	80 200,59 2,1 630 234,28 13,53 **Hg** $[Xe]4f^{14} 5d^{10} 6s^2$ Mercury	81 204,37 3,1 1.746 577 11,85 **Tl** $[Xe]4f^{14} 5d^{10} 6s^2p^1$ Thallium	82 207,2 4,2 2.023 600,6 11,4 **Pb** $[Xe]4f^{14} 5d^{10} 6s^2p^2$ Lead	83 208,9804 3,5 1.837 544,52 9,8 **Bi** $[Xe]4f^{14} 5d^{10} 6s^2p^3$ Bismuth	84 (209) 4,2 1.235 527 9,4 **Po** $[Xe]4f^{14} 5d^{10} 6s^2p^4$ Polonium	85 (210) ±1,3,5,7 610 575 — **At** $[Xe]4f^{14} 5d^{10} 6s^2p^5$ Astatine	86 (222) 4,2 211 202 9,91† **Rn** $[Xe]4f^{14} 5d^{10} 6s^2p^6$ Radon
87 (223) 1 950 300 — **Fr** $[Rn]7s^1$ Francium	88 226,0254 2 1.809 973 5 **Ra** $[Rn]7s^2$ Radium	89 227,0278 3 3.473 1.323 10,07 **Ac**** $[Rn]6d^1 7s^2$ Actinium	104 (261) — — — — **Unq** $[Rn]5f^{14} 6d^2 7s^2$ ()	105 (262) — — — — **Unp** $[Rn]5f^{14} 6d^3 7s^2$ ()	106 (263) — — — — **Unh** $[Rn]5f^{14} 6d^4 7s^2$ ()												

The A & B subgroup designations, applicable to the elemetens in rows 4, 5, 6, and 7, are those recommended by the International Union of Pure and Applied Chemistry. It should be noted that some authors and organizations use the opposite convention in distinguishing these subgroups.

The latest IUPAC notation uses numbers from 1 to 18.

* Estimates
** or Argilium
*** or Cassiopium

* LANTHANIDES	58 140,12 3,4 3.699 1.071 6,78 **Ce** $[Xe]4f^1 5d^1 6s^2$ Cerium	59 140,9077 3,4 3.785 1.204 6,77 **Pr** $[Xe]4f^3 6s^2$ Praseodymium	60 144,24 3 3.341 1.289 7,00 **Nd** $[Xe]4f^4 6s^2$ Neodymium	61 (145) 3 3.785 1.204 6,475 **Pm** $[Xe]4f^5 6s^2$ Promethium	62 150,4 3,2 2.064 1.345 7,54 **Sm** $[Xe]4f^6 6s^2$ Samarium	63 151,96 3,2 1.870 1.090 5,26 **Eu** $[Xe]4f^7 6s^2$ Europium	64 157,25 3 3.539 1.585 7,89 **Gd** $[Xe]4f^7 5d^1 6s^2$ Gadolinium	65 158,9254 3,4 3.496 1.630 8,27 **Tb** $[Xe]4f^9 6s^2$ Terbium	66 162,50 3 2.835 1.682 8,54 **Dy** $[Xe]4f^{10} 6s^2$ Dysprosium	67 164,9304 3 2.968 1.743 8,80 **Ho** $[Xe]4f^{11} 6s^2$ Holmium	68 167,26 3 3.136 1.795 9,05 **Er** $[Xe]4f^{12} 6s^2$ Erbium	69 168,9342 3,2 2.220 1.818 9,33 **Tm** $[Xe]4f^{13} 6s^2$ Thulium	70 173,04 3,2 1.467 1.097 6,98 **Yb** $[Xe]4f^{14} 6s^2$ Ytterbium	71 174,967 3 3.668 1.936 9,84 **Lu** $[Xe]4f^{14} 5d^1 6s^2$ Lutetium***
** ACTINIDES	90 232,0381 4 5.061 2.028 11,7 **Th** $[Rn]6d^2 7s^2$ Thorium	91 231,0359 5,4 — — 15,4 **Pa** $[Rn]5f^2 6d^1 7s^2$ Protactinium	92 238,029 6,5,4,3 4.407 1.405 18,90 **U** $[Rn]5f^3 6d^1 7s^2$ Uranium	93 237,0482 6,5,4,3 — 910 20,4 **Np** $[Rn]5f^4 6d^1 7s^2$ Neptunium	94 (244) 6,5,4,3 3.503 913 19,8 **Pu** $[Rn]5f^6 7s^2$ Plutonium	95 (243) 6,5,4,3 2.880 1.268 13,6 **Am** $[Rn]5f^7 7s^2$ Americium	96 (247) 3 — 1.340 13,51 **Cm** $[Rn]5f^7 6d^1 7s^2$ Curium	97 (247) 4,3 — — — **Bk** $[Rn]5f^9 7s^2$ Berkelium	98 (251) 3 — 900 — **Cf** $[Rn]5f^{10} 7s^2$ Californium	99 (252) — — — — **Es** $[Rn]5f^{11} 7s^2$ Einsteinium	100 (257) — — — — **Fm** $[Rn]5f^{12} 7s^2$ Fermium	101 (258) — — — — **Md** $[Rn]5f^{13} 7s^2$ Mendelevium	102 (259) — — — — **No** $[Rn]5f^{14} 7s^2$ Nobelium	103 (260) — — — — **Lr** $[Rn]5f^{14} 6d^1 7s^2$ Lawrencium

† For the gas phase (noted by the symbol ◢) the units is g/l, for T=273K and P=1atm.

Table 4 Theoretical values of the energy levels$-E_{n's}$,$-E_{n'p}$, $-E_{n'-1,\,d}$, and $V_1 \equiv \left(E_{n'p} - E_{n's}\right)/4$ for the highest partially or fully occupied subshell (From Harrison' s book [20])

Non-transition elements

C	
19.37	$\leftarrow -E_{n's}$
11.07	$\leftarrow -E_{n'p}$
2.08	$\leftarrow V_1$

Transition elements

Fe	
7.08	$\leftarrow E_{n's}$
16.54	$\leftarrow E_{n'-1,d}$

H																	***He*** 24,97 -	
Li 5,34 - -	***Be*** 8,41 - -											***B*** 13,46 8,43 1,26	***C*** 19,37 11,07 2,08	***N*** 26,22 13,84 3,10	***O*** 34,02 16,72 4,33	***F*** 42,78 19,86 5,73	***Ne*** 52,51 23,13 7,35	$n' = 2$
Na 4,95 - -	***Mg*** 6,88 - -											***Al*** 10,70 5,71 1,25	***Si*** 14,79 7,58 1,80	***P*** 19,22 9,54 2,42	***S*** 24,01 11,60 3,10	***Cl*** 29,19 13,78 3,85	***Ar*** 34,75 16,08 4,67	$n' = 3$
K 4,01 - -	***Ca*** 5,32 - -	***Sc*** 5,72 9,35	***Ti*** 6,04 11,04	***V*** 6,32 12,55	***Cr*** 6,59 13,94	***Mn*** 6,84 15,27	***Fe*** 7,08 16,54	***Co*** 7,31 17,77	***Ni*** 7,52 18,96	***Cu*** 6,49 13,36	***Zn*** 7,96 4 0,99	***Ga*** 11,55 5,67 1,47	***Ge*** 15,15 7,33 1,96	***As*** 18,91 8,98 2,48	***Se*** 22,86 10,68 3,05	***Br*** 27,00 12,43 3,64	***Kr*** 31,37 14,26 4,28	$n' = 4$
Rb 3,75 - -	***Sr*** 4,85 - -	***Y*** 5,34 6,80	***Zr*** 5,68 8,46	***Nb*** 5,95 10,03	***Mo*** 6,19 11,56	***Tc*** 6,39 13,08	***Ru*** 6,58 14,59	***Rh*** 6,75 16,16	***Pd*** 6,91 17,66	***Ag*** 5,98 14,62	***Cd*** 7,21 4 0,803	***In*** 10,14 5,37 1,19	***Sn*** 13,04 6,76 1,57	***Sb*** 16,02 8,14 1,97	***Te*** 19,12 9,54 2,40	***I*** 22,34 10,97 2,84	***Xe*** 25,69 12,44 3,31	$n' = 5$
Cs 3,36 - -	***Ba*** 4,29 - -	***La*** 4,35 -	***Hf*** 5,72 8,14	***Ta*** 5,98 9,57	***W*** 6,19 10,96	***Re*** 6,38 12,35	***Os*** 6,52 13,73	***Ir*** 6,71 15,13	***Pt*** 6,85 16,55	***Au*** 6,01 14,17	***Hg***(α) 7,10 4,11 0,75	***Tl*** 9,82 5,23 1,15	***Pb*** 12,48 6,53 1,49	***Bi*** 15,19 7,79 1,85	***Po*** 17,96 9,05 2,23	***At*** 20,82 10,33 2,62	***Rn*** 23,78 11,64 3,04	$n' = 6$
Fr	***Ra***	***Ac***																

Ce	***Pr***	***Nd***	***Pm***	***Sm***	***Eu***	***Gd***	***Tb***	***Dy***	***Ho***	***Er***	***Tm***	***Yb***	***Lu***
Th	***Pa***	***U***	***Np***	***Pu***	***Am***	***Cm***	***Bk***	***Cf***	***Es***	***Fm***	***Md***	***No***	***Lr***