

Appendix

A Directory of Characters

In the following tables we list the Examples where characters with a given period on a field with a given discriminant $D < 0$ occur. We begin with the most frequent discriminants -3 , -4 , -8 , -24 . Thereafter the discriminants are ordered according to their absolute values. Each table is ordered in ascending absolute values of the character periods.

$$D = -3$$

period	Example(s)
1	11.1, 11.4, 18.17, 18.18, 20.30, 25.7, 26.27, 26.28, 26.33, 26.36
$1 + \omega$	9.7, 11.4, 18.15, 20.26, 26.19, 26.24, 26.33
2	11.11, 18.16, 20.28, 26.13, 26.16, 26.27, 26.33, 26.36
3	11.5, 11.13
$2(1 + \omega)$	9.3, 9.9, 15.14, 18.13, 20.24, 20.26, 26.6, 26.16, 26.19, 26.24, 26.33
4	11.21, 11.22, 25.6, 26.13, 26.16, 26.25, 26.33, 26.36
$4 + \omega, 4 + \bar{\omega}$	12.3
$2(2 + \omega), 2(2 + \bar{\omega})$	16.17
6	11.1, 11.7, 11.9, 11.15, 11.20, 14.4, 25.11
$4(1 + \omega)$	9.1, 9.2, 10.16, 10.23, 11.21, 11.22, 13.7, 13.15, 13.24, 15.14, 26.3, 26.6, 26.16, 26.24, 26.25, 26.33
$3(2 + \omega), 3(2 + \bar{\omega})$	16.13, 16.18
8	25.5, 25.8, 25.19, 25.20, 26.16

period	Example(s)
12	11.17, 16.20, 18.4, 20.24, 20.26, 20.28, 20.30, 25.1, 25.10, 25.15
$8(1 + \omega)$	13.2, 13.11, 13.12, 13.18, 13.24, 13.27, 15.3, 15.17, 19.4, 25.9, 26.6, 26.16, 29.12
$6(2 + \omega), 6(2 + \bar{\omega})$	16.19
16	25.2, 25.20
$10(1 + \omega)$	14.8
$8(2 + \omega), 8(2 + \bar{\omega})$	30.12
24	18.8, 20.12, 25.3, 25.16, 25.18, 25.26, 25.27, 26.7, 26.17, 26.23, 26.32
$4(5 + 2\omega), 4(5 + 2\bar{\omega})$	12.9
$16(1 + \omega)$	13.12, 13.30, 15.5, 19.3, 19.4
$8(3 + \omega), 8(3 + \bar{\omega})$	31.9
$12(2 + \omega), 12(2 + \bar{\omega})$	16.21
$20(1 + \omega)$	20.6
$8(3 + 2\omega), 8(3 + 2\bar{\omega})$	31.15
$8(4 + \omega), 8(4 + \bar{\omega})$	23.18
$24(1 + \omega)$	20.17
48	25.4, 25.31, 25.37, 25.38
$8(5 + 2\omega), 8(5 + 2\bar{\omega})$	22.5, 22.16
$32(1 + \omega)$	15.6, 15.26, 15.27
$8(7 + \omega), 8(7 + \bar{\omega})$	21.13
$16(4 + \omega), 16(4 + \bar{\omega})$	23.23
$16(5 + 2\omega), 16(5 + 2\bar{\omega})$	22.22
$64(1 + \omega)$	19.11
$16(7 + \omega), 16(7 + \bar{\omega})$	21.14

$D = -4$

period	Example(s)
1	10.21, 13.5, 18.19, 24.31, 26.29, 26.32, 26.37, 29.14
$1 + i$	10.6, 10.21, 15.11, 17.14, 20.25, 26.18, 26.26, 26.32, 26.37
2	9.5
$2 \pm i$	24.31
$2(1 + i)$	10.7, 10.9
3	10.15, 18.15, 18.19, 26.20, 26.23, 26.29, 26.32, 29.14
4	13.5, 13.6, 15.11, 15.21, 24.26, 26.14, 26.18
$3(1 + i)$	10.17, 18.12, 18.14, 20.25, 26.5, 26.18, 26.32
$2(2 \pm i)$	12.1, 12.18, 24.25, 24.29
$4(1 + i)$	10.1, 10.2, 10.18, 13.17, 13.22, 15.28, 15.30
6	9.1, 9.2, 10.23, 11.18, 11.21, 11.22

period	Example(s)
$3(2 \pm i)$	17.10, 24.5
$2(3 \pm 2i)$	22.25
8	13.3, 13.23, 15.10, 15.15, 15.29, 15.30, 19.6
$6(1 + i)$	10.12, 18.3
$4(2 \pm i)$	24.1, 24.26
9	20.11
$8(1 + i)$	19.6, 19.7
12	13.2, 13.9, 15.3, 15.17, 25.12, 26.5, 26.14, 26.18, 29.12
$4(1 \pm 3i)$	17.1
$9(1 + i)$	20.16
$6(2 \pm i)$	12.17, 16.14, 16.16
16	15.12, 15.18, 15.24
$4(4 \pm i)$	22.3
$12(1 + i)$	10.5, 10.19, 10.24, 13.25, 13.28, 17.13, 18.6, 20.9, 20.13, 25.14
$8(2 \pm i)$	24.10
18	14.1, 14.5
$6(3 \pm i)$	17.9
20	29.2
$4(1 \pm 5i)$	17.6
$4(5 \pm 3i)$	17.7
24	13.4, 13.31, 15.5, 15.19, 15.22, 15.23, 19.3, 24.17, 25.13, 25.22, 25.23, 25.24, 29.7
$18(1 + i)$	18.7, 20.16
$12(2 \pm i)$	24.11
$20(1 + i)$	20.5
28	29.1
$8(3 \pm 2i)$	22.18
30	20.6
$6(5 \pm i)$	17.23
32	19.9
$8(4 \pm i)$	22.10
$24(1 + i)$	15.6
36	25.28, 26.7, 26.17, 26.23, 26.32, 29.8
$12(3 \pm i)$	17.11, 17.12, 24.13, 24.16, 27.10
40	29.3
$12(3 \pm 2i)$	22.20
$4(9 \pm 7i)$	28.4
$4(11 \pm 3i)$	28.4
48	15.13, 15.20, 15.25, 15.27
$36(1 + i)$	18.10, 20.20, 20.21, 25.29, 25.30, 25.35
$4(13 \pm i)$	28.6
$4(11 \pm 7i)$	28.6
$24(2 \pm i)$	24.18, 24.19

period	Example(s)
$12(5 \pm i)$	17.22
$12(5 \pm 3i)$	17.26
72	25.33, 25.35, 29.10
$60(1 + i)$	20.8
$24(3 \pm 2i)$	22.21
$24(4 \pm i)$	22.11
120	29.4, 29.5

$$D = -8$$

period	Example(s)
1	15.2, 26.30, 26.35
$\sqrt{-2}$	13.13, 15.9, 15.30, 19.8, 26.9, 26.12, 26.35
$1 \pm \sqrt{-2}$	26.35
2	13.13, 15.2, 15.30, 19.8, 26.9, 26.12, 26.30, 26.35
$2\sqrt{-2}$	13.13, 13.14, 13.20
3	26.22, 26.30
$2(1 \pm \sqrt{-2})$	26.9, 26.35
4	10.1, 10.2, 10.11, 15.30
$3 \pm 2\sqrt{-2}$	22.15
$3\sqrt{-2}$	26.1, 26.12
$2(2 \pm \sqrt{-2})$	25.8, 25.19, 25.20
$4\sqrt{-2}$	13.3, 13.8, 13.23, 15.15, 15.29, 15.30, 26.9, 26.12
6	26.1, 26.12, 26.22, 26.30
$4(1 \pm \sqrt{-2})$	18.1, 18.5, 27.1
8	15.1, 15.2, 15.4, 19.6, 19.7
$2(3 \pm 2\sqrt{-2})$	22.15
$6\sqrt{-2}$	25.9
$4(2 \pm \sqrt{-2})$	25.2, 25.20, 26.9
12	18.6, 20.9, 20.13, 20.19, 25.14
$4(3 \pm \sqrt{-2})$	17.4
$8(1 \pm \sqrt{-2})$	25.21
$6(2 + \sqrt{-2})$	25.16
16	19.1, 19.2
$12\sqrt{-2}$	25.22, 26.1, 26.12, 29.7
$4(2 \pm 3\sqrt{-2})$	23.6
20	20.5
$12(1 + \sqrt{-2})$	18.9, 18.10, 25.35, 27.1
$4(4 \pm 3\sqrt{-2})$	22.13
28	20.4

period	Example(s)
$4(1 \pm 5\sqrt{-2})$	28.15
$4(7 \pm \sqrt{-2})$	28.15
$12(2 + \sqrt{-2})$	25.31, 25.35, 25.39
36	20.10, 20.18

$$D = -24$$

period	Example(s)
1	26.31, 26.34, 27.18
$\sqrt{-2}$	26.8, 26.11, 26.34
$\sqrt{3}$	26.21, 26.31
2	26.31, 27.17
$\sqrt{-6}$	26.2, 26.11, 26.21, 26.31
$2\sqrt{-2}$	26.8, 26.11, 26.34
3	13.16, 27.13
$2\sqrt{3}$	13.16, 13.26, 26.2, 26.11, 26.21, 26.31
4	18.1, 18.5, 27.1
$\sqrt{3} \pm 3\sqrt{-2}$	23.17
$3\sqrt{3}$	25.11
$4\sqrt{-2}$	25.21, 26.8, 26.11
6	27.14
$4\sqrt{3}$	10.5, 10.13, 10.20, 10.24, 17.13, 20.19
8	27.15
$6\sqrt{-2}$	25.17
$4\sqrt{-6}$	13.4, 13.10, 13.11, 13.12, 13.29, 15.19, 15.22, 15.23, 19.3, 19.4, 24.17, 25.24, 29.7
$6\sqrt{3}$	25.10
$4(1 \pm \sqrt{-6})$	28.9
$2(2\sqrt{3} \pm 3\sqrt{-2})$	24.8
12	18.2, 18.9, 18.11, 20.20, 25.30, 27.1
$4(2 \pm \sqrt{-6})$	30.2
$2(6 \pm \sqrt{-6})$	23.17
$8\sqrt{3}$	15.7
$4(3 \pm \sqrt{-6})$	17.2, 17.12, 27.9
$12\sqrt{-2}$	25.32, 25.35, 25.36, 25.39, 25.40, 25.41
$4(4 \pm \sqrt{-6})$	31.1
$8\sqrt{-6}$	15.27
$12\sqrt{3}$	20.14, 20.15, 20.22
$4(2\sqrt{3} \pm 3\sqrt{-2})$	24.12, 24.15, 24.16, 24.18, 24.20, 24.21
$4(3 \pm 2\sqrt{-6})$	17.21

period	Example(s)
24	26.2, 26.11, 27.16
$4(6 \pm \sqrt{-6})$	23.4, 23.22, 23.25, 23.26
$16\sqrt{3}$	19.5
$4(4\sqrt{3} \pm 3\sqrt{-2})$	23.11
$20\sqrt{3}$	20.8
$16\sqrt{-6}$	19.10
$20\sqrt{-6}$	29.4

 $D = -7$

period	Example(s)
1	12.4, 20.1
$\frac{1}{2}(1 \pm \sqrt{-7})$	17.19
2	23.27
$\frac{1}{2}(5 \pm \sqrt{-7})$	23.16, 27.7
3	12.3, 16.18
4	23.5, 23.27
$2(1 \pm \sqrt{-7})$	27.6
8	23.16
$\frac{3}{2}(5 \pm \sqrt{-7})$	23.17
9	20.2
16	23.3, 23.20, 23.28
$8\sqrt{-7}$	29.1
24	23.17
48	23.4, 23.26

 $D = -11$

period	Example(s)
2	12.6
4	23.1
16	23.2, 23.7
$4(3 \pm \sqrt{-11})$	31.23

 $D = -15$

period	Example(s)
1	16.1, 30.8
$\frac{1}{2}(\sqrt{3} \pm \sqrt{-5})$	30.8
$\sqrt{3}$	12.12
2	30.8
3	16.2, 30.1
$\frac{1}{2}(\sqrt{3} \pm 3\sqrt{-5})$	17.10, 24.5

 $D = -19$

period	Example(s)
$\frac{1}{2}(1 \pm \sqrt{-19})$	16.11
6	12.11
12	21.2
24	21.13
48	21.14

period	Example(s)
$\frac{1}{2}(9 \pm \sqrt{-15})$	24.6
6	30.1
8	30.6
$2(3 + \sqrt{-15})$	17.9
$8\sqrt{3}$	24.6, 24.11
16	30.2
24	30.3
$16\sqrt{3}$	24.14, 24.22
48	30.4

$D = -20$

period	Example(s)
1	24.4, 24.28, 24.29
$\sqrt{2}$	17.15, 24.27
2	12.1, 12.18, 24.25, 24.29
$\sqrt{2}(1 \pm \sqrt{-5})$	16.15
4	24.1, 24.3
6	16.16
$2\sqrt{-10}$	20.7
8	24.2, 24.7, 24.9
$4(1 \pm \sqrt{-5})$	28.7
$4(3 \pm \sqrt{-5})$	28.1
18	20.3

$D = -23$

period	Example(s)
1	12.8
$\frac{1}{2}(3 \pm \sqrt{-23})$	21.3
8	21.3, 21.4, 21.9
$4(1 - \sqrt{-23})/\Lambda_{23},$ $4(1 + \sqrt{-23})/\bar{\Lambda}_{23}$	31.18
16	21.1, 21.9

$$D = -35$$

period	Example(s)
1	31.22
2	16.7, 31.22
4	31.19, 31.21
16	31.20

$$D = -39$$

period	Example(s)
1	31.10
$\Lambda_{39}, \bar{\Lambda}_{39}$	31.10
$\sqrt{-3}$	12.15
2	31.10
3	16.5, 31.6
$\frac{1}{2}(3 \pm \sqrt{-39})$	17.24
6	31.6
8	31.9
$4\sqrt{-3}\Lambda_{39}$	17.23, 17.24
$8\sqrt{-3}$	22.20
24	31.7
$16\sqrt{-3}$	22.23, 22.24
48	31.8

$$D = -40$$

period	Example(s)
1	24.30, 27.12
$\sqrt{-2}$	24.23
2	24.23, 24.30, 27.12
3	24.6
4	17.1
$4\sqrt{-2}$	24.10, 24.24
8	27.8
$6\sqrt{-2}$	24.6
12	17.2, 27.9
$4\sqrt{-10}$	29.3
$4(1 \pm \sqrt{-10})$	28.3
$4(2 \pm \sqrt{-10})$	31.20
$12\sqrt{-2}$	24.12, 24.14, 24.16, 24.20, 24.22
$12\sqrt{5}$	20.8
24	27.11

$$D = -51$$

period	Example(s)
$8\sqrt{3}$	22.8
$2(3 \pm \sqrt{-51})$	16.10
$16\sqrt{3}$	22.12

$D = -52$

period	Example(s)
1	22.26
$\sqrt{2}$	17.25
2	22.25
4	22.7
6	12.9
8	22.4, 22.17
$6\sqrt{2}$	17.23
12	22.5
24	22.6, 22.19, 22.20

 $D = -55$

period	Example(s)
1	31.24
3	16.8
8	31.23

 $D = -56$

period	Example(s)
1	23.16, 27.7
2	27.6
$2\sqrt{2}$	23.16
4	17.3
$4\sqrt{2}$	23.3, 23.19, 23.20, 23.28
8	27.5

 $D = -68$

period	Example(s)
2	12.10, 22.14
4	22.1, 22.3
$4\sqrt{2}$	17.7
8	22.2, 22.9, 22.10

 $D = -84$

period	Example(s)
1	30.13
3	30.9
$2\sqrt{3}$	12.13
4	30.12
$2\sqrt{6}$	17.17
6	16.3
$4\sqrt{6}$	17.18, 23.24
12	30.10
$8\sqrt{3}$	23.21, 23.22, 23.24, 23.25
$4(3 \pm \sqrt{-21})$	28.2
24	30.11

 $D = -88$

period	Example(s)
1	23.15, 27.4
$\sqrt{-2}$	23.14
2	23.14, 23.15, 27.4
4	17.4
$4\sqrt{-2}$	23.6, 23.14
8	27.2
12	17.5
$12\sqrt{-2}$	23.10
24	27.3

$D = -91$

period	Example(s)
6	16.12

$D = -95$

period	Example(s)
1	16.11

$D = -104$

period	Example(s)
4	17.6
$4\sqrt{2}$	22.18
$4\Lambda_{26}, 4\bar{\Lambda}_{26}$	28.13

$D = -120$

period	Example(s)
4	28.7
$2\sqrt{-6}$	24.8
$4\sqrt{-3}$	17.11, 24.13, 24.16, 27.10
$4\sqrt{-6}$	24.15, 24.19, 24.21
12	28.8
$4\sqrt{-30}$	29.5
$4(3 \pm \sqrt{-30})$	28.5

$D = -132$

period	Example(s)
1	31.5
$2\sqrt{3}$	12.14
4	31.4
$2\sqrt{6}$	17.20
6	16.4
$4\sqrt{3}$	23.8
8	31.1
12	31.2
$8\sqrt{3}$	23.9, 23.12
24	31.3

$D = -136$

period	Example(s)
1	22.15
$2\sqrt{2}$	22.15
4	17.7
$4\sqrt{2}$	22.10, 22.13
12	17.8

$D = -152$

period	Example(s)
$\frac{4}{\Lambda_{38}}(2 - \sqrt{-38}),$ $\frac{4}{\Lambda_{38}}(2 + \sqrt{-38})$	31.12

$D = -168$

period	Example(s)
4	28.9
$2\sqrt{6}$	23.18
$4\sqrt{3}$	17.18, 23.24
$4\sqrt{6}$	23.22, 23.23, 23.24
12	28.10

$D = -184$

period	Example(s)
1	21.3
$2\sqrt{2}$	21.3, 21.9
$4\sqrt{2}$	21.4, 21.9
$12\sqrt{2}$	21.7, 21.8

$D = -228$

period	Example(s)
1	31.16
3	31.11
4	31.15
$2\sqrt{6}$	17.27
6	16.6
$4\sqrt{3}$	21.10
8	31.12
12	31.13
$8\sqrt{3}$	21.11
24	31.14

$D = -260$

period	Example(s)
2	16.9

$D = -264$

period	Example(s)
4	28.11
$4\sqrt{3}$	17.5, 17.21
$4\sqrt{6}$	23.10, 23.11, 23.13
12	28.12

$D = -276$

period	Example(s)
4	31.18
$8\sqrt{3}$	21.5, 21.7, 21.8
12	31.17

$D = -280$

period	Example(s)
4	28.1
12	28.2

$D = -312$

period	Example(s)
4	28.13
$2\sqrt{6}$	22.16
$4\sqrt{3}$	17.22
$4\sqrt{6}$	22.21, 22.22
12	28.14

$D = -340$

period	Example(s)
6	16.10

$D = -408$

period	Example(s)
4	28.15
$2\sqrt{6}$	22.8
$4\sqrt{3}$	17.26
$4\sqrt{6}$	22.11, 22.12
12	28.16

$D = -440$

period	Example(s)
4	28.3

$D = -456$

period	Example(s)
4	28.17
$2\sqrt{6}$	21.13
$4\sqrt{3}$	17.28
$4\sqrt{6}$	21.12, 21.14
12	28.18

$D = -520$

period	Example(s)
4	28.4
12	28.5

$D = -552$

period	Example(s)
4	28.19
$4\sqrt{6}$	21.6, 21.7, 21.8
12	28.20

$D = -680$

period	Example(s)
4	28.6

In the following tables we list the examples where Hecke characters on real quadratic fields occur. For the most frequent discriminants 8, 12 and 24 we arrange the tables as before where, however, the character periods are not listed according to their absolute values, but rather according to the absolute values of their algebraic norms. For other discriminants D we just list the values of D and the numbers of the examples, but do not indicate periods of characters.

These tables will also display all our examples of identities of theta series on three distinct quadratic fields.

$D = 8$

period	Example(s)
$3 - \sqrt{2}$	23.16, 27.7
4	10.1, 15.28, 15.30
$5 \pm \sqrt{2}$	21.3
$2(3 \pm \sqrt{2})$	27.6
$4\sqrt{2}$	13.3, 15.29
$2(2 + 3\sqrt{2})$	23.16
8	19.6, 19.7
$6\sqrt{2}$	13.11, 19.4
12	18.6, 20.9, 20.13, 25.14
$2(2 \pm 5\sqrt{2})$	21.3
$4(2 \pm 3\sqrt{2})$	23.3, 23.20
$4(5 \pm 2\sqrt{2})$	17.7
$12\sqrt{2}$	13.12, 15.23, 19.4, 25.22, 29.7
20	20.5
$4(4 \pm 5\sqrt{2})$	22.10
$4(2 \pm 5\sqrt{2})$	21.9
$12(3 + \sqrt{2})$	17.18
$24\sqrt{2}$	15.27
$6(4 \pm 5\sqrt{2})$	22.8
36	18.10, 25.35
$12(2 \pm 3\sqrt{2})$	23.22, 23.24
$36\sqrt{2}$	25.35
$12(4 \pm 5\sqrt{2})$	22.12
$12(2 \pm 5\sqrt{2})$	21.7

 $D = 12$

period	Example(s)
$2\sqrt{3}$	9.1
$4(1 + \sqrt{3})$	18.1, 18.5, 27.1
$4\sqrt{3}$	13.2, 15.3, 29.12
8	25.21
$4(3 + \sqrt{3})$	20.19
12	26.7, 26.17, 26.23, 26.32
$8\sqrt{3}$	15.5, 19.3
$12(1 + \sqrt{3})$	18.9, 27.1
$10\sqrt{3}$	20.6
$2(9 + \sqrt{3})$	17.23

period	Example(s)
$4(2 \pm 3\sqrt{3})$	31.18
$8(3 + \sqrt{3})$	15.6
$4(1 \pm 3\sqrt{3})$	28.13
24	25.39
$4(3 \pm 4\sqrt{3})$	22.20
$4(3 \pm 5\sqrt{3})$	17.5
$8(6 \pm \sqrt{3})$	23.10
$8(9 \pm 2\sqrt{3})$	21.7, 21.8

$D = 24$

period	Example(s)
$2(2 + \sqrt{6})$	25.8, 25.19
$3 \pm 2\sqrt{6}$	24.6
$2\sqrt{6}$	25.9
$4(2 + \sqrt{6})$	25.2
$4(3 + \sqrt{6})$	10.5, 17.13
$6(2 + \sqrt{6})$	25.16
$4(1 \pm \sqrt{6})$	28.7
$4\sqrt{6}$	13.4, 15.22, 15.23, 24.17, 25.24, 29.7
$2(6 \pm \sqrt{6})$	24.6
$4(9 \pm 4\sqrt{6})$	17.12
$12(2 + \sqrt{6})$	25.31, 25.35
$8\sqrt{6}$	15.27
$12(3 + \sqrt{6})$	20.20
$2(6 \pm 5\sqrt{6})$	21.13
$4(6 \pm \sqrt{6})$	24.14, 24.18, 24.22
$4(4 \pm 3\sqrt{6})$	31.12
$20(3 + \sqrt{6})$	20.8
$4(6 \pm 5\sqrt{6})$	21.14
$4(12 \pm \sqrt{6})$	21.8
$20\sqrt{6}$	29.4

<i>D</i>	Example(s)
5	12.1, 24.1, 24.8, 24.21, 24.25, 31.23
13	22.25, 31.9
17	22.3, 22.13, 22.15
21	12.3
28	28.9, 29.1, 30.12
40	17.1, 24.10, 29.3, 30.2
44	17.4, 17.21, 23.6, 23.11, 28.3
56	23.18, 23.22, 23.23, 23.25, 28.1, 31.20
60	16.10, 17.2, 17.9, 17.10, 20.8, 24.11, 24.12, 24.16, 24.20, 27.9
76	31.15
88	31.1
104	17.6, 22.16, 22.18, 22.22
120	17.11, 24.13, 24.16, 24.19, 27.10, 28.2, 29.5
136	17.7, 22.10
152	21.13, 21.14
156	12.9, 17.23, 22.5, 22.20, 28.5
168	23.4, 23.17, 23.26
204	28.15
312	17.22, 22.21
408	17.26, 22.11
520	28.4
680	28.6

B Index of Notations

$A_{\mathfrak{p}}$	the group of one-units modulo \mathfrak{p}^r , for a prime ideal \mathfrak{p}	§6.1
d	a square-free positive integer	
D	the discriminant of an imaginary quadratic field, $D = -d$ or $D = -4d$	§5.2
$e(z)$	$= \exp(2\pi iz) = e^{2\pi iz}$	
$E_k(z)$	Eisenstein series of weight k for the modular group	§1.5
$E_{k, N, \delta}(z)$	Eisenstein series for the group $\Gamma^*(N)$, with k even, $\delta = \pm 1$	§1.6
$E_{k, P, \delta_i}(z)$	Eisenstein series for the group $\Gamma^*(P)$, with $P \neq 2$ prime, $(-1)^k = (\frac{-1}{P})$, $\delta = \pm 1$	§1.6
\mathbb{H}	$= \{x + iy \in \mathbb{C} \mid x \in \mathbb{R}, y > 0\}$, the upper half plane	§1.1
$J(L, z)$	$= cz + d$ for $L = \begin{pmatrix} * & * \\ c & d \end{pmatrix}$ in $SL_2(\mathbb{R})$	§1.3
\mathcal{J}_d	a system of integral ideal numbers for $\mathbb{Q}(\sqrt{-d})$	§5.5
$(\mathcal{J}_d/(M))^\times$	group of coprime residues modulo M	§5.5
k	the weight of a modular form	§1.4
K	an algebraic number field	
$\mathcal{K}(N)$	the cone of holomorphic eta products of level N	§2.5
$\mathcal{K}^*(N)$	the cone of holomorphic eta products for $\Gamma^*(N)$	§3.5
$\mathcal{M}(\Gamma, k, v)$	vector space of modular forms	§1.4
$\mathcal{M}(\Gamma_0(N), k, \chi)$	vector space of modular forms	§1.7
N	a positive integer, usually the level of an eta product	
N	$= N_{K/\mathbb{Q}}$, the norm function for ideals in a number field K	
\mathcal{O}_d	the ring of integers in $\mathbb{Q}(\sqrt{-d})$	§5.2
$r_k(n)$	the number of representations of n as a sum of k squares	§10.5
R^\times	the group of units in a ring R	
s	the numerator of an eta product	§2.1
$S(N, k)$	the simplex of holomorphic eta products of level N and weight k	§3.1
$S(N, k)$	the simplex of holomorphic eta products of level N and weight $\leq k$	§3.1
$S(N, k)^{\text{pr}}$	projection of $S(N, k)$	§3.1
$S^*(N, k)$	the simplex of holomorphic eta products of weight k for $\Gamma^*(N)$	§3.5
$S^*(N, k)^{\text{pr}}$	the projection of $S^*(N, k)$	§3.5
$\mathcal{S}(\Gamma, k, v)$	vector space of cusp forms	§1.4
$\mathcal{S}(\Gamma_0(N), k, \chi)$	vector space of cusp forms	§1.7
$\text{sgn}(x)$	the sign of a real number $x \neq 0$	§1.3
t	the denominator of an eta product	§2.1
T_m	the m th Hecke operator	§1.7
T^T	the transpose of a matrix T	
v_η	the multiplier system of η	§1.3
W_N	the Fricke involution, $z \mapsto -1/(Nz)$	§1.6
Z_n	the cyclic group of order n	§6.1
$\delta, \varepsilon, \nu, \sigma, \kappa$	signs which can independently take the values 1 and -1	
Γ_1	the modular group	§1.3
$\Gamma_0(N)$	the Hecke congruence group of level N	§1.6
$\Gamma^*(N)$	the Fricke group of level N	§1.6
$\Delta(z)$	the discriminant function	§1.5
η	the Dedekind eta function	§1.1
$\theta(z)$	$= \sum_{n=-\infty}^{\infty} e^{\pi i n^2 z}$, the Jacobi theta function	§1.2
$\Theta_k(\xi, \cdot)$	Hecke theta series of weight k and character ξ on some field	§5.2
$\Theta_k(K, \xi, \cdot)$	theta series as before, indicating the field K	§5.2
$\Theta_k(D, \chi, \cdot)$	Hecke theta series of weight k and character χ on the quadratic number field with discriminant D	§5.5

ξ, ζ	roots of unity, values of characters	
ξ, Ξ	characters on real quadratic fields, often with subscripts and/or constructs, such as ξ^* , $\tilde{\chi}_{\delta, \varepsilon}, \dots$	
$\sigma_l(N)$	sum of the l th powers of the positive divisors of N	§1.5
$\tau(N)$	$= \sigma_0(N)$, the number of positive divisors of N	§1.5
$\tau(n)$	the Ramanujan numbers	§1.5
φ	the Euler function; or (more frequently) a character	§5.5
χ	(sometimes) a Dirichlet character	
$\chi, \psi, \varphi, \phi, \rho$	characters on imaginary quadratic fields, often adorned with subscripts and/or constructs, such as $\tilde{\chi}_{\delta, \varepsilon, \nu}$ or $\hat{\psi}_\delta$	
ω	$= e(1/6) = \frac{1}{2}(1 + \sqrt{-3})$	
$(\frac{\varepsilon}{d})$	the Legendre–Jacobi–Kronecker symbol	§1.1
$[1^{a_1}, 2^{a_2}, \dots]$	short notation for an eta product $\eta^{a_1}(z)\eta^{a_2}(2z) \dots$, frequently written as a fraction in brackets with positive exponents in nominator and denominator	§2.1
$[x]$	Gauss bracket, or floor: the largest integer $\leq x$	
$\lceil x \rceil$	ceiling: the smallest integer $\geq x$	
R^\times	the group of units in a ring R	
μ'	$= a - b\sqrt{d}$, the conjugate of a real quadratic irrational number $\mu = a + b\sqrt{d}$	
$\#B$	the number of elements in a finite set B	

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