A Tensor Operations in Orthogonal Curvilinear Coordinate Systems

A.1 Change of Coordinate System

The vector and tensor operations we have discussed in the foregoing chapters were performed solely in rectangular coordinate system. It should be pointed out that we were dealing with quantities such as velocity, acceleration, and pressure gradient that are independent of any coordinate system within a certain frame of reference. In this connection it is necessary to distinguish between a coordinate system and a frame of reference. The following example should clarify this distinction. In an absolute frame of reference, the flow velocity vector may be described by the rectangular Cartesian coordinate x_i:

$$\boldsymbol{V} = \boldsymbol{V}(x_1,\ x_2,\ x_3) = \boldsymbol{V}(\boldsymbol{X}) \tag{A.1}$$

It may also be described by a cylindrical coordinate system, which is a non-Cartesian coordinate system:

$$\boldsymbol{V} = \boldsymbol{V}(x,\ r,\ \theta) \tag{A.2}$$

or generally by any other non-Cartesian or curvilinear coordinate ξ_i that describes the flow channel geometry:

$$\boldsymbol{V} = \boldsymbol{V}(\xi_1,\ \xi_2,\ \xi_3) \tag{A.3}$$

By changing the coordinate system, the flow velocity vector will not change. It remains invariant under any transformation of coordinates. This is true for any other quantities such as acceleration, force, pressure or temperature gradient. The concept of invariance, however, is generally no longer valid if we change the frame of reference. For example, if the flow particles leave the absolute frame of reference and enter the relative frame of reference, for example a moving or rotating frame, its velocity will experience a change. In this Chapter, we will pursue the concept of quantity invariance and discuss the fundamentals that are needed for coordinate transformation.

A.2 Co- and Contravariant Base Vectors, Metric Coefficients

As we saw in the previous chapter, a vector quantity is described in Cartesian coordinate system x_i by its components:

M.T. Schobeiri: Fluid Mechanics for Engineers, pp. 475–487.
© Springer Berlin Heidelberg 2010

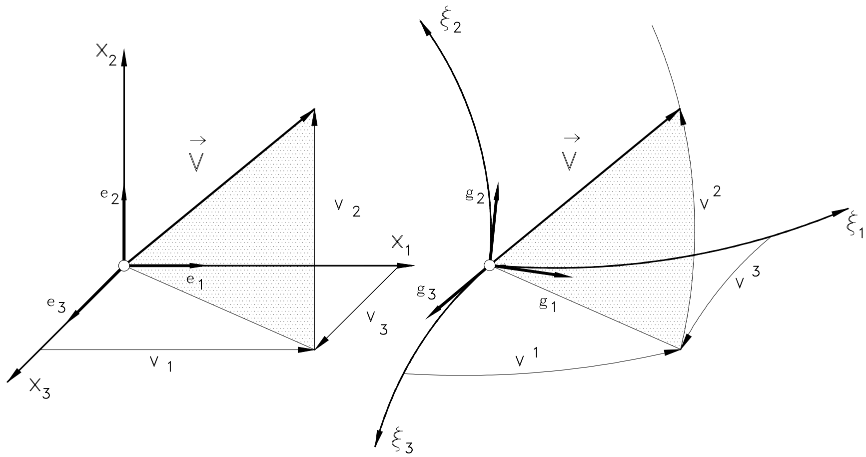

Fig. A.1: Base vectors in a Cartesian (left) and in a generalized orthogonal curvilinear coordinate system (right)

$$\boldsymbol{V} = \boldsymbol{e_i} V_i = \boldsymbol{e_1} V_1 + \boldsymbol{e_2} V_2 + \boldsymbol{e_3} V_3 \tag{A.4}$$

with e_i as orthonormal unit vectors (Fig. A.1 left). The same vector transformed into the curvilinear coordinate system ξ_k (Fig. A.1 right) is represented by:

$$\boldsymbol{V} = \boldsymbol{g_k} V^k = \boldsymbol{g_1} V^1 + \boldsymbol{g_2} V^2 + \boldsymbol{g_3} V^3 \tag{A.5}$$

where $\mathbf{g}_k$ are the base vectors and $\mathbf{V}^k$ the components of **V** with respect to the base $\mathbf{g}_k$ in a curvilinear coordinate system. For curvilinear coordinate system, we place the indices diagonally for summing convenience. Unlike the Cartesian base vectors $\mathbf{e}_i$, that are orthonormal vectors (of unit length and mutually orthogonal), the base vectors $\mathbf{g}_k$ do not have unit lengths. The base vectors $\mathbf{g}_k$ represent the rate of change of the position vector **x** with respect to the curvilinear coordinates ξ_i.

$$\boldsymbol{g_k} = \frac{\partial \boldsymbol{x}}{\partial \xi_k} = \frac{\partial(\boldsymbol{e}_i x_i)}{\partial \xi_k} \tag{A.6}$$

Since in a Cartesian coordinate system the unit vectors $\mathbf{e}_i$, are not functions of the coordinates x_i, Eq. (A.6) can be written as:

$$\boldsymbol{g}_k = \boldsymbol{e}_i \frac{\partial x_i}{\partial \xi_k} \tag{A.7}$$

Similarly, the *reciprocal base* vector $\mathbf{g}^k$ defined as:

$$\boldsymbol{g}^j = \boldsymbol{e}_m \frac{\partial \xi_j}{\partial x_m} \tag{A.8}$$

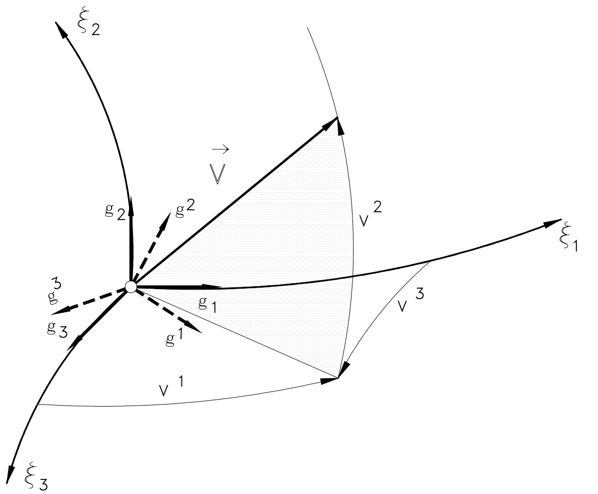

Fig. A.2: Co- and contravariant base vectors

As shown in Fig. A.2, the covariant base vectors $\mathbf{g}_2$, $\mathbf{g}_2$, and $\mathbf{g}_3$ are tangent vectors to the mutually orthogonal curvilinear coordinates ξ_1, ξ_2, and ξ_3. The reciprocal base vectors $\mathbf{ga}^1$, $\mathbf{g}^2$, $\mathbf{g}^3$, however, are orthogonal to the planes described by $\mathbf{g}_2$ and $\mathbf{g}_3$, $\mathbf{g}_3$ and $\mathbf{g}_1$, and $\mathbf{g}_1$ and $\mathbf{g}_2$, respectively. These base vectors are interrelated by:

$$\boldsymbol{g}_k \cdot \boldsymbol{g}^j = \boldsymbol{e}_i \cdot \boldsymbol{e}_m \frac{\partial x_i}{\partial \xi_k} \frac{\partial \xi_j}{\partial x_m} = \delta_{im} \frac{\partial x_i}{\partial \xi_k} \frac{\partial \xi_j}{\partial x_m} = \frac{\partial \xi_j}{\partial \xi_k} \equiv \delta_k^j \tag{A.9}$$

where $\mathbf{g}_k$ and $\mathbf{g}^j$ are referred to as the covariant and contravariant base vectors, respectively. The new Kronecker delta $\delta_k{}^j$ from Eq. (A.9) has the values:

$$\boldsymbol{g}_k \cdot \boldsymbol{g}^j = \delta_k^j, \quad \delta_k^j = 1 \text{ for } k = j, \quad \delta_k^j = 0 \text{ for } k \neq j$$

The vector $\mathbf{V}$ written relative to its contravariant base is:

$$\boldsymbol{V} = \boldsymbol{g}^k V_k = \boldsymbol{g}^1 V_1 + \boldsymbol{g}^2 V_2 + \boldsymbol{g}^3 V_3 \tag{A.10}$$

Similarly, the components V_k and V^k are called the covariant and contravariant components, respectively. The scalar product of covariant respectively contravariant base vectors results in the covariant and covariant metric coefficients:

$$g_{ij} = \boldsymbol{g}_i \cdot \boldsymbol{g}_j, \quad g^{ij} = \boldsymbol{g}^i \cdot \boldsymbol{g}^j \tag{A.11}$$

The mixed metric coefficient is defined as

$$g_i^j = \boldsymbol{g}_i \cdot \boldsymbol{g}^j \tag{A.12}$$

The covariant base vectors can be expressed in terms of the contravariant base vectors. First we assume that:

$$\begin{aligned} \boldsymbol{g}^1 &= A^{11}\boldsymbol{g}_1 + A^{12}\boldsymbol{g}_2 + A^{13}\boldsymbol{g}_3 \\ \boldsymbol{g}^2 &= A^{21}\boldsymbol{g}_1 + A^{22}\boldsymbol{g}_2 + A^{23}\boldsymbol{g}_3 \\ \boldsymbol{g}^3 &= A^{31}\boldsymbol{g}_1 + A^{32}\boldsymbol{g}_2 + A^{33}\boldsymbol{g}_3 \end{aligned} \tag{A.13}$$

Generally the contravariant base vector can be written as

$$\boldsymbol{g}^i = A^{ij}\boldsymbol{g}_j \tag{A.14}$$

To find a direct relation between the base vectors, first the coefficient matrix A^{ij} must be determined. To do so, we multiply Eq. (A.14) with $\mathbf{g}^k$ scalarly:

$$\boldsymbol{g}^i \cdot \boldsymbol{g}^k = A^{ij}\boldsymbol{g}_j \cdot \boldsymbol{g}^k$$

This leads to $g^{ik} = A^{ij}\delta_j^k$. The right hand side is different from zero only if $j = k$. That means:

$$g^{ik} = A^{ik} \tag{A.16}$$

Introducing Eq. (A.16) into (A.14) results in a relation that expresses the contravariant base vectors in terms of covariant base vectors:

$$\boldsymbol{g}^i = g^{ij}\boldsymbol{g}_j \tag{A.17}$$

The covariant base vector can also be expressed in terms of contravariant base vectors in a similar way:

$$\boldsymbol{g}_k = g_{kl}\boldsymbol{g}^l \tag{A.18}$$

Multiply Eq. (A.l8) with (A.17) establishes a relationship between the covariant and contravariant metric coefficients:

$$\boldsymbol{g}^i \cdot \boldsymbol{g}_k = g^{ij}g_{kl}\boldsymbol{g}_j \cdot \boldsymbol{g}^l, \quad \text{and} \quad \delta_k^i = g^{ij}g_{kl}\delta_j^l \tag{A.19}$$

Applying the Kronecker delta on the right hand side results in:

$$g^{ij}g_{kj} = \delta_k^i \tag{A.20}$$

A.3 Physical Components of a Vector

As mentioned previously, the base vectors $\mathbf{g}_i$ or $\mathbf{g}^j$ are not unit vectors. Consequently the co- and contravariant vector components V_j or V^1 do not reflect the physical components of vector **V**. To obtain the physical components, first the corresponding unit vectors must be found. They can be obtained from:

$$\boldsymbol{g}_i^* = \frac{\boldsymbol{g}_i}{|\boldsymbol{g}_i|} = \frac{\boldsymbol{g}_i}{\sqrt{\boldsymbol{g}_i \cdot \boldsymbol{g}_i}} = \frac{\boldsymbol{g}_i}{\sqrt{g_{(ii)}}} \tag{A.21}$$

Similarly, the contravariant unit vectors are:

$$\boldsymbol{g}^{*i} = \frac{\boldsymbol{g}^i}{|\boldsymbol{g}^i|} = \frac{\boldsymbol{g}^i}{\sqrt{\boldsymbol{g}^i \cdot \boldsymbol{g}^i}} = \frac{\boldsymbol{g}^i}{\sqrt{g^{(ii)}}} \tag{A.22}$$

where $\mathbf{g}_i^*$, represents the unit base vector, $|\mathbf{g}^i|$ the absolute value of the base vector. The expression (ii) denotes that no summing is carried out, whenever the indices are enclosed within parentheses. The vector can now be expressed in terms of its unit base vectors and the corresponding physical components:

$$\boldsymbol{V} = \boldsymbol{g}_i V^i = \boldsymbol{g}_i^* V^{*i} = \frac{\boldsymbol{g}_i}{\sqrt{g_{(ii)}}} V^{*i} \tag{A.23}$$

Thus the covariant and contravariant physical components can be easily obtained from:

$$V_i^* = \sqrt{g^{(ii)}}\, V_i, \quad V^{*i} = \sqrt{g_{(ii)}}\, V^i \tag{A.24}$$

A.4 Derivatives of the Base Vectors, Christoffel Symbols

In a curvilinear coordinate system, the base vectors are generally functions of the coordinates itself. This fact must be considered while differentiating the base vectors. Consider the derivative:

$$\boldsymbol{g}_{i,j} \equiv \frac{\partial \boldsymbol{g}_i}{\partial \xi_j} = \frac{\partial}{\partial \xi_j}\left(e_k \frac{\partial x_k}{\partial \xi_i}\right) = e_k \frac{\partial^2 x_k}{\partial \xi_j \partial \xi_i} \tag{A.25}$$

Similar to Eq. (A.7), the unit vector $\mathbf{e}_k$ can be written:

$$\boldsymbol{e}_k = \boldsymbol{g}_n \frac{\partial \xi_n}{\partial x_k} \tag{A.26}$$

Introducing Eq. (A.26) into (A.25) yields:

$$\boldsymbol{g}_{i,j} = \frac{\partial^2 x_k}{\partial \xi_j \partial \xi_i} \frac{\partial \xi_n}{\partial x_k} \boldsymbol{g}_n \equiv \Gamma_{ij}^n \boldsymbol{g}_n = \Gamma_{ijn} \boldsymbol{g}^n \tag{A.27}$$

with Γ_{ijn}, and $\Gamma_{ij}^{\ n}$ as the Christoffel symbol of first and second kind, respectively with the definition:

$$\Gamma_{ijn} = \frac{\partial^2 x_k}{\partial \xi_i \partial \xi_j} \frac{\partial \xi_k}{\partial x_n}, \quad \Gamma_{ij}^n = \frac{\partial^2 x_k}{\partial \xi_j \partial \xi_i} \frac{\partial \xi_n}{\partial x_k} \tag{A.28a}$$

From (A.28a) follows that the Christoffel symbols of the second kind is related the first kind by:

$$\Gamma_{ij}^{k} = \Gamma_{ijm} g^{mk} \tag{A.28b}$$

Since the Christoffel symbols convertible by using the metric coefficients, for the sake of simplicity, in what follows, we use the second kind. The derivative of contravariant base vector is:

$$\boldsymbol{g}^{j}_{,i} \equiv \frac{\partial \boldsymbol{g}^{j}}{\partial \xi_i} = -\Gamma_{ik}^{j} \boldsymbol{g}^{k} \tag{A.29}$$

The Christoffel symbols are then obtained by expanding Eq. (A.28a):

$$\Gamma_{ml}^{k} = \Gamma_{lm}^{k} = \frac{1}{2} g^{kn} \left(g_{mn,l} + g_{nl,m} - g_{lm,n}\right) \tag{A.30a}$$

$$\Gamma_{ml}^{k} = \Gamma_{lm}^{k} = \frac{1}{2} g^{(kk)} \left(g_{mk,l} + g_{kl,m} - g_{lm,k}\right) \tag{A.30b}$$

In Eq. (A.30a), the Christoffel symbols are symmetric in their lower indices. Furthermore, the fact that the only non-zero elements of the metric coefficients are the diagonal elements allowed the modification of the first equation in (A.30a) to arrive at t(A.30b). Again, note that a repeated index in parentheses in an expression such as $g^{(kk)}$ does not subject to summation.

A.5 Spatial Derivatives in Curvilinear Coordinate System

The differential operator ∇, Nabla, is in curvilinear coordinate system defined as:

$$\nabla = \boldsymbol{g}^{i} \frac{\partial}{\partial \xi_i} \tag{A.31}$$

A.5.1 Application of ∇ to Tensor Functions

In this chapter, the operator ∇ will be applied to different arguments such as zeroth, first and second order tensors. If the argument is a zeroth order tensor which is a scalar quantity such as pressure or temperature, the results of the operation is the gradient of the scalar field which is a vector quantity:

$$\nabla p = \boldsymbol{g}^{i} \frac{\partial p}{\partial \xi_i} \equiv \boldsymbol{g}^{i} p_{,i} \tag{A.32}$$

The abbreviation ",i " refers to the derivative of the argument, in this case p, with respect to the coordinate ξ_i . If the argument is a first order tensor such as a velocity vector, the order of the resulting tensor depends on the operation character between the operator ∇ and the argument. For divergence and curl of a vector using the chain rule, the differentiations are:

$$\nabla \cdot \boldsymbol{V} = \left(\boldsymbol{g}^i \frac{\partial}{\partial \xi_i}\right) \cdot \left(\boldsymbol{g}_j V^j\right) = \boldsymbol{g}^i \cdot \left(\frac{\partial \boldsymbol{g}_j}{\partial \xi_i} V^j + \frac{\partial \boldsymbol{V}^j}{\partial \xi_i} \boldsymbol{g}_j\right) \tag{A.33a}$$

$$\nabla \times \boldsymbol{V} = \left(\boldsymbol{g}^i \frac{\partial}{\partial \xi_i}\right) \times \left(\boldsymbol{g}^j V_j\right) = \boldsymbol{g}^i \times \boldsymbol{g}^j V_{j,i} + \boldsymbol{g}^i \times \boldsymbol{g}^j{}_{,i} V_j \tag{A.33b}$$

Implementing the Christoffel symbol, the results of the above operations are the divergence and the curl of the vector $\boldsymbol{V}$. It should be noticed that a scalar operation leads to a contraction of the order of tensor on which the operator is acting. The scalar operation in (A.33a) leads to:

$$\nabla \cdot \boldsymbol{V} = V^i_{,i} + V^J \Gamma^i_{ij} \tag{A.34a}$$

The vector operation yields the rotation or curl of a vector field as:

$$\nabla \times \boldsymbol{V} = \boldsymbol{g}^i \times \boldsymbol{g}^j (V_{j,i} - \Gamma^k_{ij} V_k) = \frac{1}{\sqrt{g}} \varepsilon^{ijk} \boldsymbol{g}_k (V_{j,i} - \Gamma^k_{ij} V_k) \tag{A.34b}$$

with ε^{ijk} as the permutation symbol that functions similar to the one for Cartesian coordinate system and $\sqrt{g} = \sqrt{|g_{ij}|}$.

The gradient of a first order tensor such as the velocity vector $\boldsymbol{V}$ is a second order tensor. Its index notation in a curvilinear coordinate system is:

$$\nabla \boldsymbol{V} = \boldsymbol{g}^i \boldsymbol{g}_j \left(V^j_{,i} + V^k \Gamma^j_{ik}\right) \tag{A.35}$$

A scalar operation that involves ∇ and a second order tensor, such as the stress tensor $\boldsymbol{\Pi}$ or deformation tensor $\mathbf{D}$, results in a first order tensor which is a vector:

$$\nabla \cdot \Pi = \left(\boldsymbol{g}^m \frac{\partial}{\partial \xi_m}\right) \cdot \left(\boldsymbol{g}_i \boldsymbol{g}_j \pi^{ij}\right) = \boldsymbol{g}^m \cdot \left(\boldsymbol{g}_k \boldsymbol{g}_l\right)\left(\pi^{kl}_{,m} + \pi^{nl} \Gamma^k_{nm} + \pi^{kn} \Gamma^l_{nm}\right) \tag{A.36}$$

The right hand side of (A.36) is reduced to:

$$\nabla \cdot \Pi = \boldsymbol{g}_j \left(\pi^{mj}_{,m} + \pi^{nj} \Gamma^m_{nm} + \pi^{mn} \Gamma^j_{mn}\right) \tag{A.37}$$

By calculating the shear forces using the Navier-Stokes equation, the second derivative, the Laplace operator Δ, is needed:

$$\Delta = \nabla \cdot \nabla = \nabla^2 = \left(\boldsymbol{g}^i \frac{\partial}{\partial \xi_i}\right) \cdot \left(\boldsymbol{g}^j \frac{\partial}{\partial \xi_j}\right) \tag{A.38}$$

This operator applied to the velocity vector yields:

$$\Delta \boldsymbol{V} = \boldsymbol{g}_m g^{ik}\left[V^m_{,ik} + V^n_{,i} \Gamma^m_{nk} + V^n_{,k} \Gamma^m_{ni} - V^m_{,j} \Gamma^j_{ik} + V^p \left(\Gamma^n_{pi} \Gamma^m_{nk} - \Gamma^j_{ik} \Gamma^m_{pj} + \Gamma^m_{pi,k}\right)\right] \tag{A.39}$$

A.6 Application Example 1: Inviscid Incompressible Flow Motion

As the first application example, the equation of motion for an inviscid incompressible and steady low is transformed into a cylindrical coordinate system, where it is decomposed in its three components r, θ, z. The coordinate invariant version of the equation is written as:

$$\boldsymbol{V}\cdot\nabla\boldsymbol{V} = -\frac{1}{\rho}\nabla p \tag{A.40}$$

The transformation and decomposition procedure is shown in the following steps.

A.6.1 Equation of Motion in Curvilinear Coordinate Systems

The second order tensor on the left hand side can be obtained using Eq. (A.35):

$$\nabla\boldsymbol{V} = \boldsymbol{g}^i\boldsymbol{g}_j\left(V^j_{,i} + V^k\Gamma^j_{ik}\right) \tag{A.41}$$

The scalar multiplication with the velocity vector **V** leads to:

$$\boldsymbol{V}\cdot\nabla\boldsymbol{V} = \boldsymbol{g}_m V^m\cdot\boldsymbol{g}^i\boldsymbol{g}_j\left(V^j_{,i} + V^k\Gamma^j_{ik}\right) \tag{A.42}$$

Introducing the mixed Kronecker delta:

$$\boldsymbol{V}\cdot\nabla\boldsymbol{V} = \delta^i_m\boldsymbol{g}_j V^m\left(V^j_{,i} + V^k\Gamma^j_{ik}\right) \tag{A.43}$$

For an orthogonal curvilinear coordinate system the mixed Kronecker delta is:

$$\begin{aligned}\delta^i_m &= 1 \quad \text{for} \quad i = m\\ \delta^i_m &= 0 \quad \text{for} \quad i \neq m\end{aligned} \tag{A.44}$$

Taking this into account, Eq. (A.43) yields:

$$\boldsymbol{V}\cdot\nabla\boldsymbol{V} = \boldsymbol{g}_j V^i\left(V^j_{,i} + V^k\Gamma^j_{ik}\right) \tag{A.45}$$

Rearranging the indices

$$\boldsymbol{V}\cdot\nabla\boldsymbol{V} = \boldsymbol{g}_i\left(V^j V^i_{,j} + V^j V^k\Gamma^i_{kj}\right) \tag{A.46}$$

The pressure gradient on the right hand side of Eq. (A.40) is calculated form Eq. (A.32):

$$\nabla p = \boldsymbol{g}^i\frac{\partial p}{\partial\xi_i} = \boldsymbol{g}^i p_{,i} \tag{A.47}$$

Replacing the contravariant base vector with the covariant one using Eq. (A.47) leads to:

$$\nabla p = \boldsymbol{g}^i \frac{\partial p}{\partial \xi_i} = \boldsymbol{g}_i g^{ji} p_{,j} \tag{A.48}$$

Incorporating Eqs. (A.46) and (A.48) into Eq. (A.40) yields:

$$\boldsymbol{g}_i \left(V^j V^i_{,j} + V^j V^k \Gamma^i_{kj} \right) = -\frac{1}{\rho} \boldsymbol{g}_i g^{ji} p_{,j} \tag{A.49}$$

In i-direction, the equation of motion is:

$$V^j V^i_{,j} + V^j V^k \Gamma^i_{kj} = -\frac{1}{\rho} g^{ji} p_{,j} \tag{A.50}$$

A.6.2 Special Case: Cylindrical Coordinate System

To transfer Eq. (A.40) in any arbitrary curvilinear coordinate system, first the coordinate system must be specified. The cylinder coordinate system is related to the Cartesian coordinate system is given by:

$$x_1 = r\cos\Theta\,,\; x_2 = r\sin\Theta,\; x_3 = z \tag{A.51}$$

The curvilinear coordinate system is represented by:

$$\xi_1 = r,\; \xi_2 = \Theta,\; \xi_3 = z \tag{A.52}$$

A.6.3 Base Vectors, Metric Coefficients

The base vectors are calculated from Eq. (A.7).

$$\boldsymbol{g}_k = \boldsymbol{e}_i \frac{\partial x_i}{\partial \xi_k} \tag{A.53}$$

Equation (A.53) decomposed in its components yields:

$$\begin{aligned}
\boldsymbol{g}_1 &= \boldsymbol{e}_1 \frac{\partial x_1}{\partial \xi_1} + e_2 \frac{\partial x_2}{\partial \xi_1} + e_3 \frac{\partial x_3}{\partial \xi_1} \\
\boldsymbol{g}_2 &= e_1 \frac{\partial x_1}{\partial \xi_2} + e_2 \frac{\partial x_2}{\partial \xi_2} + e_3 \frac{\partial x_3}{\partial \xi_2} \\
\boldsymbol{g}_3 &= e_1 \frac{\partial x_1}{\partial \xi_3} + e_2 \frac{\partial x_2}{\partial \xi_3} + e_3 \frac{\partial x_3}{\partial \xi_3}
\end{aligned} \tag{A.54}$$

The differentiation of the Cartesian coordinates yields:

$$\begin{aligned} \boldsymbol{g}_1 &= \boldsymbol{e}_1 \cos\theta + \boldsymbol{e}_2 \sin\theta \\ \boldsymbol{g}_2 &= -\boldsymbol{e}_1 r\sin\theta + \boldsymbol{e}_2 r\cos\theta \\ \boldsymbol{g}_3 &= \boldsymbol{e}_3 \end{aligned} \tag{A.55}$$

The co- and contravariant metric coefficients are:

$$\left(g_{ij}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \left(g^{ij}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A.56}$$

The contravariant base vectors are obtained from:

$$\begin{aligned} \boldsymbol{g}^i &= g^{ij}\boldsymbol{g}_j \\ \boldsymbol{g}^1 &= g^{11}\boldsymbol{g}_1 + g^{12}\boldsymbol{g}_2 + g^{13}\boldsymbol{g}_3 \\ \boldsymbol{g}^2 &= g^{21}\boldsymbol{g}_1 + g^{22}\boldsymbol{g}_2 + g^{23}\boldsymbol{g}_3 \\ \boldsymbol{g}^3 &= g^{31}\boldsymbol{g}_1 + g^{32}\boldsymbol{g}_2 + g^{33}\boldsymbol{g}_3 \end{aligned} \tag{A.57a}$$

Since the mixed metric coefficient are zero, (A.57a) reduces to:

$$\boldsymbol{g}^1 = g^{11}\boldsymbol{g}_1 , \ \boldsymbol{g}^2 = g^{22}\boldsymbol{g}_2, \ \boldsymbol{g}^3 = g^{33}\boldsymbol{g}_3 \tag{A.57b}$$

A.6.4 Christoffel Symbols

The Christoffel symbols are calculated from Eq. (A.30)

$$\Gamma^k_{ml} = \Gamma^k_{lm} = \frac{1}{2} g^{(kk)} \left(g_{mk,l} + g_{kl,m} - g_{lm,k}\right) \tag{A.58}$$

To follow the calculation procedure, one zero- element and one non-zero element are calculated:

$$\begin{aligned} \Gamma^1_{11} &= \frac{1}{2} g^{11} \left(\frac{\partial g_{11}}{\partial \xi_1} + \frac{\partial g_{11}}{\partial \xi_1} - \frac{\partial g_{11}}{\partial \xi_1} \right) = 0 \\ \Gamma^1_{22} &= \frac{1}{2} g^{11} \left(\frac{\partial g_{21}}{\partial \xi_2} + \frac{\partial g_{12}}{\partial \xi_2} - \frac{\partial g_{22}}{\partial \xi_1} \right) = -r \end{aligned} \tag{A.59}$$

All other elements are calculated similarly. They are shown in the following matrices:

$$\left(\Gamma^1_{lm}\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -r & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \left(\Gamma^2_{lm}\right) = \begin{pmatrix} 0 & 1/r & 0 \\ 1/r & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \left(\Gamma^3_{lm}\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.60}$$

Introducing the non-zero Christoffel symbols into Eq. (A.50), the components in g_l, g_2, and g_3 directions are:

$$V^1 V^1_{,1} + V^2 V^1_{,2} + V^3 V^1_{,3} + \Gamma^1_{22} V^2 V^2 = -\frac{1}{\rho} g^{11} p_{,1} \tag{A.61}$$

$$V^1 V^2_{,1} + V^2 V^2_{,2} + V^3 V^2_{,3} + 2\Gamma^2_{21} V^2 V^1 = -\frac{1}{\rho} g^{22} p_{,2} \tag{A.63}$$

$$V^1 V^3_{,1} + V^2 V^3_{,2} + V^3 V^3_{,3} = -\frac{1}{\rho} g^{33} p_{,3} \tag{A.62}$$

A.6.5 Introduction of Physical Components

The physical components can be calculated from Eqs. (A.21) and (A.24):

$$V^*_i = \sqrt{g^{(ii)}}\, V_i, \quad V^{*i} = \sqrt{g_{(ii)}}\, V^i$$

$$V^{*1} = \sqrt{g_{(11)}}\, V_1, \quad V^{*2} = \sqrt{g_{(22)}}\, V^2; \quad V^{*3} = \sqrt{g_{33}}\, V^3 \tag{A.64}$$

$$V^{*1} = \sqrt{1}\, V^1; V^{*2} = \sqrt{r^2}\, V^2; \quad V^{*3} = \sqrt{1}\, V^3$$

The V^i -components expressed in terms of V^{*i} are:

$$V^1 = V^{*1}\ ;\ V^2 = \frac{1}{r} V^{*2}\ ;\ V^3 = V^{*3} \tag{A.65}$$

Introducing Eqs.(A.65) into (A.61), (A.62), and (A.63) results in:

$$V^{*1} V^{*1}_{,1} + \frac{V^{*2}}{r} V^{*1}_{,2} + V^{*3} V^{*1}_{,3} + \Gamma^1_{22} \frac{V^{*2} V^{*2}}{r^2} = -\frac{1}{\rho} g^{11} p_{,1} \tag{A.66}$$

$$V^{*1} \frac{V^{*2}_{,1}}{r} - V^{*1} \frac{V^{*2}}{r^2} + \frac{V^{*2}}{r^2} V^{*2}_{,2} + V^{*3} \frac{V^{*2}_{,3}}{r} + \frac{2}{r} \Gamma^2_{21} V^{*2} V^{*1} = -\frac{1}{\rho} g^{22} p_{,2} \tag{A.67}$$

$$V^{*1} V^{*3}_{,1} + \frac{V^{*2}}{r} V^{*3}_{,2} + V^{*3} V^{*3}_{,3} = -\frac{1}{\rho} g^{33} p_{,2} \tag{A.68}$$

According to the definition:

$$\xi_1 = r;\ \xi_2 = \Theta;\ \xi_3 = z \tag{A.69}$$

the physical components of the velocity vectors are:

$$V^{*1} = V_r;\ V^{*2} = V_\Theta;\ V^{*3} = V_z \tag{A.70}$$

and insert these relations into Eqs. (A.66) to (A.68), the resulting components in r, Θ, and z directions are:

$$\begin{aligned}
&V_r \frac{\partial V_r}{\partial r} + \frac{V_\Theta}{r}\frac{\partial V_r}{\partial \Theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\Theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}\\
&V_r \frac{\partial V_\Theta}{\partial r} + \frac{V_\Theta}{r}\frac{\partial V_\Theta}{\partial \Theta} + V_z \frac{\partial V_\Theta}{\partial z} + \frac{V_r V_\Theta}{r} = -\frac{1}{\rho}\frac{\partial p}{r\partial \Theta}\\
&V_r \frac{\partial V_z}{\partial r} + \frac{V_\Theta}{r}\frac{\partial V_z}{\partial \Theta} + V_z \frac{\partial V_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z}
\end{aligned} \tag{A.71}$$

A.7 Application Example 2: Viscous Flow Motion

As the second application example, the Navier-Stokes equation of motion for a viscous incompressible flow is transferred into a cylindrical coordinate system, where it is decomposed in its three components r, θ, z. The coordinate invariant version of the equation is written as:

$$\boldsymbol{V} \cdot \nabla \boldsymbol{V} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{V} \tag{A.72}$$

The second term on the right hand side of Eq. (A.72) exhibits the shear stress force. It was treated in section A.5, Eq. (A.39) and is the only term that has been added to the equation of motion for inviscid flow, Eq. (A.40).

A.7.1 Equation of Motion in Curvilinear Coordinate Systems

The transformation and decomposition procedure is similar to the example in section A. 6. Therefore, a step by step derivation is not necessary.

$$\begin{aligned}
\boldsymbol{g}_i\left(V^j V^i_{,j} + V^j V^k \Gamma^i_{kj}\right) = -\frac{1}{\rho}\boldsymbol{g}_i g^{ji} p_{,j} + \nu \boldsymbol{g}_m\Big[V^m_{,ik} +{}&\\
V^n_{,i}\Gamma^m_{nk} + V^n_{,k}\Gamma^m_{ni} - V^m_{,j}\Gamma^j_{ik} +{}&\\
V^p\left(\Gamma^n_{pi}\Gamma^m_{nk} - \Gamma^j_{ik}\Gamma^m_{pj} + \Gamma^m_{pi,k}\right)\Big]g^{ik}&
\end{aligned} \tag{A.73}$$

A.7.2 Special Case: Cylindrical Coordinate System

Using the Christoffel symbols from section A.6.4 and the physical components from A.6.5, and inserting the corresponding relations these relations into Eqs. (A.73), the resulting components in r, Θ, and z directions are:

$$V_r \frac{\partial V_r}{\partial r} + \frac{V_\Theta}{r}\frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\Theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} +$$
$$\nu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \Theta^2} + \frac{\partial^2 V_r}{\partial z^2} - 2\frac{\partial V_\Theta}{r^2 \partial \Theta} + \frac{\partial V_r}{r \partial r} - \frac{V_r}{r^2}\right) \tag{A.74}$$

$$V_r \frac{\partial V_\Theta}{\partial r} + \frac{V_\Theta}{r}\frac{\partial V_\Theta}{\partial \Theta} + V_z \frac{\partial V_\Theta}{\partial z} + \frac{V_r V_\Theta}{r} = -\frac{1}{\rho}\frac{\partial p}{r \partial \Theta}$$
$$+ \nu\left(\frac{\partial^2 V_\Theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V_\Theta}{\partial \Theta^2} + \frac{\partial^2 V_\Theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial V_r}{r^2 \partial \Theta} + \frac{1}{r}\frac{\partial V_\Theta}{\partial r} - \frac{V_\Theta}{r^2}\right) \tag{A.75}$$
$$V_r \frac{\partial V_z}{\partial r} + \frac{V_\Theta}{r}\frac{\partial V_z}{\partial z} + V_z \frac{\partial V_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$
$$+ \nu\left[\frac{\partial^2 V_z}{\partial r^2} + \frac{\partial^2 V_z}{r^2 \partial \Theta^2} + \frac{\partial^2 V_z}{\partial z^2} + \frac{1}{r}\frac{\partial V_z}{\partial r}\right]$$

References

1. Aris, R.: Vector, Tensors and the Basic Equations of Fluid Mechanics. Prentice-Hall, Englewood Cliffs (1962)
2. Brand, L.: Vector and Tensor Analysis. John Wiley and Sons, New York (1947)
3. Klingbeil, E.: Tensorrechnung für Ingenieure. Bibliographisches Institut, Mannheim (1966)
4. Lagally, M.: Vorlesung über Vektorrechnung, 3rd edn. Akademische Verlagsgesellschaft, Leipzig (1944)
5. Vavra, M.H.: Aero-Thermodynamics and Flow in Turbomachines. John Wiley & Sons, Chichester (1960)

B Physical Properties of Dry Air

Table B.1

Enthalpy *h*, specific heat at constant pressure c_p, entropy *s*, viscosity μ and thermal conductivity κ as a function of temperature *T* pressure p = 1 bar.

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
0.000	0.010	1.003	6.774	17.294	24.210
10.000	10.043	1.003	6.811	17.744	24.893
20.000	20.080	1.004	6.845	18.190	25.571
30.000	30.121	1.004	6.879	18.632	26.243
40.000	40.167	1.005	6.912	19.069	26.910
50.000	50.219	1.005	6.943	19.503	27.572
60.000	60.277	1.006	6.974	19.933	28.229
70.000	70.343	1.007	7.004	20.359	28.880
80.000	80.417	1.008	7.033	20.781	29.527
90.000	90.500	1.009	7.061	21.199	30.169
100.000	100.593	1.010	7.088	21.613	30.806
110.000	110.697	1.011	7.115	22.024	31.439
120.000	120.812	1.012	7.141	22.431	32.067
130.000	130.940	1.013	7.166	22.834	32.690
140.000	141.080	1.015	7.191	23.234	33.309
150.000	151.235	1.016	7.216	23.630	33.924
160.000	161.404	1.018	7.239	24.023	34.534
170.000	171.588	1.019	7.263	24.412	35.140
180.000	181.788	1.021	7.285	24.798	35.742
190.000	192.004	1.022	7.308	25.180	36.340
200.000	202.238	1.024	7.329	25.559	36.934
210.000	212.489	1.026	7.351	25.935	37.524
220.000	222.759	1.028	7.372	26.308	38.110
230.000	233.047	1.030	7.393	26.677	38.692
240.000	243.355	1.032	7.413	27.043	39.271
250.000	253.683	1.034	7.433	27.407	39.846
260.000	264.032	1.036	7.452	27.767	40.417
270.000	274.401	1.038	7.472	28.124	40.985
280.000	284.791	1.040	7.491	28.478	41.549
290.000	295.203	1.042	7.509	28.829	42.110
300.000	305.637	1.044	7.528	29.177	42.667

M.T. Schobeiri: Fluid Mechanics for Engineers, pp. 489–497.
© Springer Berlin Heidelberg 2010

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
300.000	305.637	1.044	7.528	29.177	42.667
310.000	316.093	1.047	7.546	29.523	43.221
320.000	326.572	1.049	7.564	29.865	43.772
330.000	337.074	1.051	7.581	30.205	44.320
340.000	347.598	1.054	7.598	30.542	44.865
350.000	358.146	1.056	7.615	30.877	45.406
360.000	368.718	1.058	7.632	31.209	45.945
370.000	379.313	1.061	7.649	31.538	46.481
380.000	389.932	1.063	7.665	31.864	47.013
390.000	400.575	1.065	7.681	32.188	47.543
400.000	411.242	1.068	7.697	32.510	48.070
410.000	421.933	1.070	7.713	32.829	48.595
420.000	432.648	1.073	7.729	33.145	49.116
430.000	443.388	1.075	7.744	33.459	49.635
440.000	454.151	1.078	7.759	33.771	50.151
450.000	464.939	1.080	7.774	34.081	50.665
460.000	475.751	1.082	7.789	34.388	51.177
470.000	486.587	1.085	7.804	34.693	51.685
480.000	497.448	1.087	7.818	34.995	52.192
490.000	508.332	1.090	7.833	35.296	52.696
500.000	519.240	1.092	7.847	35.594	53.197
510.000	530.172	1.094	7.861	35.890	53.697
520.000	541.128	1.097	7.875	36.184	54.194
530.000	552.107	1.099	7.889	36.476	54.688
540.000	563.110	1.101	7.902	36.766	55.181
550.000	574.135	1.104	7.916	37.054	55.671
560.000	585.184	1.106	7.929	37.340	56.160
570.000	596.256	1.108	7.942	37.624	56.646
580.000	607.351	1.111	7.955	37.907	57.130
590.000	618.468	1.113	7.968	38.187	57.612
600.000	629.607	1.115	7.981	38.465	58.092
610.000	640.769	1.117	7.994	38.742	58.570
620.000	651.952	1.119	8.006	39.017	59.046
630.000	663.157	1.122	8.019	39.290	59.521
640.000	674.384	1.124	8.031	39.561	59.993
650.000	685.631	1.126	8.044	39.831	60.464
660.000	696.900	1.128	8.056	40.099	60.932
670.000	708.190	1.130	8.068	40.365	61.399
680.000	719.500	1.132	8.080	40.630	61.864
690.000	730.830	1.134	8.091	40.893	62.327
700.000	742.180	1.136	8.103	41.155	62.789

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
710.000	753.550	1.138	8.115	41.415	63.249
720.000	764.940	1.140	8.126	41.673	63.707
730.000	776.349	1.142	8.138	41.930	64.163
740.000	787.777	1.144	8.149	42.186	64.618
750.000	799.223	1.146	8.160	42.440	65.071
760.000	810.689	1.147	8.172	42.692	65.522
770.000	822.172	1.149	8.183	42.944	65.972
780.000	833.674	1.151	8.194	43.193	66.420
790.000	845.193	1.153	8.204	43.442	66.866
800.000	856.730	1.155	8.215	43.689	67.311
810.000	868.284	1.156	8.226	43.935	67.754
820.000	879.855	1.158	8.237	44.180	68.196
830.000	891.443	1.160	8.247	44.423	68.636
840.000	903.047	1.161	8.258	44.665	69.075
850.000	914.669	1.163	8.268	44.906	69.511
860.000	926.306	1.165	8.278	45.146	69.947
870.000	937.959	1.166	8.289	45.384	70.381
880.000	949.627	1.168	8.299	45.621	70.813
890.000	961.311	1.169	8.309	45.857	71.243
900.000	973.011	1.171	8.319	46.093	71.672
910.000	984.725	1.172	8.329	46.326	72.100
920.000	996.454	1.174	8.339	46.559	72.526
930.000	1.008.198	1.175	8.348	46.791	72.950
940.000	1.019.956	1.177	8.358	47.022	73.373
950.000	1.031.728	1.178	8.368	47.251	73.794
960.000	1.043.515	1.179	8.377	47.480	74.213
970.000	1.055.315	1.181	8.387	47.708	74.631
980.000	1.067.129	1.182	8.396	47.934	75.047
990.000	1.078.956	1.183	8.406	48.160	75.462
1.000.000	1.090.796	1.185	8.415	48.385	75.875
1.010.000	1.102.650	1.186	8.424	48.609	76.286
1.020.000	1.114.516	1.187	8.434	48.832	76.696
1.030.000	1.126.395	1.189	8.443	49.054	77.104
1.040.000	1.138.287	1.190	8.452	49.275	77.511
1.050.000	1.150.191	1.191	8.461	49.495	77.915
1.060.000	1.162.108	1.192	8.470	49.714	78.318
1.070.000	1.174.036	1.193	8.479	49.932	78.719
1.080.000	1.185.977	1.195	8.488	50.150	79.119
1.090.000	1.197.929	1.196	8.496	50.367	79.516
1.100.000	1.209.893	1.197	8.505	50.583	79.912

Enthalpy *h* , specific heat at constant pressure c_p, entropy *s*, viscosity μ and thermal conductivity κ as a function of temperature *T* pressure p = 5.0 bar.

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
0.000	0.0100	1,003	6.12	17.294	24.210
10.000	10.043	1.003	6.349	17.744	24.893
20.000	20.080	1.004	6.383	18.190	25.571
30.000	30.121	1.004	6.417	18.632	26.243
40.000	40.167	1.005	6.450	19.069	26.910
50.000	50.219	1.005	6.481	19.503	27.572
60.000	60.277	1.006	6.512	19.933	28.229
70.000	70.343	1.007	6.542	20.359	28.880
80.000	80.417	1.008	6.571	20.781	29.527
90.000	90.500	1.009	6.599	21.199	30.169
100.000	100.593	1.010	6.626	21.613	30.806
110.000	110.697	1.011	6.653	22.024	31.439
120.000	120.812	1.012	6.679	22.431	32.067
130.000	130.940	1.013	6.704	22.834	32.690
140.000	141.080	1.015	6.729	23.234	33.309
150.000	151.235	1.016	6.754	23.630	33.924
160.000	161.404	1.018	6.777	24.023	34.534
170.000	171.588	1.019	6.801	24.412	35.140
180.000	181.788	1.021	6.823	24.798	35.742
190.000	192.004	1.022	6.846	25.180	36.340
200.000	202.238	1.024	6.868	25.559	36.934
210.000	212.489	1.026	6.889	25.935	37.524
220.000	222.759	1.028	6.910	26.308	38.110
230.000	233.048	1.030	6.931	26.677	38.692
240.000	243.356	1.032	6.951	27.043	39.271
250.000	253.684	1.034	6.971	27.407	39.846
260.000	264.032	1.036	6.990	27.767	40.417
270.000	274.401	1.038	7.010	28.124	40.985
280.000	284.791	1.040	7.029	28.478	41.549
290.000	295.203	1.042	7.047	28.829	42.110
300.000	305.637	1.044	7.066	29.177	42.667

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
310.000	316.093	1.047	7.084	29.523	43.221
320.000	326.572	1.049	7.102	29.865	43.772
330.000	337.074	1.051	7.119	30.205	44.320
340.000	347.598	1.054	7.136	30.542	44.865
350.000	358.146	1.056	7.154	30.877	45.406
360.000	368.718	1.058	7.170	31.209	45.945
370.000	379.313	1.061	7.187	31.538	46.481
380.000	389.932	1.063	7.203	31.864	47.013
390.000	400.575	1.065	7.220	32.188	47.543
400.000	411.242	1.068	7.235	32.510	48.070
410.000	421.933	1.070	7.251	32.829	48.595
420.000	432.648	1.073	7.267	33.145	49.116
430.000	443.388	1.075	7.282	33.459	49.635
440.000	454.151	1.078	7.297	33.771	50.151
450.000	464.939	1.080	7.312	34.081	50.665
460.000	475.751	1.082	7.327	34.388	51.177
470.000	486.587	1.085	7.342	34.693	51.685
480.000	497.448	1.087	7.356	34.995	52.192
490.000	508.332	1.090	7.371	35.296	52.696
500.000	519.240	1.092	7.385	35.594	53.197
510.000	530.172	1.094	7.399	35.890	53.697
520.000	541.128	1.097	7.413	36.184	54.194
530.000	552.107	1.099	7.427	36.476	54.688
540.000	563.110	1.101	7.440	36.766	55.181
550.000	574.135	1.104	7.454	37.054	55.671
560.000	585.184	1.106	7.467	37.340	56.160
570.000	596.256	1.108	7.480	37.624	56.646
580.000	607.351	1.111	7.493	37.907	57.130
590.000	618.468	1.113	7.506	38.187	57.612
600.000	629.607	1.115	7.519	38.465	58.092
610.000	640.769	1.117	7.532	38.742	58.570
620.000	651.952	1.119	7.545	39.017	59.046
630.000	663.157	1.122	7.557	39.290	59.521
640.000	674.384	1.124	7.569	39.561	59.993
650.000	685.631	1.126	7.582	39.831	60.464
660.000	696.900	1.128	7.594	40.099	60.932
670.000	708.190	1.130	7.606	40.365	61.399
680.000	719.500	1.132	7.618	40.630	61.864
690.000	730.830	1.134	7.630	40.893	62.327
700.000	742.180	1.136	7.641	41.155	62.789

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
710.000	753.550	1.138	7.653	41.415	63.249
720.000	764.940	1.140	7.664	41.673	63.707
730.000	776.349	1.142	7.676	41.930	64.163
740.000	787.777	1.144	7.687	42.186	64.618
750.000	799.223	1.146	7.698	42.440	65.071
760.000	810.689	1.147	7.710	42.692	65.522
770.000	822.172	1.149	7.721	42.944	65.972
780.000	833.674	1.151	7.732	43.193	66.420
790.000	845.193	1.153	7.743	43.442	66.866
800.000	856.730	1.155	7.753	43.689	67.311
810.000	868.284	1.156	7.764	43.935	67.754
820.000	879.855	1.158	7.775	44.180	68.196
830.000	891.443	1.160	7.785	44.423	68.636
840.000	903.047	1.161	7.796	44.665	69.075
850.000	914.669	1.163	7.806	44.906	69.511
860.000	926.306	1.165	7.816	45.146	69.947
870.000	937.959	1.166	7.827	45.384	70.381
880.000	949.627	1.168	7.837	45.621	70.813
890.000	961.311	1.169	7.847	45.857	71.243
900.000	973.011	1.171	7.857	46.093	71.672
910.000	984.725	1.172	7.867	46.326	72.100
920.000	996.454	1.174	7.877	46.559	72.526
930.000	1.008.198	1.175	7.887	46.791	72.950
940.000	1.019.956	1.177	7.896	47.022	73.373
950.000	1.031.728	1.178	7.906	47.251	73.794
960.000	1.043.515	1.179	7.916	47.480	74.213
970.000	1.055.315	1.181	7.925	47.708	74.631
980.000	1.067.129	1.182	7.934	47.934	75.047
990.000	1.078.956	1.183	7.944	48.160	75.462
1.000.000	1.090.796	1.185	7.953	48.385	75.875
1.010.000	1.102.650	1.186	7.963	48.609	76.286
1.020.000	1.114.516	1.187	7.972	48.832	76.696
1.030.000	1.126.395	1.189	7.981	49.054	77.104
1.040.000	1.138.287	1.190	7.990	49.275	77.511
1.050.000	1.150.191	1.191	7.999	49.495	77.915
1.060.000	1.162.108	1.192	8.008	49.714	78.318
1.070.000	1.174.036	1.193	8.017	49.932	78.719
1.080.000	1.185.977	1.195	8.026	50.150	79.119
1.090.000	1.197.929	1.196	8.035	50.367	79.516
1.100.000	1.209.893	1.197	8.043	50.583	79.912

Enthalpy *h* , specific heat at constant pressure c_p, entropy *s*, viscosity μ and thermal conductivity κ as a function of temperature *T* pressure p = 10 bar.

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
0.000	0.010	1.003	6.114	17.294	24.210
10.000	10.043	1.003	6.150	17.744	24.893
20.000	20.080	1.004	6.184	18.190	25.571
30.000	30.121	1.004	6.218	18.632	26.243
40.000	40.167	1.005	6.251	19.069	26.910
50.000	50.219	1.005	6.282	19.503	27.572
60.000	60.277	1.006	6.313	19.933	28.229
70.000	70.343	1.007	6.343	20.359	28.880
80.000	80.417	1.008	6.372	20.781	29.527
90.000	90.500	1.009	6.400	21.199	30.169
100.000	100.593	1.010	6.427	21.613	30.806
110.000	110.697	1.011	6.454	22.024	31.439
120.000	120.812	1.012	6.480	22.431	32.067
130.000	130.940	1.013	6.506	22.834	32.690
140.000	141.080	1.015	6.530	23.234	33.309
150.000	151.235	1.016	6.555	23.630	33.924
160.000	161.404	1.018	6.578	24.023	34.534
170.000	171.588	1.019	6.602	24.412	35.140
180.000	181.788	1.021	6.624	24.798	35.742
190.000	192.004	1.022	6.647	25.180	36.340
200.000	202.238	1.024	6.669	25.559	36.934
210.000	212.489	1.026	6.690	25.935	37.524
220.000	222.759	1.028	6.711	26.308	38.110
230.000	233.047	1.030	6.732	26.677	38.692
240.000	243.355	1.032	6.752	27.043	39.271
250.000	253.683	1.034	6.772	27.407	39.846
260.000	264.032	1.036	6.792	27.767	40.417
270.000	274.401	1.038	6.811	28.124	40.985
280.000	284.791	1.040	6.830	28.478	41.549
290.000	295.203	1.042	6.848	28.829	42.110
300.000	305.637	1.044	6.867	29.177	42.667

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
310.000	316.093	1.047	6.885	29.523	43.221
320.000	326.572	1.049	6.903	29.865	43.772
330.000	337.074	1.051	6.920	30.205	44.320
340.000	347.598	1.054	6.938	30.542	44.865
350.000	358.146	1.056	6.955	30.877	45.406
360.000	368.718	1.058	6.971	31.209	45.945
370.000	379.313	1.061	6.988	31.538	46.481
380.000	389.932	1.063	7.004	31.864	47.013
390.000	400.575	1.065	7.021	32.188	47.543
400.000	411.242	1.068	7.037	32.510	48.070
410.000	421.933	1.070	7.052	32.829	48.595
420.000	432.648	1.073	7.068	33.145	49.116
430.000	443.388	1.075	7.083	33.459	49.635
440.000	454.151	1.078	7.098	33.771	50.151
450.000	464.939	1.080	7.113	34.081	50.665
460.000	475.751	1.082	7.128	34.388	51.177
470.000	486.587	1.085	7.143	34.693	51.685
480.000	497.448	1.087	7.158	34.995	52.192
490.000	508.332	1.090	7.172	35.296	52.696
500.000	519.240	1.092	7.186	35.594	53.197
510.000	530.172	1.094	7.200	35.890	53.697
520.000	541.128	1.097	7.214	36.184	54.194
530.000	552.107	1.099	7.228	36.476	54.688
540.000	563.109	1.101	7.241	36.766	55.181
550.000	574.135	1.104	7.255	37.054	55.671
560.000	585.184	1.106	7.268	37.340	56.160
570.000	596.256	1.108	7.281	37.624	56.646
580.000	607.350	1.111	7.295	37.907	57.130
590.000	618.468	1.113	7.307	38.187	57.612
600.000	629.607	1.115	7.320	38.465	58.092
610.000	640.768	1.117	7.333	38.742	58.570
620.000	651.952	1.119	7.346	39.017	59.046
630.000	663.157	1.122	7.358	39.290	59.521
640.000	674.383	1.124	7.370	39.561	59.993
650.000	685.631	1.126	7.383	39.831	60.464
660.000	696.900	1.128	7.395	40.099	60.932
670.000	708.190	1.130	7.407	40.365	61.399
680.000	719.500	1.132	7.419	40.630	61.864
690.000	730.830	1.134	7.431	40.893	62.327
700.000	742.180	1.136	7.442	41.155	62.789

T [C]	h [kJ/kg]	C_p [kJ/kg K]	s [kJ/kg K]	μ [kg/ms]10^6	κ [J/msK]10^3
710.000	753.550	1.138	7.454	41.415	63.249
720.000	764.940	1.140	7.465	41.673	63.707
730.000	776.349	1.142	7.477	41.930	64.163
740.000	787.776	1.144	7.488	42.186	64.618
750.000	799.223	1.146	7.499	42.440	65.071
760.000	810.688	1.147	7.511	42.692	65.522
770.000	822.172	1.149	7.522	42.944	65.972
780.000	833.673	1.151	7.533	43.193	66.420
790.000	845.193	1.153	7.544	43.442	66.866
800.000	856.730	1.155	7.554	43.689	67.311
810.000	868.284	1.156	7.565	43.935	67.754
820.000	879.855	1.158	7.576	44.180	68.196
830.000	891.443	1.160	7.586	44.423	68.636
840.000	903.047	1.161	7.597	44.665	69.075
850.000	914.668	1.163	7.607	44.906	69.511
860.000	926.305	1.165	7.617	45.146	69.947
870.000	937.958	1.166	7.628	45.384	70.381
880.000	949.627	1.168	7.638	45.621	70.813
890.000	961.311	1.169	7.648	45.857	71.243
900.000	973.010	1.171	7.658	46.093	71.672
910.000	984.725	1.172	7.668	46.326	72.100
920.000	996.454	1.174	7.678	46.559	72.526
930.000	1008.198	1.175	7.688	46.791	72.950
940.000	1019.956	1.177	7.697	47.022	73.373
950.000	1031.728	1.178	7.707	47.251	73.794
960.000	1043.514	1.179	7.717	47.480	74.213
970.000	1055.315	1.181	7.726	47.708	74.631
980.000	1067.128	1.182	7.736	47.934	75.047
990.000	1078.955	1.183	7.745	48.160	75.462
1000.000	1090.796	1.185	7.754	48.385	75.875
1010.000	1102.650	1.186	7.764	48.609	76.286
1020.000	1114.516	1.187	7.773	48.832	76.696
1030.000	1126.396	1.189	7.782	49.054	77.104
1040.000	1138.287	1.190	7.791	49.275	77.510
1050.000	1150.191	1.191	7.800	49.495	77.915
1060.000	1162.108	1.192	7.809	49.714	78.318
1070.000	1174.037	1.193	7.818	49.932	78.719
1080.000	1185.977	1.195	7.827	50.150	79.119
1090.000	1197.929	1.196	7.836	50.367	79.516
1100.000	1209.893	1.197	7.844	50.583	79.912
1.110.000	1.221.869	1.198	7.853	50.798	80.306

Index