

# Logics: The Grammars

This Appendix lists the logics which are investigated in this treatise. Some logics which are discussed merely for illustrating a particular point are omitted.

## Basic Modal Language: $\mathfrak{M}_b(\tau, P)$

The *basic modal language*  $\mathfrak{M}_b(\tau, P)$  is given by the syntax

$$\phi ::= p \mid \top \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \Delta(\phi_1, \dots, \phi_{\text{ar}(\Delta)}),$$

where  $\tau$  is a set of modal operators,  $\Delta \in \tau$  is a modal operator of arity  $\text{ar}(\Delta)$ , and  $p \in P$  with  $P$  a set of propositional letters: defined in Section 2.3 on page 76.

## The Extended Modal Language: $\mathfrak{M}_s(\tau, P)$

The extended modal language  $\mathfrak{M}_s(\tau, P)$  is defined through the syntax

$$\phi ::= p \mid \top \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \Delta_q(\phi_1, \dots, \phi_{\text{ar}(\Delta)}),$$

where  $q \in \mathbb{Q} \cap [0, 1]$  is a rational number,  $\tau$  is a set of modal operators,  $\Delta \in \tau$  is a modal operator of arity  $\text{ar}(\Delta)$ , and  $p \in P$  with  $P$  a set of propositional letters: defined in Section 2.3 on page 77.

## Hennessy-Milner Logic: $\mathfrak{L}(\mathbf{Act}, \mathbb{Q} \cap [0, 1])$

The formulas for  $\mathfrak{L}(\mathbf{Act}, \mathbb{Q} \cap [0, 1])$  are given through

$$\top \mid \phi_1 \wedge \phi_2 \mid \langle a \rangle_r \phi,$$

$a \in \mathbf{Act}$  is an action,  $r \in [0, 1] \cap \mathbb{Q}$ : Section 2.3.1 on page 79.

**Variante**  $\mathfrak{L}(\mathbf{Act}, [0, 1])$ :  $r$  may take arbitrary real values in the unit interval  $[0, 1]$ ; the variant defined in Section 2.6 on page 100, see also page 133. The discussion in Section 3.4 is based on it.

## Continuous Time Stochastic Logic With Fixed-Point Operators: $\mu\mathbf{CSL}(\mathbf{AP}, \mathbf{SV}, \mathbf{PV})$

This logic is usually abbreviated as  $\mu\mathbf{CSL}$ .

- *State formulas*:

$$\phi ::= \top \mid a \mid Z \mid \neg\phi \mid \phi \wedge \phi' \mid \mathcal{S}_{\times p}(\phi) \mid \mathcal{P}_{\times p}(\psi).$$

$a \in \mathbf{AP}$  is an atomic proposition,  $Z \in \mathbf{SV}$  is a state variable,  $\psi$  is a path formula,  $\times \in \{<, \leq, >, \geq\}$  is a relational operator, and  $p \in [0, 1]$  is a rational number.

- *Path formulas*:

$$\psi ::= P \mid \neg\psi \mid \psi \wedge \psi' \mid \mathcal{X}^I \psi \mid \phi \mathcal{U}^I \phi' \mid \mu P.\psi.$$

$P \in \mathbf{PV}$  is a path variable,  $\phi, \phi'$  are state formulas,  $I \subseteq \mathbb{R}_+$  is a closed interval with rational bounds.

These operators are available in  $\mu\mathbf{CSL}$ :

1.  $\mathcal{S}_{\times p}(\phi)$ : steady state probability.
2.  $\mathcal{P}_{\times p}(\psi)$ : path quantifier.
3.  $\mathcal{X}^I \phi$ : next operator;  $I$  is an interval.
4.  $\phi_1 \mathcal{U}^I \phi_2$ : until-operator;  $I$  is an interval.

The syntax von  $\mu\mathbf{CSL}$  is defined in Section 2.4.1 on page 84.

## Coalgebraic Logic (Left Case): $\mathfrak{L}^b(\Lambda, \Theta, \Gamma)$

$\Lambda$  and  $\Theta$  are at most countable sets of predicate liftings resp.  $\mathfrak{T}$ -predicate liftings (Definition 4.2.2).  $\mathfrak{L}^b = \mathfrak{L}^b(\Lambda, \Theta, \Gamma)$  is defined through

- *State formulas*:  $\phi ::= \top \mid \phi_1 \wedge \phi_2 \mid \langle \lambda \rangle \phi \mid \langle \vartheta \rangle \psi$  with  $\lambda \in \Lambda$  a predicate lifting and  $\vartheta \in \Theta$  a  $\mathfrak{T}$ -predicate lifting.
- *$\mathfrak{T}$ -formulas*:  $\psi ::= \widetilde{\top} \mid \psi_1 \wedge \psi_2 \mid b(\phi_1, \dots, \phi_{\text{ar}(b)})$  with  $b \in \Gamma$  a bridge operator with arity  $\text{ar}(b)$  (Definition 4.2.6).

$\mathfrak{L}^b$  is defined in Section 4.3.2 on page 167.

The logic  $\mathfrak{L}^b(\emptyset, \Theta, \Gamma)$  is discussed as an application of left coalgebras to modal logic in Section 4.3.4, it is given on page 182.

## Coalgebraic Logic (Right Case): $\mathfrak{L}^\sharp(\Lambda, \text{Br}, \text{Inf}, \mathbf{V})$

$\Lambda, \text{Br}$  and  $\text{Inf}$  are countable sets of  $\mathfrak{T}^R$ -predicate liftings, bridge operators and infinitesimal operators (Definition 4.4.17), resp.,  $\mathbf{V}$  is a countable set of variables.

The formulas of the (*full*) logic  $\mathfrak{L}^\sharp = \mathfrak{L}^\sharp(\Lambda, \text{Br}, \text{Inf}, \mathbf{V})$  are given through

$$\phi ::= \top \mid \phi_1 \wedge \phi_2 \mid \langle \lambda \rangle \phi \mid \beta(\phi_1, \dots, \phi_{\partial(\beta)}) \mid x \mid \iota x. \phi$$

with  $\lambda \in \Lambda$ ,  $\beta \in \text{Br}$ ,  $x \in \mathbf{V}$  and  $\iota \in \text{Inf}$ . The *basic* or *kernel logic* is the special case  $\mathfrak{L}_b^\sharp = \mathfrak{L}^\sharp(\Lambda, \emptyset, \emptyset, \emptyset)$ , given by the grammar

$$\phi ::= \top \mid \phi_1 \wedge \phi_2 \mid \langle \lambda \rangle \phi$$

for  $\lambda \in \Lambda$ . These logics are defined in Section 4.4.2 on page 193.

# References

- [1] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions*. Dover Publications, New York, 1965.
- [2] W. Arveson. *An Invitation to  $C^*$ -Algebra*. Graduate Texts in Mathematics. Springer-Verlag, New York, 1976.
- [3] C. Baier, B. Haverkort, H. Hermanns, and J.-P. Katoen. Model-checking algorithms for continuous time Markov chains. *IEEE Trans. Softw. Eng.*, 29(6):524 – 541, June 2003.
- [4] M. Barr and C. Wells. *Category Theory for Computing Science*. Les Publications CRM, Montreal, 1999.
- [5] H. Bauer. *Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie*. Walter de Gruyter, Berlin, 1968.
- [6] P. Billingsley. *Convergence of Probability Measures*. John Wiley & Sons, New York, 1st edition, 1968.
- [7] P. Billingsley. *Convergence of Probability Measures*. John Wiley & Sons, New York, 2nd edition, 1999.
- [8] P. Blackburn, M. de Rijke, and Y. Venema. *Modal Logic*. Number 53 in Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, Cambridge, UK, 2001.
- [9] P. Blackburn, J. van Benthem, and F. Wolter, editors. *Handbook of Modal Logic*, volume 3 of *Studies in Logic and Practical Reasoning*. Elsevier, Amsterdam, 2007.
- [10] M. M. Bonsangue and A. Kurz. Duality of logics of transition systems. In V. Sassone, editor, *Proc. FOSSACS'05*, volume 3441 of *LNCS*, pages 455–469, 2005.
- [11] F. Borceux. *Handbook of Categorical Algebra 1: Basic Category Theory*, volume 50 of *Encyclopedia of Mathematics and Its Applications*. Cambridge University Press, Cambridge, UK, 1994.
- [12] F. Borceux. *Handbook of Categorical Algebra 2: Categories and Structures*, volume 51 of *Encyclopedia of Mathematics and Its Applications*. Cambridge University Press, Cambridge, UK, 1994.

- [13] S. Burris and H. P. Sankappanavar. *A Course in Universal Algebra*. Springer-Verlag (The Millennium Edition), 1981.
- [14] C. Castaing and M. Valadier. *Convex Analysis and Measurable Multifunctions*. Number 580 in Lect. Notes Math. Springer-Verlag, Berlin, Heidelberg, New York, 1977.
- [15] C. Cirstea and D. Pattinson. Modular construction on modal logics. In P. Gardner and N. Yoshida, editors, *Proc. CONCUR'04*, number 3170 in Lect. Notes Comp. Sci., pages 258 – 275, 2004.
- [16] E. M. Clarke, O. Grumberg, and D. A. Peled. *Model Checking*. The MIT Press, Cambridge, MA, 1999.
- [17] J. Desharnais, A. Edalat, and P. Panangaden. Bisimulation of labelled Markov-processes. *Information and Computation*, 179(2):163 – 193, 2002.
- [18] E.-E. Doberkat. Eilenberg-Moore algebras for stochastic relations. *Information and Computation*, 204:1756 – 1781, 2006.
- [19] E.-E. Doberkat. Semi-pullbacks for stochastic relations over analytic spaces. *Math. Struct. Comp. Sci.*, 15:647 – 670, 2005.
- [20] E.-E. Doberkat. *Stochastic Relations. Foundations for Markov Transition Systems*. Chapman & Hall/CRC Press, Boca Raton, New York, 2007.
- [21] E.-E. Doberkat. Stochastic coalgebraic logic: Bisimilarity and behavioral equivalence. *Ann. Pure Appl. Logic*, 155:46 – 68, 2008.
- [22] E.-E. Doberkat. Stochastic relations: congruences, bisimulations and the Hennessy-Milner theorem. *SIAM J. Computing*, 35(3):590 – 626, 2006.
- [23] E.-E. Doberkat. The converse of a probabilistic relation. *J. Logic and Algebraic Progr.*, 62(1):133 – 154, 2004.
- [24] E.-E. Doberkat. Factoring stochastic relations. *Information Processing Letters*, 90(4):161 – 166, May 2004.
- [25] E.-E. Doberkat. Kleisli morphisms and randomized congruences for the Giry monad. *J. Pure Appl. Alg.*, 211:638–664, 2007.
- [26] E.-E. Doberkat. Erratum and Addendum: Eilenberg-Moore algebras for stochastic relations. *Information and Computation*, 206:1476 – 1484, 2008.
- [27] E.-E. Doberkat. Semi-pullbacks and bisimulations in categories of stochastic relations. In *Proc. ICALP'03*, volume 2719 of *Lect. Notes Comp. Sci.*, pages 996 – 1007, Berlin, 2003. Springer-Verlag.
- [28] E.-E. Doberkat. Lattice properties of congruences for stochastic relations. Technical Report 178, Chair for Software Technology, Technische Universität Dortmund, March 2009.
- [29] E.-E. Doberkat and Ch. Schubert. Coalgebraic logic for stochastic right coalgebras. *Ann. Pure Appl. Logic*, 159:268–284, 2009.
- [30] L. Dubins and D. Freedman. Measurable sets of measures. *Pac. J. Math.*, 14:1211 – 1222, 1964.
- [31] A. Edalat. Semi-pullbacks and bisimulation in categories of Markov processes. *Math. Struct. Comp. Science*, 9(5):523 – 543, 1999.

- [32] G. A. Edgar. *Integral, Probability, and Fractal Measures*. Springer-Verlag, New York, 1998.
- [33] R. Engelking. *General Topology*, volume 6 of *Sigma Series in Pure Mathematics*. Heldermann-Verlag, Berlin, revised and completed edition edition, 1989.
- [34] M. Fisz. *Wahrscheinlichkeitsrechnung und Mathematische Statistik*. VEB Deutscher Verlag der Wissenschaften, Berlin, 1971.
- [35] M. Fitting. Modal proof theory. In P. Blackburn, J. van Benthem, and F. Wolter, editors, *Handbook of Modal Logic*, volume 3 of *Studies in Logic and Practical Reasoning*, pages 85–138. Elsevier, Amsterdam, 2007.
- [36] D. H. Fremlin. *Measure Theory*, volume 1 – 4. Torres Fremlin, Colchester, 2000 – 2003.
- [37] E. Gamma, R. Helm, R. Johnson, and J. Vlissides. *Design Patterns — Elements of Reusable Object-Oriented Software*. Professional Computing Series. Addison-Wesley, Reading, Mass., 1994.
- [38] M. Giry. A categorical approach to probability theory. In *Categorical Aspects of Topology and Analysis*, number 915 in Lect. Notes Math., pages 68 – 85, Berlin, 1981. Springer-Verlag.
- [39] R. Goldblatt. Deduction systems for coalgebras over measurable spaces. *Journal of Logic and Computation*, (in print), 2008.
- [40] R. Goldblatt. *Mathematics of Modality*. Number 43 in CSLI Lecture Notes. CSLI Publications, Stanford, 1993.
- [41] I. P. Goulden and D. M. Jackson. *Combinatorial Enumeration*. Wiley-Interscience Series in Discrete Mathematics. John Wiley & Sons, New York, 1983.
- [42] G. Grätzer. *Universal Algebra*. The University Series in Higher Mathematics. Van Nostrand, Princeton, N.J., 1968.
- [43] P. R. Halmos. *Measure Theory*. Van Nostrand Reinhold, New York, 1950.
- [44] M. Hennessy and R. Milner. On observing nondeterminism and concurrency. In *Proc. ICALP'80*, number 85 in Lect. Notes Comp. Sci., pages 395 – 409, Berlin, 1980. Springer-Verlag.
- [45] E. Hewitt and K. R. Stromberg. *Real and Abstract Analysis*. Springer-Verlag, Berlin, Heidelberg, New York, 1965.
- [46] C. J. Himmelberg and F. Van Vleck. Some selection theorems for measurable functions. *Can. J. Math.*, 21:394 – 399, 1969.
- [47] K. Hinderer. *Foundations of Non-stationary Dynamic Programming with Discrete Time Parameter*. Number 33 in Lect. Notes Op. Res. Math. Syst. Springer-Verlag, Berlin, 1970.
- [48] C. Jones and G. Plotkin. A probabilistic powerdomain of evaluations. In *Proc. LICS'89*, pages 186 – 195, IEEE Computer Society Press, 1989.
- [49] A. S. Kechris. *Classical Descriptive Set Theory*. Graduate Texts in Mathematics. Springer-Verlag, Berlin, Heidelberg, New York, 1994.
- [50] J. Keener and J. Sneyd. *Mathematical Physiology*. Interdisciplinary Applied Mathematics. Springer-Verlag, 2004.

- [51] K. Keimel. The monad of probability measures over compact ordered spaces and its Eilenberg-Moore algebras. Preprint. TU Darmstadt, July 2008.
- [52] D. E. Knuth. *The Art of Computer Programming. Vol. I, Fundamental Algorithms*. Addison-Wesley, Reading, Mass., 2 edition, 1973.
- [53] C. Kupke, A. Kurz, and Y. Venema. Stone coalgebras. *Theor. Comp. Sci.*, 327(1 - 2):109 – 134, 2004.
- [54] C. Kupke, A. Kurz, and Y. Venema. Completeness of the finitary Moss logic. In C. Areces and R. Goldblatt, editors, *Adv. Mod. Logic AiML'08*, volume 193 – 218. College Publications, 2008.
- [55] K. Kuratowski. *Topology*, volume I. PWN – Polish Scientific Publishers and Academic Press, Warsaw and New York, 1966.
- [56] A. Kurz. Specifying coalgebras with modal logic. *Theor. Comp. Sci.*, 260:119 – 138, 2001.
- [57] S. Mac Lane. *Categories for the Working Mathematician*. Graduate Texts in Mathematics. Springer-Verlag, Berlin, 1997.
- [58] K. G. Larsen and A. Skou. Bisimulation through probabilistic testing. *Information and Computation*, 94:1 – 28, 1991.
- [59] G. W. Mackey. Borel structure in groups and their duals. *Trans. Am. Math. Soc.*, 85:134 – 165, 1957.
- [60] E. Moggi. Notions of computation and monads. *Information and Computation*, 93:55 – 92, 1991.
- [61] L. M. Moss. Coalgebraic logic. *Ann. Pure Appl. Logic*, 96:277 – 317, 1999.
- [62] L. M. Moss and I. D. Viglizzo. Harsanyi type spaces and final coalgebras constructed from satisfied theories. *ENTCS*, pages 279 – 295, 2004.
- [63] L. M. Moss and I. D. Viglizzo. Final coalgebras for functors on measurable spaces. *Inf. Comput.*, 204:610–636, 2006.
- [64] P. Panangaden. Probabilistic relations. In C. Baier, M. Huth, M. Kwiatkowska, and M. Ryan, editors, *Proc. PROBMIV*, pages 59 – 74, 1998.
- [65] K. R. Parthasarathy. *Probability Measures on Metric Spaces*. Academic Press, New York, 1967.
- [66] D. Pattinson. Expressive logics for coalgebras via terminal sequence induction. *Notre Dame J. Formal Logic*, 45(1):19 – 33, 2004.
- [67] D. Pumplün. Positively convex modules and ordered normed linear spaces. *J. Convex Analysis*, 10(1):109 – 127, 2003.
- [68] D. Pumplün. *Elemente der Kategorientheorie*. Spektrum Akademischer Verlag, Heidelberg, 1999.
- [69] B. v. Querenburg. *Mengentheoretische Topologie*. Springer -Lehrbuch. Springer-Verlag, Berlin, 3rd edition, 2001.
- [70] U. Rieder. Bayesian dynamic programming. *Adv. Appl. Prob.*, 7:330 – 348, 1975.
- [71] W. Rudin. *Real and Complex Analysis*. Tata McGraw-Hill, 2nd edition, 1974.

- [72] J. J. M. M. Rutten. Universal coalgebra: a theory of systems. *Theor. Comp. Sci.*, 249(1):3 – 80, 2000. Special issue on modern algebra and its applications.
- [73] S. Schneider and S. Thomas. Countable Borel equivalence relations. Lecture Notes, Appalachian Set Theory Seminar, Athens, OH, November 2007.
- [74] L. Schröder. Expressivity of coalgebraic modal logic: the limits and beyond. *Theor. Comp. Sci.*, 390:230–247, 2008.
- [75] L. Schröder and D. Pattinson. Modular algorithms for heterogeneous modal logics. In *Proc. ICALP*, number 4596 in Lect. Notes Comp. Sci., pages 459–471, 2007.
- [76] Ch. Schubert. Coalgebraic modal logic over analytic spaces. Technical Report 170, Chair for Software Technology, Technical University of Dortmund, Dezember 2007.
- [77] A. N. Shiryaev. *Probability*, volume 95 of *Graduate Texts in Mathematics*. Springer-Verlag, Berlin, Heidelberg, New York, second edition, 1996.
- [78] A. Sokolova. *Coalgebraic Analysis of Probabilistic Systems*. PhD thesis, Department of Computer Science, University of Eindhoven, 2005.
- [79] S. M. Srivastava. *A Course on Borel Sets*. Graduate Texts in Mathematics. Springer-Verlag, Berlin, 1998.
- [80] Y. Sun. Economics and nonstandard analysis. In P. A. Loeb and M. Wolff, editors, *Nonstandard Analysis for the Working Mathematician*, Mathematics and Its Applications, pages 259 – 305. Kluwer Academic Publishers, Dordrecht, 2000.
- [81] I. D. Viglizzo. Final sequences and final coalgebras for measurable spaces. In *Proc. CALCO*, number 3629 in Lect. Notes Comp. Sci., pages 395 – 407. Springer-Verlag, 2005.
- [82] D. H. Wagner. A survey of measurable selection theorems. *SIAM J. Control Optim.*, 15(5):859 – 903, August 1977.
- [83] H. S. Wilf. *generatingfunctionology*. Academic Press, Boston, 1990.
- [84] D. J. Wilkinson. *Stochastic Modelling for Systems Biology*. Mathematical and Computational Biology Series. Chapman & Hall/CRC-Press, Boca Raton, New York, 2006.
- [85] J. Worrell. On the final sequence of a finitary set functor. *Theor. Comp. Sci.*, 338(1–3):184–199, 2005.
- [86] C. Zhou. *Complete Deductive Systems for Probabilistic Logic with Application to Harsanyi Type spaces*. PhD thesis, Department of Mathematics, University of Indiana, 2007.



# List of Symbols

$\mathbf{C}(a, b)$	2	$\delta_{i,j}$	62
$\eta : \mathfrak{F} \xrightarrow{\bullet} \mathfrak{G}$	2	$\rightarrow_a$	70
$\sigma(\mathcal{M}_0)$	4	$c \propto c'$	73
$\eta_\alpha$	7	$\tau = (O, \mathbf{ar})$	76
$\chi_A$	8	$\mathbf{ar}$	76
$\mathcal{F}(N, \mathcal{N})$	9	$\llbracket \phi \rrbracket$	77
$\mathcal{B}(X, \mathcal{T}), \mathcal{B}(X)$	10	$F_\Delta(\mathcal{K})$	78
$X^\infty$	13	$Th_{\mathcal{K}}(s)$	78
$\ker(f)$	21	$\mathbf{Act}$	79
$\exists R(G)$	23	$R^\smile$	79
$\forall R(G)$	23	$\times p$	85
$\mathfrak{P}(N, \mathcal{N}), \mathfrak{S}(N, \mathcal{N})$	26	$\mathcal{S}_{\times p}(\phi)$	85
$\mu_1 \otimes \mu_2$	26	$\mathcal{P}_{\times p}(\psi)$	85
$\mathcal{C}(X)$	29	$\mathcal{X}^I \phi$	85
$\rightarrow_w$	29	$\phi_1 U^I \phi_2$	85
$\mathcal{M}^\bullet$	32	$\sigma @ t$	85
$K : (M, \mathcal{M}) \rightsquigarrow (N, \mathcal{N})$	33	$\zeta_{\mathcal{M}, \omega_{\mathcal{M}}}$	91
$\mathfrak{S}(f)(\mu)$	34	$r_{\mathcal{K}}$	102
$\mathbf{U}(x \mid a, h)$	35	$\mathbf{e}, \mathbf{m}$	115
$\mathbf{N}(x \mid m, \sigma)$	36	$\underline{g} * f$	117
$\mathbf{P}(n \mid \lambda)$	38	$\overline{K}$	118
$\mathbf{B}(i \mid n, p)$	39	$K \curvearrowright L$	120
$D_x, D^y$	40	$\mathcal{K} \curvearrowright^e \mathcal{L}$	142
$\Sigma(\mathcal{B}(X), \rho)$	47	<b>ANL, BOR</b>	159
$\times_{n \in \mathbb{N}} \rho_n$	52	$\mathfrak{B}$	159
$\alpha \diamond \beta$	53	$L\mathfrak{T}, \mathfrak{T}^R$	159
$\overline{\xi}$	56	$\mathfrak{z}$	160
$[\gamma]$	57	$\mu \blacktriangleright B$	181

# Index

$F_\sigma$ -set	12	behavioral equivalence	94
$G_\delta$ -set	12	coalgebra	4
$\mu$ CSL	84	Benthem, J. van	100
$\mu$ -operator	85	Billingsley, P.	67
behavioral equivalence	94	bisimilar	71, 94
bisimilar	94	coalgebra	3
logical equivalent models	93	states	208
next operator	84	bisimulation	70
path quantifier	84	mediator	71
steady-state	84	Blackburn, P.	79, 100
until operator	84	Bonsangue, M. M.	214
$\mu$ CSL		Borceux, F.	2, 155
morphism	86	Borel measurability	10
$\sigma$ -algebra		Borel sets	10
common events	72	invariant	47
final	6	boundary of a set	29
initial	7	bridge operator	162
product	8	Burris, S.	55, 68, 117
sum	8		
trace	7	<b>C</b>	
weak*	32	Castaing representation	24
@-operator	89	Castaing, C.	67
		category	2
<b>A</b>		Kleisli	117
Abramowitz, M.	36	Change of Variable Formula	34
admissible functor	199	Choquet representation	26
amalgamation	53, 74, 107	Clarke, E. M.	100
analytic set	17	class S	208
Arveson, W.	67, 155	coalgebra	3
atom	22, 48	behavioral equivalence	4
		bisimilar	3
<b>B</b>		dynamics	3
Baier, C.	100, 214	left	165
Barr, M.	2, 4	mediating	4
base, weak topology	29	morphism	3
Bauer, H.	115, 117	right	187
		completing functor	200

- congruence
  - randomized ..... 124
  - simulation ..... 73
  - simulation-equivalence ..... 73
- continuity ..... 10
- convolution ..... 117
- coproduct ..... 2
- cut ..... 40
  - horizontal ..... 40
  - vertical ..... 40
- cylinder sets ..... 41
- Cirstea, C. .... 157
  
- D**
- Desharnais, J. .... 67, 214
- diagonal ..... 20, 47
- Dirac measure ..... 29
- distribution
  - binomial ..... 39
  - normal ..... 36, 39
  - Poisson ..... 38
  - rectangular ..... 35
  - uniform ..... 35
- Doberkat, E.-E. .... vi, vii, 2, 16, 28, 34, 67, 68, 80, 84, 99, 100, 125, 154, 155, 185, 207, 214
- Dubins, L. .... 67
  
- E**
- Edalat, A. .... 67, 214
- Edgar, G. A. .... 67
- Engelking, R. .... 67
- equivalence relation
  - grounded ..... 57
  - lifting ..... 56
  - near-grounded ..... 57
  - smooth ..... 44
  - determining sequence ..... 44
  - spawning ..... 72
- ergodic relation ..... 142
  
- F**
- factor map ..... 7
- factor relation ..... 53
- Fisz, M. .... 35, 36
- Fitting, M. .... 100
- Freedman, D. .... 67
- Fremlin, D. H. .... 67
- function
  - indicator ..... 8
  - step ..... 8
- functor
  - absorbing ..... 177
  - admissible ..... 199
  - completing ..... 200
  - distributing ..... 176
  - probability ..... 34
  - strongly completing ..... 211
  - subprobability ..... 34, 116
  
- G**
- Gamma, E. .... 186
- generating function ..... 180
- generator
  - $\cap$ -stable ..... 40
- Giry monad ..... 116
- Giry, M. .... 154
- Goldblatt, R. .... 100, 155
- good guys ..... 40
- Goulden, I. P. .... 180
- Grätzer, G. .... 68, 117, 124
- graph of a map ..... 21
- Grumberg, O. .... 100
  
- H**
- Halmos, P. R. .... 8, 26, 67
- Haverkort, B. .... 100, 214
- Helm, R. .... 186
- Hennessy, M. .... 79, 155
- Hennessy-Milner logic ..... 79, 100
- Hermanns, H. .... 100, 214
- Hewitt, E. .... 35, 67
- Hilbert cube ..... 12, 14
- Himmelberg, C. J. .... 67
- Hinderer, K. .... 114, 155
- hom set ..... 2
- homeomorphism relation ..... 10
  
- I**
- indicator function ..... 8
- infinitesimal operator ..... 192
- invariant ..... 19
  
- J**
- Jackson, D. M. .... 180
- Johnson, R. .... 186
- Jones, C. .... 117
  
- K**
- Katoen, J.-P. .... 100, 214
- Kechris, A. S. .... 67, 155
- Keener, J. .... 115
- Keimel, K. .... 214
- Kleisli
  - category ..... 117
  - composition ..... 117
  - extension ..... 118
  - morphism ..... 117

- Knuth, D. E. . . . . 180
- Kripke model . . . . . 23
- $\mathcal{L}$ -morphism . . . . . 136
- behavioral equivalent . . . . . 82
- degenerate . . . . . 78
- distributional equivalence . . . . . 142
- logical equivalent . . . . . 82
- morfism . . . . . 102
- nondeterministic . . . . . 77
- randomized morfism . . . . . 135
- stochastic . . . . . 78
- strong morfism . . . . . 82
- strongly bisimilar . . . . . 82
- weak behavioral equivalence . . . . . 147
- Kronecker's  $\delta$  . . . . . 62
- Kupke, C. . . . . 213, 214
- Kuratowski, K. . . . . 14, 67
- Kurz, A. . . . . 213, 214
- L**
- labeled transition system . . . . . 79
- Larsen, K. G. . . . . 79, 100, 155, 214
- left coalgebra . . . . . 165
- morphism . . . . . 166
- Lipschitz condition . . . . . 31
- lives on-relation . . . . . 182
- logic
- $\mu$ CSL . . . . . 84
- coalgebraic . . . . . 167, 193
- Hennessy-Milner . . . . . 79, 100, 133, 212
- logical equivalent . . . . . 82, 93
- M**
- Mac Lane, S. . . . . 2, 155, 211, 214
- Mackey, G. W. . . . . 67
- map
- continuous . . . . . 10
- graph . . . . . 21
- kernel . . . . . 21
- measurable . . . . . 5
- Markov transition system . . . . . 79, 158
- measurable
- real-valued function . . . . . 9
- rectangle . . . . . 8
- measure
- Dirac . . . . . 29
- probability . . . . . 25
- product . . . . . 26
- projective limit . . . . . 41, 43
- projective system . . . . . 41
- subprobability . . . . . 26
- Milner, R. . . . . 79, 155
- modal language
- basic . . . . . 77
- extended . . . . . 77
- negation-free . . . . . 77
- modal logic
- arrow logic . . . . . 80
- basic temporal . . . . . 79
- modal similarity type . . . . . 77
- model . . . . . 190
- $\mu$ CSL . . . . . 85
- morphism . . . . . 86
- behavioral equivalent . . . . . 195
- bisimilar . . . . . 195
- logical equivalent . . . . . 195
- Moggi, E. . . . . 117
- monad . . . . . 115
- Giry . . . . . 116
- multiplication . . . . . 115
- unit . . . . . 115
- morfism . . . . . 102, 187
- morphism
- cospan . . . . . 4
- randomized . . . . . 120
- span . . . . . 4
- stochastic relation . . . . . 37
- Moss, L. M. . . . . 155, 157, 213
- N**
- natural transformation . . . . . 2
- next operator . . . . . 84
- P**
- Panangaden, P. . . . . 67, 154, 214
- Parthasarathy, K. R. . . . . 67
- path quantifier . . . . . 84
- Pattinson, D. . . . . 157, 158, 197, 213
- Peled, D. A. . . . . 100
- Plotkin, G. . . . . 117
- point-affine . . . . . 64
- positive convex . . . . . 62
- partition . . . . . 62
- structure . . . . . 62
- predicate lifting . . . . . 160
- probability functor . . . . . 34
- product . . . . . 2
- projective limit . . . . . 88
- projective system . . . . . 88
- pullback . . . . . 3
- semi- . . . . . 3
- weak . . . . . 3
- Pumplün, D. . . . . 2, 62
- Q**
- Querenburg, B. v. . . . . 67
- R**
- randomized congruence . . . . . 124

grounded ..... 124  
 randomized morphism ..... 120  
 near-grounded ..... 126  
 reader  
 gentle ..... 132  
 patient ..... vii  
 relation  
 $\mathcal{C}$ -measurable ..... 24  
 measurable ..... 23  
 Castaing representation ..... 24  
 satisfaction ..... 76  
 stochastic  
 converse ..... 79  
 direct sum ..... 73  
 strong inverse ..... 23  
 weak inverse ..... 23  
 weakly measurable ..... 23  
 Rieder, U. .... 67  
 right coalgebra ..... 187  
 morfism ..... 187  
 Rijke, M. de ..... 79, 100  
 Rudin, W. .... 35  
 Rutten, J. J. M. M. .... 100, 124, 176

## S

Sankappanavar, H. P. .... 55, 68, 117  
 satisfaction ..... 77  
 Schneider, S. .... 68  
 Schröder, L. .... 157, 197, 198, 213  
 Schubert, Ch. .... 208, 214  
 selector ..... 24  
 semi-pullback ..... 3  
 separation ..... 171, 196  
 strong ..... 172  
 set  
 analytic ..... 17  
 Borel ..... 4, 10  
 clopen ..... 15  
 co-analytic ..... 17  
 cylinder ..... 8, 52  
 measurable ..... 4  
 Shiryayev, A. N. .... 45, 115  
 similarity type ..... 77  
 simulation-equivalence ..... 73  
 Skou, A. .... 79, 100, 155, 214  
 Sneyd, J. .... 115  
 Sokolova, A. .... 158, 165, 186, 213  
 space  
 analytic ..... 18  
 measurable ..... 4  
 separable ..... 20  
 metric ..... 11  
 complete ..... 13  
 separable ..... 13

metrizable ..... 11  
 Polish ..... 13  
 Borel sets ..... 4  
 Standard Borel ..... 13  
 topological ..... 9  
 base ..... 9  
 Borel sets ..... 10  
 compact ..... 14  
 continuity ..... 10  
 dense ..... 12  
 Hausdorff ..... 9  
 homeomorphism ..... 10  
 subbase ..... 9  
 Srivastava, S. M. .... 16, 67, 68, 155  
 Standard Borel cover ..... 24  
 states  
 bisimilar ..... 208  
 steady-state ..... 84  
 Stegun, I. A. .... 36  
 stochastic relation ..... 33  
 bisimilar ..... 71  
 congruence ..... 53  
 converse ..... 80  
 ergodic ..... 142  
 morphism ..... 37  
 Stromberg, K. R. .... 35, 67  
 strong inverse ..... 23  
 subprobability ..... 26  
 support ..... 148  
 subprobability functor ..... 34, 116  
 substitution ..... 89, 191  
 Sun, Y. .... 114  
 support ..... 148

## T

theorem  
 $\pi$ - $\lambda$  ..... 5  
 Alexandrov ..... 14, 43  
 Blackwell-Mackey ..... 22  
 Bolzano-Weierstraß ..... 15  
 Lusin ..... 19  
 Monotone Convergence ..... 26  
 Portmanteau ..... 29  
 Souslin ..... 19  
 Tihonov ..... 14  
 theory ..... 78  
 path ..... 90  
 state ..... 90  
 Thomas, S. .... 68  
 topology ..... 9  
 base ..... 9  
 Borel sets ..... 10  
 compact ..... 14  
 dense ..... 12

initial .....	10	Vlissides, J. ....	186
metric .....	11	<b>W</b>	
product .....	10	Wagner, D. H. ....	67, 68
subbase .....	9	weak inverse .....	23
subspace .....	10	weak pullback .....	3
sum .....	10	weak topology .....	29
weak .....	29	base .....	29
transition system		Hutchinson distance .....	31
Markov .....	79	Prohorov metric .....	30
nondeterministic .....	79	weak* $\sigma$ -algebra .....	32
<b>U</b>		Wells, C. ....	2, 4
universal relation .....	47	Wilf, H. S. ....	180
until operator .....	84	Wilkinson, D. J. ....	115
<b>V</b>		Wolter, F. ....	100
Valadier, M. ....	67	Worrell, J. ....	213
valuation .....	190	<b>Y</b>	
Venema, Y. ....	79, 100, 213, 214	Yoneda Lemma .....	4
Viglizzo, I. D. ....	155, 213	<b>Z</b>	
Vleck, F. Van .....	67	Zhou, C. ....	155