Logics: The Grammars

This Appendix lists the logics which are investigated in this treatise. Some logics which are discussed merely for illustrating a particular point are omitted.

Basic Modal Language: $\mathfrak{M}_{\mathfrak{b}}(\tau, P)$

The *basic modal language* $\mathfrak{M}_{\mathfrak{b}}(\tau, P)$ is given by the syntax

$$\phi ::= p \mid \top \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \triangle(\phi_1, \ldots \phi_{\mathfrak{ar}(\triangle)}),$$

where τ is a set of modal operators, $\triangle \in \tau$ is a modal operator of arity $\mathfrak{ar}(\triangle)$, and $p \in P$ with P a set of propositional letters: defined in Section 2.3 on page 76.

The Extended Modal Language: $\mathfrak{M}_{\mathfrak{s}}(\tau, P)$

The extended modal language $\mathfrak{M}_{\mathfrak{s}}(\tau, P)$ is defined through the syntax

$$\phi ::= p \mid \top \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \triangle_q(\phi_1, \ldots \phi_{\mathfrak{ar}(\triangle)}),$$

where $q \in \mathbb{Q} \cap [0, 1]$ is a rational number, τ is a set of modal operators, $\triangle \in \tau$ is a modal operator of arity $\mathfrak{ar}(\triangle)$, and $p \in P$ with P a set of propositional letters: defined in Section 2.3 on page 77.

E.-E. Doberkat, *Stochastic Coalgebraic Logic*, Monographs in Theoretical Computer Science. An EATCS Series, DOI 10.1007/978-3-642-02995-0,

Hennessy-Milner Logic: $\mathfrak{L}(\mathsf{Act}, \mathbb{Q} \cap [0,1])$

The formulas for $\mathfrak{L}(\mathsf{Act}, \mathbb{Q} \cap [0,1])$ are given through

$$\top \mid \phi_1 \wedge \phi_2 \mid \langle a \rangle_r \phi,$$

$a \in \mathsf{Act}$ is an action, $r \in [0,1] \cap \mathbb{Q}$: Section 2.3.1 on page 79.
Variant $\mathfrak{L}(\mathsf{Act}, [0,1])$: r may take arbitrary real values in the unit interval $[0,1]$; the variant defined in Section 2.6 on page 100, see also page 133. The discussion in Section 3.4 is based on it.

Continuous Time Stochastic Logic With Fixed-Point Operators: μCSL(AP, SV, PV)

This logic is usually abbreviated as μ**CSL**.

- *State formulas*:

$$\phi ::= \top \mid a \mid Z \mid \neg\phi \mid \phi \wedge \phi' \mid \mathcal{S}_{\ltimes p}(\phi) \mid \mathcal{P}_{\ltimes p}(\psi).$$

  $a \in \mathsf{AP}$ is an atomic proposition, $Z \in \mathsf{SV}$ is a state variable, ψ is a path formula, $\ltimes \in \{<, \leq, >, \geq\}$ is a relational operator, and $p \in [0,1]$ is a rational number.
- *Path formulas*:

$$\psi ::= P \mid \neg\psi \mid \psi \wedge \psi' \mid \mathcal{X}^I \psi \mid \phi\, \mathcal{U}^I \phi' \mid \mu P.\psi.$$

  $P \in \mathsf{PV}$ is a path variable, ϕ, ϕ' are state formulas, $I \subseteq \mathbb{R}_+$ is a closed interval with rational bounds.

  These operators are available in μ**CSL**:

1. $\mathcal{S}_{\ltimes p}(\phi)$: steady state probability.
2. $\mathcal{P}_{\ltimes p}(\psi)$: path quantifier.
3. $\mathcal{X}^I \phi$: next operator; I is an interval.
4. $\phi_1\, \mathcal{U}^I \phi_2$: until-operator; I is an interval.

The syntax von μ**CSL** is defined in Section 2.4.1 on page 84.

Coalgebraic Logic (Left Case):$\mathfrak{L}^\flat(\Lambda, \Theta, \Gamma)$

Λ and Θ are at most countable sets of predicate liftings resp. $\mathfrak{T}$-predicate liftings (Definition 4.2.2). $\mathfrak{L}^\flat = \mathfrak{L}^\flat(\Lambda, \Theta, \Gamma)$ is defined through

- *State formulas*: $\phi ::= \top \mid \phi_1 \wedge \phi_2 \mid \langle\lambda\rangle\phi \mid \langle\vartheta\rangle\psi$ with $\lambda \in \Lambda$ a predicate lifting and $\vartheta \in \Theta$ a $\mathfrak{T}$-predicate lifting.
- $\mathfrak{T}$-*formulas*: $\psi ::= \widetilde{\top} \mid \psi_1 \wedge \psi_2 \mid b(\phi_1, \ldots, \phi_{\mathfrak{ar}(b)})$ with $b \in \Gamma$ a bridge operator with arity $\mathfrak{ar}(b)$ (Definition 4.2.6).

$\mathfrak{L}^\flat$ is defined in Section 4.3.2 on page 167.

The logic $\mathfrak{L}^\flat(\emptyset, \Theta, \Gamma)$ is discussed as an application of left coalgebras to modal logic in Section 4.3.4, it is given on page 182.

Coalgebraic Logic (Right Case): $\mathfrak{L}^\sharp(\Lambda, \mathsf{Br}, \mathsf{Inf}, \mathsf{V})$

Λ, Br and Inf are countable sets of $\mathfrak{T}^{\mathrm{R}}$-predicate liftings, bridge operators and infinitesimal operators (Definition 4.4.17), resp., V is a countable set of variables.

The formulas of the *(full) logic* $\mathfrak{L}^\sharp = \mathfrak{L}^\sharp(\Lambda, \mathsf{Br}, \mathsf{Inf}, \mathsf{V})$ are given through

$$\phi ::= \top \mid \phi_1 \wedge \phi_2 \mid \langle\lambda\rangle\phi \mid \beta(\phi_1, \ldots, \phi_{\partial(\beta)}) \mid x \mid \iota x.\phi$$

with $\lambda \in \Lambda$, $\beta \in \mathsf{Br}$, $x \in \mathsf{V}$ and $\iota \in \mathsf{Inf}$. The *basic* or *kernel logic* is the special case $\mathfrak{L}^\sharp_b = \mathfrak{L}^\sharp(\Lambda, \emptyset, \emptyset, \emptyset)$, given by the grammar

$$\phi ::= \top \mid \phi_1 \wedge \phi_2 \mid \langle\lambda\rangle\phi$$

for $\lambda \in \Lambda$. These logics are defined in Section 4.4.2 on page 193.

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List of Symbols

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