Appendix

A The equivalent equation of the functional stationary condition (2-45)

In order to derive the equivalent equation of the stationary condition (2-45), the integration by parts formula (2-53) is used firstly to develop the expression of the functional variation $\delta\Pi_3$.

The area integration terms in $\delta\Pi_3$ are

$$\begin{aligned}
&\iint_{\Omega_a+\Omega_b} \{[D(\kappa_x+\mu\kappa_y)-M_x]\delta\kappa_x+[D(\kappa_y+\mu\kappa_x)-M_y]\delta\kappa_y \\
&\quad +2[D(1-\mu)\kappa_{xy}-M_{xy}]\delta\kappa_{xy}\}\mathrm{d}x\mathrm{d}y \\
&\quad -\iint_{\Omega_a+\Omega_b}\left[\left(\frac{\partial^2 w}{\partial x^2}+\kappa_x\right)\delta M_x+\left(\frac{\partial^2 w}{\partial y^2}+\kappa_y\right)\delta M_y+2\left(\frac{\partial^2 w}{\partial x\partial y}+\kappa_{xy}\right)\delta M_{xy}\right]\mathrm{d}x\mathrm{d}y \\
&\quad -\iint_{\Omega_a+\Omega_b}\left[\left(\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}\right)+q\right]\delta w\mathrm{d}x\mathrm{d}y
\end{aligned}$$

The line integration terms in $\delta\Pi_3$ are

$$\begin{aligned}
&\int_{C_{3a}+C_{3b}}\left(Q_n+\frac{\partial M_{ns}}{\partial s}-\bar{V}_n\right)\delta w\mathrm{d}s-\int_{C_{1a}+C_{2a}+C_{1b}+C_{2b}}(w-\bar{w})\delta\left(Q_n+\frac{\partial M_{ns}}{\partial s}\right)\mathrm{d}s \\
&\quad -\int_{C_{2a}+C_{3a}+C_{2b}+C_{3b}}(M_n-\bar{M}_n)\frac{\partial\delta w}{\partial n}\mathrm{d}s+\int_{C_{1a}+C_{1b}}\left(\frac{\partial w}{\partial n}-\bar{\psi}_n\right)\delta M_n\mathrm{d}s \\
&\quad +\int_{C_{ab}}\left\{(M_n^{(b)}-M_n^{(a)})\left(\frac{\partial\delta w}{\partial n}\right)^{(a)}+\left[\left(Q_n+\frac{\partial M_{ns}}{\partial s}\right)^{(b)}+\left(Q_n+\frac{\partial M_{ns}}{\partial s}\right)^{(a)}\right]\delta w^{(a)}\right. \\
&\quad \left.+\left[\left(\frac{\partial w}{\partial n}\right)^{(b)}+\left(\frac{\partial w}{\partial n}\right)^{(a)}\right]\delta M_n^{(b)}-(w^{(b)}-w^{(a)})\delta\left(Q_n+\frac{\partial M_{ns}}{\partial s}\right)^{(b)}\right\}\mathrm{d}s
\end{aligned}$$

The corner point and the node terms in $\delta\Pi_3$ are

$$- \sum_{A_{1a}+A_{1b}} (w - \bar{w})\Delta\delta M_{ns} + \sum_{A_{2a}+A_{2b}} (\Delta M_{ns} - \bar{R})\delta w$$

$$- \sum_{J_1} [(w^{(b)} - \bar{w})(\Delta\delta M_{ns})^{(b)} + (w^{(a)} - \bar{w})(\Delta\delta M_{ns})^{(a)}]$$

$$+ \sum_{J_2} \{[(\Delta M_{ns})^{(a)} + (\Delta M_{ns})^{(b)} - \bar{R}]\delta w^{(a)} - (w^{(b)} - w^{(a)})(\Delta\delta M_{ns})^{(b)}\}$$

Since the variation of the field variables can be arbitrarily and independently performed, based on the stationary condition (2-45) it can be known that the area integration terms, the line integration terms, the corner point and the node terms in $\delta\Pi_3$ should be zero, respectively. From the zero condition of the area integration terms in $\delta\Pi_3$, the field Eq. (2-46) within Ω_a and Ω_b can be derived. From the zero condition of the line integration terms in $\delta\Pi_3$, the boundary condition (2-47) on the boundary lines C_a and C_b and the interface condition (2-48) on the interface C_{ab} can be derived. From the zero conditions of the corner point and the node terms in $\delta\Pi_3$, the corner point condition (2-49) on the boundary line and the node condition (2-50) on the interface can be derived.

B The node conditions derived from the stationary condition (2-77)

Two different node types, J_1 and J_2, are considered here, respectively. Then, the related node conditions can be derived from the stationary condition (2-77) $\delta\Pi = 0$.

Firstly, consider the node J_1 with supports; the node terms in $\delta\Pi$ related to the node J_1 are composed of the following three terms.

(1) In the potential energy variation $\sum_{e_p} \delta\Pi_p^{(e_p)}$ of all the potential energy elements e_p around the node, the corresponding node term is $\sum_{e_p} (\Delta M_{ns})^{(e_p)} \delta w^{(e_p)}$.

(2) In the complementary energy variation $-\sum_{e_c} \delta\Pi_c^{(e_c)}$ of all the complementary energy elements e_c around the node, the corresponding node term is $-\sum_{e_c} w^{(e_c)} (\Delta\delta M_{ns})^{(e_c)}$.

(3) In δG_1, the corresponding node terms are

$$-\sum_{e_p} [(w^{(e_p)} - \bar{w})(\Delta\delta M_{ns})^{(e_p)} + (\Delta M_{ns})^{(e_p)} \delta w^{(e_p)}] + \sum_{e_c} \bar{w}(\Delta\delta M_{ns})^{(e_c)}$$

By superposition, the node terms related to the node J_1 in $\delta\Pi$ can be obtained

$$-\sum_{e}(w^{(e)}-\overline{w})(\Delta\delta M_{ns})^{(e)} \quad \text{(Sum of all elements } e \text{ around the node } J_1\text{)}$$

Thereby, from the stationary condition $\delta\Pi = 0$, the interface condition at the node J_1 can be obtained as:

$$w^{(e)} = \overline{w} \quad (e \text{ denotes each element around the node } J_1)$$

Secondly, consider the node J_2 without supports. In $\delta\Pi$, the node terms related to the node J_2 are composed of the following three terms.

(1) In the potential variation $\sum_{e_\mathrm{p}}\delta\Pi_\mathrm{p}^{(e_\mathrm{p})}$ of all the potential energy elements e_p around the node, the corresponding node term is $\sum_{e_\mathrm{p}}(\Delta M_{ns})^{(e_\mathrm{p})}\delta w^{(e_\mathrm{p})}$.

(2) In the complementary variation $-\sum_{e_\mathrm{c}}\delta\Pi_\mathrm{c}^{(e_\mathrm{c})}$ of all the complementary energy elements e_c around the node, the corresponding node term is $-\sum_{e_\mathrm{c}} w^{(e_\mathrm{c})}(\Delta\delta M_{ns})^{(e_\mathrm{c})}$.

(3) In δG_2, the corresponding node terms are

$$\left[\sum_{e}(\Delta M_{ns})^{(e)}-\overline{R}\right]\delta w^{(a)}+w^{(a)}\sum_{e}(\Delta\delta M_{ns})^{(e)}$$
$$-\sum_{e_\mathrm{p}}[w^{(e_\mathrm{p})}(\Delta\delta M_{ns})^{(e_\mathrm{p})}+(\Delta M_{ns})^{(e_\mathrm{p})}\delta w^{(e_\mathrm{p})}]$$

By superposition, the node terms related to the node J_2 in $\delta\Pi$ can be obtained

$$\left[\left(\sum_{e}\Delta M_{ns}^{(e)}-\overline{R}\right)\right]\delta w^{(a)}-\sum_{e}(w^{(e)}-w^{(a)})(\Delta\delta M_{ns})^{(e)}$$

Thereby, from the stationary condition $\delta\Pi = 0$, the interface condition at the node J_2 can be obtained as:

$$\sum_{e}\Delta M_{ns}^{(e)}-\overline{R}=0 \quad (\text{for the node } J_2)$$

$$w^{(e)} = w^{(a)} \quad (\text{for each element } e \text{ around the node } J_2,\ e\neq a)$$

C l_{1j} and γ_{1j} in Eq. (13-137)

$$l_{1j}=\frac{c_1a_{22}-c_2a_{12}}{a_{11}a_{22}-a_{12}a_{21}}, \quad \gamma_{1j}=\frac{c_2a_{11}-c_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}$$

in which

$$a_{11}=\left[-(\lambda_j+2)+\frac{\lambda_j\mu+1}{1-\mu}\right]\cos(\lambda_j+2)\frac{\alpha}{2}$$

$$a_{12}=\left[-K_{1j}\lambda_j+\frac{\lambda_j\mu+1}{1-\mu}\right]\cos\frac{\lambda_j\alpha}{2}$$

$$a_{21}=2(\lambda_j+1)$$

$$a_{22}=\lambda_j(1+K_{1j})$$

$$c_1=\left[\frac{\lambda_j\mu+1}{1-\mu}\frac{X+G}{(X+G)(\lambda_j+2)+2G}-\frac{\mu}{1-\mu}\right]\cos\frac{\lambda_j\alpha}{2}$$

$$c_2=\frac{(X+G)\lambda_j}{(X+G)(\lambda_j+2)+2G}\sin\frac{\lambda_j\alpha}{2}$$

D s_{1j} and t_{1j} in Eq. (13-144)

$$s_{1j}=\frac{d_1e_{22}-d_2e_{12}}{e_{11}e_{22}-e_{12}e_{21}},\quad t_{1j}=\frac{d_2e_{11}-d_1e_{21}}{e_{11}e_{22}-e_{12}e_{21}}$$

in which

$$e_{11}=\left[-(\lambda_j+2)+\frac{\lambda_j\mu+1}{1-\mu}\right]\sin\frac{1}{2}(\lambda_j+2)\alpha$$

$$e_{12}=\left[K_{1j}\lambda_j+\frac{\lambda_j\mu+1}{1-\mu}\right]\sin\frac{\lambda_j\alpha}{2}$$

$$e_{21}=2(\lambda_j+1)$$

$$e_{22}=\lambda_j(1+K_{1j})$$

$$d_1=-\left[\frac{\lambda_j\mu+1}{1-\mu}\frac{X+G}{(X+G)(\lambda_j+2)+2G}-\frac{\mu}{1-\mu}\right]\sin\frac{\lambda_j\alpha}{2}$$

$$d_2=\frac{(X+G)\lambda_j}{(X+G)(\lambda_j+2)+2G}\sin\frac{\lambda_j\alpha}{2}$$