A

Appendix: Further Topics

In this final chapter we briefly indicate some concepts and developments in nonsmooth analysis that have not been treated in the preceding text. For a comprehensive discussion we refer to the monograph [189] by Rockafellar and Wets (finite-dimensional theory) and the monograph [136, 137] by Mordukhovich (infinite-dimensional theory).

In the sequel, unless otherwise specified, let E be a normed vector space, $f : E \to \overline{\mathbb{R}}$ a proper functional, and $\bar{x} \in \operatorname{dom} f$.

(I) Clarke [34] defines the subdifferential

$$\partial_{\uparrow} f(\bar{x}) := \{x^* \in E^* \mid (x^*, -1) \in \mathrm{N}_C\big(\operatorname{epi} f, (\bar{x}, f(\bar{x}))\big)\}.$$

This set is always $\sigma(E^*, E)$-closed but may be empty. However, if $\bar{x}$ is a local minimizer of f, then $o \in \partial_{\uparrow} f(\bar{x})$. Moreover, Corollary 11.3.2 shows that $\partial_{\uparrow} f(\bar{x}) = \partial_{\circ} f(\bar{x})$ for any $\bar{x} \in E$ whenever f is locally L-continuous on E.

(II) Rockafellar [184] (see also [186, 189]) defines the *(regular) subderivative* $f^{\uparrow}(\bar{x}, \cdot) : E \to \overline{\mathbb{R}}$ of f at $\bar{x}$ as

$$f^{\uparrow}(\bar{x}, y) := \lim_{\epsilon \downarrow 0} \limsup_{(x,\alpha) \to_f \bar{x}} \inf_{z \in \mathrm{B}(y,\epsilon)} \frac{f(x + \tau z) - \alpha}{\tau}.$$

In this context, $(x, \alpha) \to_f \bar{x}$ means $(x, \alpha) \in \operatorname{epi} f$, $x \to \bar{x}$, and $\alpha \to f(\bar{x})$. Rockafellar shows that $f^{\uparrow}(\bar{x}, \cdot)$ is l.s.c. and that $f^{\uparrow}(\bar{x}, o) = -\infty$ if and only if $\partial_{\uparrow} f(\bar{x}) = \emptyset$. If $f^{\uparrow}(\bar{x}, o) > -\infty$, then $f^{\uparrow}(\bar{x}, o) = 0$, $f^{\uparrow}(\bar{x}, \cdot)$ is sublinear, and

$$\operatorname{epi} f^{\uparrow}(\bar{x}, \cdot) = \mathrm{T}_C\big(\operatorname{epi} f, (\bar{x}, f(\bar{x}))\big).$$

It then follows that $f^{\uparrow}(\bar{x}, \cdot)$ is the support functional of the Clarke subdifferential, i.e.,

$$\partial_{\uparrow} f(\bar{x}) = \{x^* \in E^* \mid \langle x^*, y \rangle \le f^{\uparrow}(\bar{x}, y) \quad \forall\, y \in E\}.$$

The functional f is said to be *directionally Lipschitz* at $\bar{x}$ if $\operatorname{epi} f$ is epi-Lipschitz at $(\bar{x}, f(\bar{x}))$. Moreover, f is said to be *subdifferentially regular* if

$$f^{\uparrow}(\bar{x}, y) = \underline{f}_H(\bar{x}, y) \quad \forall\, y \in E;$$

this is equivalent to $\operatorname{epi} f$ being a tangentially regular set. If f is locally L-continuous around $\bar{x}$, then f is subdifferentially regular if and only if f is regular in the sense of Clarke. Rockafellar shows that the sum rule

$$\partial_{\uparrow}(f+g)(\bar{x}) \subseteq \partial_{\uparrow} f(\bar{x}) + \partial_{\uparrow} g(\bar{x}) \tag{$\diamond$}$$

holds if g is directionally Lipschitz at $\bar{x}$ and $\operatorname{dom} f^{\uparrow}(\bar{x}, \cdot) \cap \operatorname{int} \operatorname{dom} g^{\uparrow}(\bar{x}, \cdot)$ is nonempty. The sum rule holds with equality if, in addition, f and g are subdifferentially regular. Certain chain rules are also established.

(III) Michel and Penot [129,130] study several types of directional derivatives and associated subdifferentials. Their aim is to generalize the G-derivative rather than the strict H-derivative as does the Clarke subdifferential (cf. Remark 7.3.10).

Using the epi-limit convergence concept, Michel and Penot define, among others, the *pseudo-strict derivative* of f at $\bar{x}$ as

$$f^{\wedge}(\bar{x}, y) := \sup_{z \in E} \ \operatorname*{eLim\,sup}_{\substack{\tau \downarrow 0,\, z' \to z \\ (\bar{x}+\tau z', \alpha) \to_f \bar{x}}} \ \tfrac{1}{\tau}\big(f(\bar{x} + \tau y + \tau z') - \alpha\big).$$

The functional $f^{\wedge}(\bar{x}, \cdot)$ is shown to be l.s.c. and sublinear. If f is locally L-continuous around $\bar{x}$, then $f^{\wedge}(\bar{x}, \cdot)$ coincides with $f^{\diamond}(\bar{x}, \cdot)$. If the directional G-derivative $f_G(\bar{x}, \cdot)$ exists, is finite and sublinear, then $f^{\wedge}(\bar{x}, \cdot) = f_G(\bar{x}, \cdot)$. The associated subdifferential, defined as

$$\partial^{\wedge} f(\bar{x}) := \{x^* \in E^* \mid \langle x^*, y\rangle \le f^{\wedge}(\bar{x}, y)\ \forall\, y \in E\},$$

thus satisfies $\partial^{\wedge} f(\bar{x}) = \{f'(\bar{x})\}$ whenever f is G-differentiable at $\bar{x}$. If $\bar{x}$ is a local minimizer of f, then $o \in \partial^{\wedge} f(\bar{x})$. Under certain regularity assumptions, the sum rule $\partial^{\wedge}(f+g)(\bar{x}) \subseteq \partial^{\wedge} f(\bar{x}) + \partial^{\wedge} g(\bar{x})$ and a chain rule are established. Finally, multiplier rules are derived in [130].

(IV) Various other generalized derivative concepts were introduced and studied. Some of them are Halkin's *screen* [83], Treiman's *B-derivatives* [206,207], and Warga's *derivate containers* [213, 214]. Much was done by Hiriart-Urruty [84,85,87] in clarifying and refining these derivative concepts.

(V) Concerning sensitivity analysis, we refer in the finite-dimensional case to Klatte and Kummer [111], Luderer et al. [126], and Rockafellar and Wets [189], and in the infinite-dimensional case to Bonnans and Shapiro [16], Clarke et al. [39], and Mordukhovich [136] as well as to the literature cited in these books.

(VI) Following Borwein and Zhu [23], we started the differential analysis of lower semicontinuous functionals with Zhu's nonlocal fuzzy sum rule which, in turn, is based on the Borwein–Preiss smooth variational principle. The basis of Mordukhovich's work constitutes his extremal principle. Another fundamental result in nonsmooth analysis is a multidirectional mean value theorem (such as Theorem 9.6.2) that originally goes back to Clarke and Ledyaev [37,38]. In [39] this result is applied to establish, among others, nonsmooth implicit and inverse function theorems. Mordukhovich and Shao [140] showed the equivalence of the extremal principle and the local fuzzy sum rule, and Zhu [226] proved the equivalence of the latter to the nonlocal fuzzy sum rule and to the multidirectional mean value theorem.

(VII) Ioffe systematically investigated *approximate subdifferentials*; see, among others, [94–98, 100].

Let $\mathcal{F}$ denote the collection of all finite-dimensional vector subspaces of E. If S is a nonempty subset of E, we write

$$f_{(S)}(x) := \begin{cases} f(x) & \text{if } x \in S, \\ +\infty & \text{if } x \in E \setminus S. \end{cases}$$

Ioffe starts with the lower directional Hadamard derivative, which he calls *lower directional Dini derivative*,

$$\underline{f}_H(\bar{x}, y) := \liminf_{\tau \downarrow 0,\, z \to y} \tfrac{1}{\tau}\big(f(\bar{x} + \tau z) - f(\bar{x})\big) \quad \forall\, y \in E.$$

He then defines the *Hadamard subdifferential* (or *Dini subdifferential*)

$$\partial^- f(\bar{x}) := \{x^* \in E^* \mid \langle x^*, y\rangle \le \underline{f}_H(\bar{x}, y)\ \forall\, y \in E\}$$

and the *A-subdifferential*

$$\partial_A f(\bar{x}) := \bigcap_{L \in \mathcal{F}} \operatorname*{Lim\,sup}_{x \to_f \bar{x}} \partial^- f_{(x+L)}(x).$$

Here, the Painlevé–Kuratowski upper limit is taken with respect to the norm topology in E and the weak star topology in E^*. Observe that Ioffe utilizes the topological form of the Painlevé–Kuratowski upper limit whereas Mordukhovich utilizes the sequential form. The interrelation between the concepts of Ioffe and Mordukhovich is discussed by Mordukhovich and Shao [142], see also Mordukhovich [136].

Ioffe further defines the *G-normal cone* to the set $M \subseteq E$ at $\bar{x} \in M$ as

$$\mathrm{N}_G(M, \bar{x}) := \mathrm{cl}^* \bigcup_{\lambda > 0} \lambda \partial_A \mathrm{d}_M(\bar{x})$$

and, respectively, the *G-subdifferential* and the *singular G-subdifferential* of f at $\bar{x}$ as

$$\partial_G f(\bar{x}) := \big\{x^* \in E^* \mid (x^*, -1) \in \mathrm{N}_G\big(\operatorname{epi} f, (\bar{x}, f(\bar{x}))\big)\big\},$$
$$\partial_G^\infty f(\bar{x}) := \big\{x^* \in E^* \mid (x^*, 0) \in \mathrm{N}_G\big(\operatorname{epi} f, (\bar{x}, f(\bar{x}))\big)\big\}.$$

The G-subdifferential and the A-subdifferential coincide for any function on finite-dimensional normed vector spaces and for directionally Lipschitz functions on arbitrary normed vector spaces. For convex functionals on any normed vector space, the G-subdifferential (in contrast to the A-subdifferential) coincides with the subdifferential of convex analysis. In general, the sets $\partial_A f(\bar{x})$ and $\partial_G f(\bar{x})$ are not convex. However, they are minimal in a certain sense among all "reasonable" subdifferential constructions. In particular, for any proper functional f one has $\partial_C f(\bar{x}) = \overline{\mathrm{co}}^*\big(\partial_G f(\bar{x}) + \partial_G^\infty f(\bar{x})\big)$.

Ioffe develops an extensive calculus for these objects. For instance, if M_1 and M_2 are closed sets one of them being epi-Lipschitz at $\bar{x}$, then an appropriate regularity assumption ensures that

$$\mathrm{N}_G(M_1 \cap M_2, \bar{x}) \subseteq \mathrm{N}_G(M_1, \bar{x}) + \mathrm{N}_G(M_2, \bar{x}),$$

with equality if $\mathrm{N}_G(M_i, \bar{x}) = \mathrm{T}(M_i, \bar{x})^\circ$ for $i = 1, 2$. This result is verified with the aid of Ekeland's variational principle. It is then applied to derive the following sum rule. If the functionals f and g are l.s.c. and one of them is directionally Lipschitz at $\bar{x}$, then under the regularity assumption $\partial_G^\infty f(\bar{x}) \cap (-\partial_G^\infty g(\bar{x})) = \{o\}$, the sum rule ($\diamond$) holds with $\partial_\uparrow$ replaced by ∂_G. An analogous result is obtained for the A-subdifferential. For an extended chain rule in terms of ∂_A see Jourani and Thibault [106]. We also refer to the survey paper by Hiriart-Urruty et al. [90].

Multiplier rules in terms of G-subdifferentials were established by Glover et al. [74]. For an alternative proof of these multiplier rules and an interesting discussion of the absence of constraint qualifications we refer to Pühl [172]. Modified multiplier rules were obtained by Glover and Craven [73].

(VIII) In these lectures, we confined ourselves to *first-order* necessary optimality conditions (which are also sufficient in the convex case). For *second-order* necessary and/or sufficient conditions we refer in the smooth case to Bonnans and Shapiro [16] and the literature cited therein. Second-order optimality conditions in terms of generalized derivatives are also treated in many papers. As a small selection we refer to Ben-Tal and Zowe [14], Casas and Tröltzsch [29], Chaney [31], Cominetti [40], Mordukhovich [135], Mordukhovich [136], Mordukhovich and Outrata [138], and Rockafellar [188].

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