APPENDIX

We review some of the basic notions of probability theory below.

A1. Notations and Conventions

A probability space is a measurable space (Ω, F, P) with $P(\Omega) = 1$. Throughout this book (Ω, F, P) denotes a given complete probability space. This means that (Ω, F) is a measurable space and P is a probability measure on (Ω, F) such that each subset of a P-null set in F is in F. If F_o is a sub-σ-algebra of F the augmentation $F_o^{\sim}$ of F_o is the smallest σ-algebra containing F_o and all the P-null sets in F_o.

Elements of the σ-algebra F are called events. A measurable function $X : (\Omega, F) \to \mathbb{R}^n$ is called a n-dimensional random variable. A funtion X is called measurable if $X^{-1}(A) \in F$ for all Borel sets A in $\mathbb{R}^n$. For any random variable X $\int_\Omega X(\omega)\ P(d\omega)$, if it exists, is called the mean or expectation of X and is denoted by $E[X]$. Thus

$$E[X] = \int_\Omega X(\omega)\ P(d\omega) = \mu$$

$E[X^n]$ is called the n^{th} moment of X about zero and $E[(X-E[X])^n]$ the n^{th} central moment. The second central moment is called variance and will be often denoted by σ^2

$$\sigma^2 = E[(X-E[X])^2] = E[(X-\mu)^2]$$

For any random variable $X : \Omega \to \mathbb{R}^n$ the function $\tilde{P} : \mathbb{R}^n \to \mathbb{C}$ defined by

$$\tilde{P}(p) = E[e^{ip\cdot X}]$$

is called the characteristic function of X. Here $p \in \mathbb{R}^n$ and $p\cdot X = \sum_{i=1}^{n} p_i x_i$.

Events $B_1 \ldots B_j$ are called independent if for every $\{i_1 \ldots i_k\} \subset \{1 \ldots j\}$

$$P[\bigcap_{l=1}^{k} B_{i_l}] = \prod_{l=1}^{k} P[B_{i_l}]$$

An arbitrary family of events is called independent if every finite sub-family is independent.

A filtration is a family $\{F_t\}_{t \in I}$ of sub-σ-algebras of F such that $F_s \subset F_t$ for all $s < t$ in I. If the following two conditions are also satisfied then $\{F_t\}_{t\ I}$ is called the standard filtration:

(i) $F_t = F_{t+} = \bigcap_{s>t} F_s$ (right continuity)

(ii) F_o contains all of the P-null sets in F (completeness)

Conditions (i) and (ii) are not obligatory but simplify many technicalities. Indeed many useful theorems for continuous parameter martingales require these hypotheses.

The standard filtration $\{F_t\}_{t \in I}$ associated with a Brownian motion $\{W_t\}_{t\ I}$ is defined by

$$F_t = \sigma\{W_s | o \leq s \leq t\}^{\sim}$$

where the inclusion of the P-null sets in F_t ensure that $F_t = F_{t+}$. The physical meaning of F_t is the following: F_t is the σ-algebra of events occuring up to time t: the "past events up to t".

A2. Conditioning

Let $X : (\Omega, F) \to \mathbb{R}^n$ be a random variable such that $E[X] < +\infty$. Let F_o be a sub-σ-algebra of F. Then there exists a random variable $Y : (\Omega, F_o) \to \mathbb{R}^n$ such that for every A in F_o

$$\int_A Y(\omega)\ P(d\omega) = \int_A X(\omega)\ P(d\omega)$$

If $\bar{Y}$ is any random variable with the same properties then we have $\bar{Y} = Y$ P - almost everywhere.

Y is called the conditional expectation of X with respect (or given) F_o and is denoted by $E[X|F_o]$.

If $F_o' \subset F_o \subset F$ are σ-algebras then

$$E[E[X|F_o]|F_o'] = E[X|F_o']$$

The following proposition exhibits conditional expectation as a projection on a Hilbert space.

Proposition Let (Ω,F,P) be a probability space and F_o a sub-σ-algebra of F. The $L^2(\Omega,F_o,P)$ is a closed subspace of $L^2(\Omega,F,P)$ and if $\Pi = \Pi^* = \Pi^2$ denotes the orthogonal projector of $L^2(\Omega,F,P)$ on $L^2(\Omega,F_o,P)$ then

$$\Pi f = E[f|F_o]$$

A3. Stochastic Processes

In applications the random evolutions of a system is described by saying that the state of the system is a function of time and randomness. This leads to the following definition: A n-dimensional stochastic process X is a function $X : I \times \Omega \to \mathbb{R}^n$ where I is an interval in $\mathbb{R}^+ = [o,+\infty)$ and $X_t(\cdot) = X(t,\cdot)$ is F-measurable for each $t \in I$. Moreover we say that X has initial value X $\mathbb{R}^n$ if $X_o(\cdot) = x$ a.s. The process X will be denoted by $\{X_t\}_{t\in I}$. The mappings $X(\cdot,\omega):t\to X(t,\omega)$, $\omega\in\Omega$, are called the paths of X.

Given an increasing family $\{F_t\}_{t\in I}$ of σ-algebras on Ω the process is said to be adapted to this family if $X_t \in F_t$ for each $t \in I$ (one says sometimes non-anticipating).

An F-measurable function $\tau:\Omega \to \overline{\mathbb{R}}_+$ is called a stopping time with respect to a filtration $\{F_t\}_{t\in I}$ if and only if $\{\omega|\tau(\omega)\leq t\}\in F_t$ for each $t\in I$. If $\{F_t\}_{t\in I}$ is a standard filtration then the condition on τ is equivalent to $\{\omega|\tau(\omega) <t\} \in F_t$ for each $t\in I$. To any stopping time τ is associated a σ-algebra F_τ, which consists of all sets in $F_\infty = \bigcup_{t\in I} F_t$ satisfying

$$A \cap \{\omega|\tau(\omega) \leq t\} \quad F_t \quad \text{for all} \quad t \in I.$$

If A is any Borel set in $\mathbb{R}^n$ then

$$\tau_A = \inf\{\tau > 0|W_t \notin A\}$$

where W_t is a n-dimensional Brownian motion is a stopping time. In general if τ_1 and τ_2 are stopping times, then $\tau_1 \wedge \tau_2 = \min(\tau_1,\tau_2)$ is a stopping time. In particular if τ is a stopping time then $\tau \wedge t$ is a stopping time for every fixed time t.

One of the basic properties of Brownian motion is the strong Markov property. Loosely speaking this says that given the history of a Brownian motion W up to some finite stopping time τ the behaviour of W after that time depends only on τ and the state W_τ of W at time τ. More precisely if $f: \mathbb{R}^d \to \mathbb{R}$ is a bounded measurable function and τ is a stopping time then

$$E[1\{\omega|\tau(\omega)<+\infty\}\ f(W_{\tau+t})|F_\tau]$$

$$= 1\{\omega|\tau(\omega)<+\infty\}\ E^{W_\tau}[f(W_t)]$$

with $E^{W_\tau}[\cdot] = E[\cdot|W_\tau]$.

A4. Martingales

Let $\{F_t\}_{t\in I}$ be a filtration. A collection $M = \{M_t, F_t\}_{t\in I}$ is called a martingale if and only if

i) $M_t \in L^1$ for each t $(E[|X_t|] < +\infty \quad \forall t)$

ii) $M_s = E[M_t|F_s]$ for all $s < t$ (almost surely P)

We call M a submartingale if the "=" in (ii) is replaced by $\geq$ and a supermartingale if it is replaced by $\leq$. In other words M is a martingale if and only if both M and $-M$ are supermartingales. The intuitive significance of (ii) is that given the bahaviour of everything up to time s, the value of M at the future time is, on average equal to its value at time s. Thus M_t is the value at time t of our fortune in a game which is fair to us (the game will be unfair for a supermartingale).

A martingale $M = \{M_t, F_t\}_{t\in I}$ is called continuous if and only if

i) $\{F_t\}_{t\in I}$ is a standard filtration

ii) $\{M_t\}_{t\in I}$ has all path continuous

The basic source of continuous martingales is stochastic integrals.
A collection $M = \{M_t, F_t\}_{t\ \ I}$ is called a local martingale if and only if

i) M_o is an F_o-measurable random variable

ii) there is a sequence $\{\tau_k\}_{k\ \ \mathbb{N}}$ of stopping times such that $\tau_k \uparrow \infty$ a.s and for each k

$$M^k = \{M_{t\wedge\tau_k} - M_o,\ F_t\}_{t\in I}$$

is a martingale. The sequence $\{\tau_k\}_{k\in\mathbb{N}}$ will be called a localizing sequence for M.

A stochastic process X_t is said to be a semimartingale if X_t is expressible as a sum of a (local) martingale and a process of bounded variation.

A5. Weak Convergence and Measures on Metric Spaces

It is often useful to consider a stochastic process as a mapping of some measurable space into a space of functions with nice topological properties. Once the space is chosen the stochastic process induces a measure on the function space and the σ-algebra over which the measure is defined is relevant.

Let X be a separable metric space and $\mathcal{B}$ the σ-algebra generated by the open sets. $\mathcal{B}$ is also the smalles σ-algebra with respect to which continuous functions are measurable. The advantage of separability is that the σ-algebra $\mathcal{B}$ is generated by spheres. A measure on X will always be on $(X,\mathcal{B})$. Such a measure μ is always regular in the sense that

$$\mu(A) = \inf_{A \subset G,\ G \text{ open}} \mu(G) = \sup_{C \subset A,\ A \text{ closed}} \mu(C)$$

where A is any Borel set.

Very often it becomes necessary to construct explicitly measures on $(X,\mathcal{B})$ and to this end weak convergence is a useful tool. The problem is to construct a measure μ on $(X,\mathcal{B})$ with certain properties (like having given finite dimensional distribution,...). One constructs a sequence of measures $\{\mu_n\}_{n \in \mathbb{N}}$ having approximate properties and hopefully μ_n will have a limit μ for $n \to +\infty$ which is what we want. Even if the sequence itself does not converge a convergent subsequence will often suffice. This means that we have to define a certain notion of convergence on the space of probability measures on $(X,\mathcal{B})$.

Let M_1 be the collection of probability measures on $(X,\mathcal{B})$. A sequence $\{P_n\}_{n \in \mathbb{N}} \subset M_1$ converges weakly to P if

$$\lim_n \int f \, dP_n = \int f \, dP$$

for each bounded and continuous function f on X. The weak convergence will be denoted by $P = w - \lim_{n\to\infty} P_n$.

This notion of convergence takes advantage of the underlying topology of X, whereas the uniform convergence and the strong convergence do not. It is useful to know that weak convergence arises from a metric defined on $M_1(X)$: indeed there exists a metric d on M_1 such that d-convergence in M_1 is the same as weak convergence. It is also useful to have a condition for the conditional compactness of a set $K \subset M_1$ under weak convergence. Such a condition ensuring that every sequence $\{P_n\}_{n \in \mathbb{N}} \subset K$ has a weakly convergent subsequence exists and is moreover necessary if the metric space X is complete. We give such a condition through the following theorems

Theorem 1

Let X be a compact metric space, then $M_1(X)$ is compact under weak convergence.

Theorem 2

Let X be a metric space and $\{\mu_n\}_{n \in \mathbb{N}}$ be a sequence of measures on $(X,\mathcal{B})$ such that

i) $\mu_n(X)$ is bounded

ii) for any $\varepsilon > 0$ there exists a compact subset K_ε of X such that $\mu_n(X \backslash K_\varepsilon) < \varepsilon$ for all n.

Then the sequence $\{\mu_n\}_{n \in \mathbb{N}}$ has a weakly convergent subsequence.

Theorem 3

Let X be a complete separable metric space. Let $\mathcal{K} \subset M_1(X)$ be a compact subset with respect to the topology of weak convergence. For any $\varepsilon > 0$ there exists a compact subset K_ε such that

$$P(K_\varepsilon) \geq 1-\varepsilon$$

for all $P \in \mathcal{K}$.

For a proof see K.R. Parthasarathy, Probability measures on metric spaces, Academic Press, New York 1967.

Remark A sequence $\{P_n\}_{n \in \mathbb{N}}$ of probability measures is said to be tight if for each $\varepsilon > 0$ there exists a compact subset K_ε such that $P_n(K_\varepsilon) > 1-\varepsilon$ for all n.

For any probability measure on $\mathbb{R}^k$ there is on some probability space a random variable having that measure as its distribution. Therefore for probability measures satisfying

$$w - \lim_n P_n = P$$

there exist random variables X_n and X having these measures as distributions and satisfying

$$\lim_n X_n = X \qquad \text{(convergence in law)}$$

According to the following fundamental theorem the X_n and X can be constructed on the same probability space and moreover in such a way that

$$\lim_n X_n(\omega) = X(\omega) \qquad \forall\omega$$

a condition which is of course much stronger than the convergence in law.

Skorohod's theorem

Let P_n and P be probability measures on $\mathbb{R}^k$ and $P = \text{w-}\lim_n P_n$ Then there exists random vectors X_n and X on a common probability space $(\Omega, \mathcal{F}, \bar{P})$ such that X_n has distribution P_n, X has distribution P and

$$\lim_{n\to\infty} X_n(\omega) = X(\omega) \qquad \forall\omega \in \Omega$$

For a proof see e.g. P. Billingsley, Probability and Measure, John Wiley New York 1979.

A6. Stochastic Ito Integrals

In this section we will briefly discuss the existence of $\int_o^t f(s,\omega)dW_s(\omega)$ where $W_t(\omega)$ is 1-dimensional Brownian motion for a wide class of functions $f : [o,\infty) \times \Omega \to \mathbb{R}$.

Let $\mathcal{F}_t$ be the σ-algebra generated by $\{W_s | s \leq t\}$. We call a function of the form

$$f(t,\omega) = \sum_{j\geq 0} e_j(\omega)\, \chi_{[j\cdot 2^{-n},(j+1)2^{-n})}(t)$$

where χ denotes the characteristic function elementary if $e_j(\omega)$ is $\mathcal{F}_{j2^{-n}}$ - measurable for all j.

For elementary functions $e(t,\omega)$ we define the integral by

$$\int_s^t e(\tau,\omega)\, dW_\tau(\omega) = \sum_{j\geq 0} e_j(\omega)\, [W_{t_{j+1}} - W_{t_j}]\,(\omega)$$

Now we make the following important observation: if $e(t,\omega)$ is bounded and elementary then

$$E[(\int_s^t e(\tau,\omega)\, dW_\tau(\omega))^2] = E[\int_s^t e(\tau,\omega)^2\, d\tau]$$

From this basic isometry we get an indication of what functions we can extend the integration.

To prove this fundamental relation let $\Delta W_j = W_{t_{j+1}} - W_{t_j}$; then we have

$$\int_s^t e(\tau,\omega)\ dW_\tau = \sum_{j\geq 0} ej(\omega)\ \Delta W_j$$

Since $e_i e_j \Delta W_i$ and ΔW_j are independent for $i<j$ it follows that

$$E[e_i e_j \Delta W_i \Delta W_j] = E[e_j{}^2]\ (t_{j+1}-t_j)\,\delta_{ij}$$

which implies the basic isometry.

Let S be the class of functions $f(t,\omega)$: $\mathbb{R}\times\Omega \to \mathbb{R}$ such that

(i) $(t,\omega) \to f(t,\omega)$ is $\mathcal{B}\times\mathcal{F}$-measurable, where $\mathcal{B}$ denotes the Borel σ-algebra on $\mathbb{R}^+$

(ii) For each t the map $\omega \to f(t,\omega)$ is $\mathcal{F}_t$-measurable (i.e. f is adapted)

(iii) $E[\int_s^t f(\tau,\omega)^2\ d\tau] < +\infty$
I(f) $f \in S$ we will define the Ito integral

$$I(f) = \int_s^t f(\tau,\omega)\ dW_\tau$$

I(f) will be $\mathcal{F}$-measurable and

$$E[(I(f))^2] = E[\int_s^t f^2 d\tau]$$

I(f)is a stochastic process called the (stochastic) Ito integral based on Brownian motion.

The idea of the construction is simple: We use the basic isometry to extend in several steps the definition for elementary functions to functions in S. An important property of the Ito integral is that it is a martingale. For continuous martingales we have the following important inequality due to Doob (see e.g.[92 a]):
If M_t is a martingale such that $t \to M_t(\omega)$ is continuous a·s then

$$P[\sup_{0\leq\tau\leq t} |M_\tau| \geq \lambda] \leq \frac{1}{\lambda^p} E[|M_t|^p]$$

provided $E[|M_t|^p] < +\infty$.

Using this inequality and the fact that

$$M_t(\omega) = \int_0^t f(s,\omega)\ dW_s$$

is a martingale with respect to $\mathcal{F}_t$ we conclude that

$$P\,[\sup_{\tau\in[o,t]} |M_\tau| > \lambda] \leq \frac{1}{\lambda^2} E\,[\int_o^t f(s,\omega)^2 ds]$$

Remark

It is possible to define $\int_o^t f(s,\omega)dW_s$ for a class of functions larger than S.

We finish this section with an example:

$$\int_o^t W_s dW_s = \frac{1}{2}W_t^2 - \frac{1}{2}t$$

The extra term $-\frac{1}{2}t$ shows that the Ito stochastic integral does not behave like ordinary integrals. From this example we see that the image of the Ito integral $W_t = \int_o^t dW_s$ by the map $f(x) = \frac{1}{2}x^2$ is not again an Ito integral but a combination of a dW_s and a ds integral. We have indeed

$$\frac{1}{2}W_t^2 = \int_o^t \frac{1}{2}\,ds + \int_o^+ W_s\, dW_s$$

It turns out that if we define stochastic integrals as a sum of a dW_s and a ds integral then this family is stable under smooth maps.

A stochastic integral is a stochastic process X_t of the form

$$X_t = X_o + \int_o^t \beta(s,\omega)\, ds + \int_o^t \sigma(s,\omega)\, dW_s$$

The above equation is often written in the shorter differential form

$$dX_t = \beta\, dt + \sigma dW_t$$

Let $g(t,x) \in C^2([o,\infty) \times \mathbb{R}, \mathbb{R})$ then

$$Y_t = f(t,X_t)$$

is again a stochastic integral and

$$dY_t = \frac{\partial f}{\partial t}(t,X_t)dt + \frac{\partial f}{\partial x}(t,X_t)dX_t + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(t,X_t)\,(dX_t)^2$$

where

$$dt \cdot dt = dt \cdot dW_t = dW_t \cdot dt = 0 \qquad dW_t \cdot dW_t = dt$$

This main result is called the <u>Ito formula,</u> which is very useful for evaluating Ito integrals.

The stochastic integral $\int_o^t \sigma(s,\omega)\, dW_s(\omega)$ based on Brownian motion is due to K. Ito (1941). Stochastic calculus (Ito's formula) based on Brownian motion is carried out according to the rule $(dW_\tau)^2 = dt$.

The theory of stochastic integrals $\int_o^t \Phi(s,\omega)\, dM(s,\omega)$ based on a martingale M is due to Kunita-Watanabe (1967). They also develop a stochastic calculus based on martingale according to the rule $(dM_t)^2 = d\langle M,M\rangle_t$, where M is the so-called quadratic variation of M (see Appendix A 7).

Among spaces of martingales, which may be studied, the space of square integrable martingales is the simplest because of its Hilbert space structure but also the richest to investigate. Indeed the classical types of stochastic integrals discussed in the literature had been introduced as isomorphic transformations of some special space of square integrable martingales.

A7. Definition and Characterization of Quadratic Variation

For $t \in I \subset \mathbb{R}_+$ a partition Π_t of $[o,t]$ is a finite ordered subset $\Pi_t = \{t_0, t_1, t_2, \ldots t_k\}$ of $[o,t]$ such that $0 = t_0 < t_1 < \ldots < t_k = t$. We denote the mesh of Π_t by

$$\delta \Pi_t \equiv \max_{j=0,1,..k-1} |t_{j+1} - t_j|$$

If $\{\Pi_t^{\,n}\}_{n \in \mathbb{N}}$ is a sequence of partition of $[o,t]$, then for each n the members of $\Pi_t^{\,n}$ will be denoted by t_{jn} $j = o,1,\ldots k_n$. The main result is the following theorem.

Theorem

Let $t \in I$ and $\{\Pi_t^{\,n}\}_{n \in \mathbb{N}}$ be a sequence of partition of $[o,t]$ such that $\lim_{n \to +\infty} \delta \Pi_t^n = 0$. Suppose M is a continuous local martingale and for each n let

$$\Sigma_t^n = \sum_{t_{jn} \in \Pi_t^n} (M_{t(j+1)n} - M_{tjn})^2$$

Then

i) if M is bounded $\{\Sigma_t^n\}_{n \in \mathbb{N}}$ converges in L^2 to

$$\langle M,M\rangle_t \equiv M^2_t - M^2_o - 2 \int_o^t M \, d\, M$$

ii) $\{\Sigma_t^n\}_{n \; \mathbb{N}}$ converges in probability to $\langle M,M\rangle_t$

We call $\langle M,M\rangle_t$ the quadratic variation of M at time t and $\langle M,M\rangle = \{\langle M,M\rangle_t\}_{t \; I}$ the quadratic variation process associated with $\{M_t\}_{t \in I}$.

A process M is a Brownian motion in $\mathbb{R}$ if and only if it is a continuous local martingale with quadratic variation $\langle M,M\rangle_t$ such that

$$\langle M,M\rangle_t = t \quad a.s \quad \text{for all } t.$$

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