Appendices

A.1 Elementary Generalized Functions

Dirac Delta Function δ. Let us consider a function (Fig. A1)

$$\delta(t) = \frac{1}{\pi}\left(\frac{1}{1+t^2}\right), \tag{A.1}$$

having the maximum value at $t = 0$ and decreasing at modulus $|t|$ growth. Let us transform the function $\Phi(t)$ by increasing its value at $t = 0$ by the factor m

$$\Phi_1(mt) = \frac{1}{\pi}\frac{m}{\left[1+(mt)^2\right]}. \tag{A.2}$$

The functions $\Phi_1(mt)$ for several m are presented in Fig. A2. It can be shown that the integrals of the functions (1) and (2) are equal to unity, i.e.

$$\int_{-\infty}^{+\infty} \Phi(t)\,\mathrm{d}t = 1, \qquad \int_{-\infty}^{+\infty} \Phi_1(mt)\,\mathrm{d}t = 1.$$

As m increases indefinitely $(m \to \infty)$ we obtain the function that is named as the Dirac delta function (Fig. A3):

$$\lim_{m\to\infty} \Phi_1(mt) = \delta(t).$$

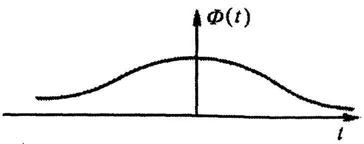

Fig. A.1.

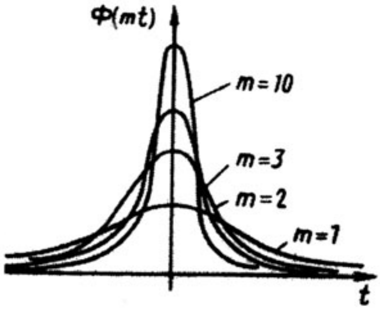

Fig. A.2.

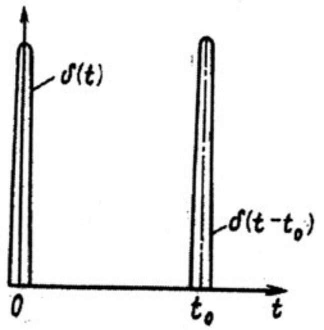

Fig. A.3.

The principal properties of the Dirac delta function are:

$$1)\ \delta(t)=\begin{cases}0 & t<0,\\ \infty & t=0,\\ 0 & t>0,\end{cases}\qquad \delta(t-t_0)=\begin{cases}0 & t<t_0,\\ \infty & t=t_0,\\ 0 & t>0;\end{cases}\tag{A.3}$$

$$2)\ \delta(t)=\delta(-t);$$

$$3)\ \int_{-\infty}^{\infty}\delta(t-t_0)\,\mathrm{d}t=1;$$

$$4)\ \delta(t)=\delta(a\varepsilon)=\frac{1}{|a|}\delta(\varepsilon),$$

where ε is a dimensionless quantity.

The integrals containing the Dirac delta function and its derivatives are

$$\int_{-\infty}^{\infty}\varphi(t)\delta(t-t_0)\,\mathrm{d}t=\varphi(t_0);$$

$$\int_{-\infty}^{\infty} \varphi(t)\dot{\delta}(t-t_0)\,dt = -\int_{-\infty}^{\infty} \dot{\varphi}(t)\delta(t-t_0)\,dt = -\dot{\varphi}(t_0)\,,$$

where $\dot{\delta}$ is the derivative of the Dirac delta function.

For the derivative of n-th order we obtain

$$\int_{-\infty}^{\infty} \delta^n(t)\,\varphi(t)\,dt = (-1)^n\,\varphi^n(t_0).$$

The derivatives of the Dirac delta function can be obtained (as easy-to-grasp presentation) as a limit of the function derivatives, i.e.

$$\lim_{m\to\infty} \Phi_1^m(mt) = \delta^n(t)\,.$$

The Heaviside Function. The integral of the Dirac delta function with variable upper limit is

$$\int_{-\infty}^{t} \delta(t-t_0)\,dt = H(t-t_0), \quad H(t-t_0) = \begin{cases} 0 & t < t_0; \\ \frac{1}{2} & t = t_0; \\ 1 & t > t_0, \end{cases} \tag{A.4}$$

where $H(t)$ is the Heaviside function (Fig. A4).

Differentiating (4) with respect to t we obtain

$$\frac{dH(t-t_0)}{dt} = \delta(t-t_0)\,. \tag{A.5}$$

The Function sign(t) (Fig. A5) is

$$\text{sign}(t) = \begin{cases} -1 & t < 0; \\ 0 & t = 0; \\ 1 & t > 0; \end{cases} \qquad \text{sign}(t-t_0) = \begin{cases} -1 & t < t_0; \\ 0 & t = t_0; \\ 1 & t > t_0; \end{cases} \tag{A.6}$$

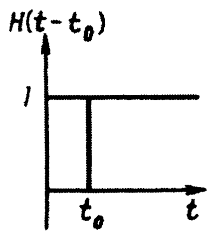

Fig. A.4.

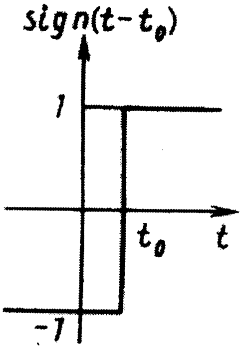

Fig. A.5.

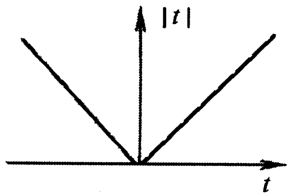

Fig. A.6.

$$\frac{\mathrm{dsign}t}{\mathrm{d}t} = 2\delta\left(t\right), \quad H(t) = \frac{1}{2}(1 + \mathrm{sign}t).$$

The function $f(t) = |t|$ (Fig. A6) is

$$|t| = t\mathrm{sign}t; \quad \frac{\mathrm{d}\left|t\right|}{\mathrm{d}t} = \mathrm{sign}t; \quad \frac{\mathrm{d}^2\left|t\right|}{\mathrm{d}t^2} = 2\delta\left(t\right). \tag{A.7}$$

A.2 Values of Integrals J_n

$$J_n = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\frac{G(i\omega)}{\left|A\left(i\omega\right)\right|^2}\mathrm{d}\omega.$$

where

$$A\left(i\omega\right) = a_0\left(i\omega\right)^n + a_1\left(i\omega\right)^{n-1} + \cdots + a_n;$$
$$G\left(i\omega\right) = b_0\left(i\omega\right)^{2n-2} + b_1\left(i\omega\right)^{2n-4} + \cdots + b_{n-1};$$

$$J_1 = \frac{b_0}{2a_0a_1}; \quad J_2 = \frac{-b_0 + \frac{a_0b_1}{a_2}}{2a_0a_1}; \quad J_3 = \frac{-a_2b_0 + a_0b_1 - \dfrac{a_0a_1b_2}{a_3}}{2a_0(a_0a_3 - a_1a_2)};$$

$$J_4 = \frac{b_0(-a_1a_4 + a_2a_3) - a_0a_3a_1 + a_0a_1b_2 + \dfrac{a_0b_3}{a_4}(a_0a_3 - a_1a_2)}{2a_0(a_0a_3^2 + a_1^2a_4 - a_1a_2a_3)};$$

$$J_5 = \frac{M_5}{2a_0\Delta_5},$$

where

$$\begin{aligned} M_5 &= b_0 - (a_0a_4a_5 + a_1a_4^2 + a_2^2a_5 - a_2a_3a_4) + a_0b_1(-a_2a_5 + a_3a_4) + \\ &+ a_0b_2(a_0a_5 - a_1a_4) + a_0b_3(-a_0a_3 + a_1a_2) + \\ &+ \frac{a_0b_4}{b_4a_5}\left(-a_0a_1a_5 + a_0a_3^2 + a_1^2a_4 - a_1a_2a_3\right); \\ \Delta_5 &= a_0^2a_5^2 - 2a_0a_1a_4a_5 - a_0a_3a_2a_5 + a_0a_3^2a_4 + a_1^2a_4 - a_1a_2a_3a_4. \end{aligned}$$

A.3 Correlation Functions and Spectral Densities Corresponding to Them

<table>
<tr><th>Correlation functions</th><th>Spectral densities</th></tr>
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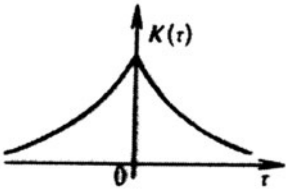
$D_0 e^{-\alpha|\tau|}$



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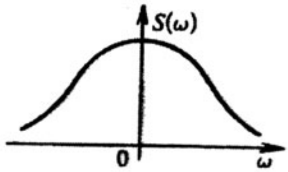
$\dfrac{2\alpha D_0}{2\pi(\alpha^2+\omega^2)}$



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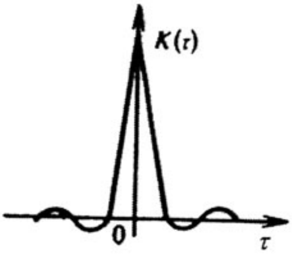
$D_0 e^{-\alpha|\tau|} \cos\beta\tau$



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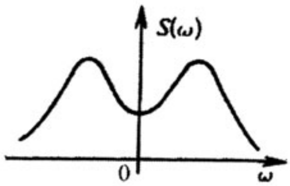
$\dfrac{2\alpha D_0}{\pi} \dfrac{\omega^2+\beta^2+\alpha^2}{(\omega^2-\beta^2-\alpha^2)^2-4\alpha^2\omega^2}$



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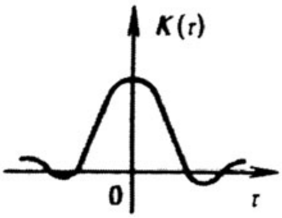
$D_0 e^{-\alpha|\tau|}\left(\cos\beta\tau+\dfrac{\alpha}{\beta}\sin\beta|\tau|\right)$



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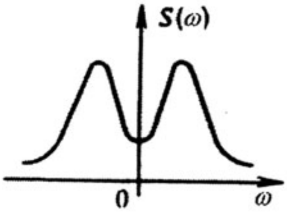
$\dfrac{2\alpha D_0}{\pi} \dfrac{\alpha^2+\beta^2}{(\omega^2-\beta^2-\alpha^2)^2-4\alpha^2\omega^2}$



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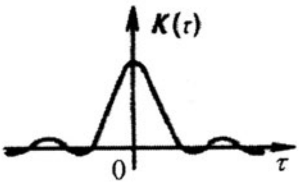
<table>
<tr><th>Correlation functions</th><th>Spectral densities</th></tr>
<tr><td>

$$D_0 \frac{\sin \omega_\beta \tau}{\omega_\beta \tau}$$



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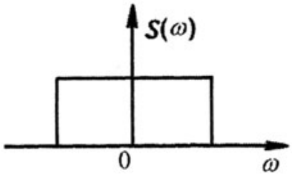
$\dfrac{D_0}{\omega_\beta}$ at $|\omega| < \omega_\beta$

0 at $|\omega| > \omega_\beta$



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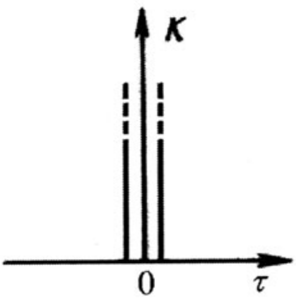
$$K(\tau) = 2\pi s_0 \delta(\tau)$$



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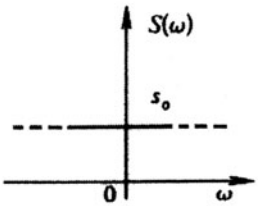

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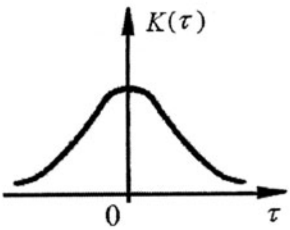
$$D_0 \exp\left\{-\frac{\alpha^2 \tau^2}{4}\right\}$$



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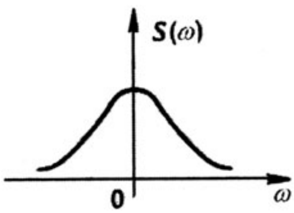
$$\frac{2D_0}{\alpha\sqrt{\pi}} \exp\left\{-\frac{\omega^2}{\alpha^2}\right\}$$



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A.4 Hiawatha Designs an Experiment

by Maurice G. Kendall

1. Hiawatha, mighty hunter
He could shoot ten arrows upwards
Shoot them witli such strength and swiftness
That the last had left the bowstring
Ere the first to earth descended.
This was commonly regarded
As a feat of skill and cunning.

2. One or two sarcastic spirits
Pointed out to him, however,
That it might be much more useful
If he sometimes hit the target.
Why not shoot a little straighter
And employ a smaller sample?

3. Hiawatha, who at college
Majored in applied statistics
Consequently felt entitled
To instruct his fellow men on
Any subject whatsoever.
Waxed exceedingly indignant
Talked about the law of error,
Talked about truncated normals.
Talked of loss of information,
Talked about his lack of bias
Pointed out that in the long run
Independent observations
Even though they missed the target
Had an average point of impact
Very near the spot he aimed at
With the possible exception
Of a set of measure zero.

4. This, they said, was rather doubtful.
Anyway, it didn't matter
What resulted in the long run:
Either he must hit the target
Much more often than at present
Or himself would have to pay for
All the arrows that he wasted.

5. Hiawatha, in a temper
Quoted parts of R.A. Fisher
Quoted Yates and quoted Finney
Quoted yards of Oscar Kempthorne
Quoted reams of Cox and Cochran
Quoted Anderson and Bancroft
Practically in extenso
Trying to impress upon them
That what actually mattered
Was to estimate the error.

6. One or two of them admitted
Such a thing might have its uses
Still, they said, he might do better
If he shot a little straighter.

7. Hiawatha, to convince them
Organized a shooting contest
Laid out in the proper manner
Of designs experimental
Recommended in the textbooks
(Mainly used for tasting tea, but
Sometimes used in other cases)
Randomized his shooting order
In factorial arrangements
Used in the theory of Galois
Fields of ideal polynomials
Got a nicely balanced layout
And successfully confounded
Second-order interactions.

8. All the other tribal marksmen
Ignorant, benighted creatures,
Of experimental set-ups
Spent their time of preparation
Putting in a lot of practice
Merely shooting at a target.

9. Thus it happened in the contest
That their scores were most impressive
With one solitary exception
This (I hate to have to say it)
Was the score of Hiawatha,
Who, as usual, shot his arrows
Shot them with great strength and swiftness
Managing to be unbiased
Not, however, with his salvo
Managing to hit the target.

10. There, they said to Hiawatha,
    That is what we all expected.

11. Hiawatha, nothing daunted,
    Called for pen and called for paper
    Did analyses of variance
    Finally produced the figures
    Showing beyond peradventure
    Everybody else was biased
    And the variance components
    Did not differ from each other
    Or from Hiawatha's
    (This last point, one should acknowledge
    Might have been much more convincing
    If he hadn't been compelled to
    Estimate has own component
    From experimental plots in
    Which the values all were missing.
    Still, they didn't understand it
    So they couldn't raise objections
    This is what so often happens
    With analyses of variance).

12. All the same, his fellow tribesmen
    Ignorant, benighted heathens.
    Took away his bow and arrows.
    Said that though my Hiawatha
    Was a brilliant statistician
    He was useless as a bowman,
    As for variance components
    Several of the more outspoken
    Made primeval observations
    Hurtfull to the finer feelings
    Even of a statistician.

13. In a corner of the forest
    Dwells alone my Hiawatha
    Permanently cogitating
    On the normal law of error
    Wondering in idle moments
    Whether an increased precision
    Might perhaps be rather better
    Even at the risk of bias
    If thereby ore, now and then, could
    Register upon the target.

The sense of the poem lies in the fact that in the case when it is required to obtain the guaranteed final result it is impossible to replace it by its probability estimation.
The theory and the numerical methods of the determination of guaranteed final results are presented in the last chapter

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