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Additional results since the first edition of these notes.

1. Proofs for the results II.3.19 and II.5.6 have now appeared (see [St₂]).
2. The problems posed in I.5.5, III.1.18 and III.3.6 have all been answered in the affirmative (in [L₁], [Ku] and [K] respectively), in the second case under the assumption that the characteristic is good. Further, in the third case the results of 3.5 and 3.7 are all true. It follows that 3.11 fails when p is bad. The answer to II.3.18 is negative: consider the adjoint group of type A₄ in characteristic 5.
3. As regards the classification of nilpotent and unipotent elements discussed in III.4, another approach is now available (see [B-C]). The results of [L₂] lead to a very different kind of classification of unipotent elements.
4. Many interesting results related to the unipotent variety and its desingularization have been obtained. In this desingularization, first given in [23], the fibre above each element consists of the Borel subgroups containing that element. (The variety X of III.3.4 is just the Lie algebra version of this desingularization.) For results about the fibres see, besides the original paper [23], the references [St₁], [St₃] and [Sp] (which is quite comprehensive and contains further references). For the connection with Kleinian (also called rational) singularities, see [Br], [St₁] and [S] (a comprehensive treatment). Finally, it has been shown that the Weyl group operates in the cohomology groups of the fibres. This leads to an a priori parametrization of the irreducible representations of Weyl groups using unipotent conjugacy classes, see [Spr₁], [Spr₂].

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