

NOTES

Firstly, we state some open problems.

1. Is every simple pli-ring a pri-ring?
2. Is every primary pli-ring with a small prime radical an epimorphic image of a primary pli-ring with large prime radical?
3. Is every primary pli-ring with large prime radical an epimorphic image of a prime pli-ring?
4. Let R be a prime pli-ring. If $R \cong M_n(A)$ and $R \cong M_n(B)$ where A, B are domains, is it true that $A \cong B$?
5. Let R be a prime pli-ring and let Q be the simple Artinian l.q. ring of R . If there exists a monomorphism $\rho: Q \rightarrow R$, it is true that $R \cong M_n(A)$ where A is a pli-domain?
6. Let A be a domain such that $M_n(A)$ is a prime pli-ring. Is A a ipli-ring?
7. Is every ideal in a prime pli-ring a product of prime ideals?
8. Does every primary pli-ring (with large prime radical) have a l.q. ring?

If the reader has read the monograph before reading these notes, he will have noticed that we have not given any illustrative examples. Here are a few references: to see that most of the complications in chapters II and III are necessary, take a glance at Jategaonkar [3,4]. To get some feeling for twisted polynomial rings, consult examples in Jacobson [3, chapter 3] and examples in Divinski [1]. For an example of a nasty pli-domain, see Cohn [3]. For some reasonable left Noetherian ipli-rings, see Clark [1]. To get convinced that general left Noetherian rings are appalling, consult Small [4].

REFERENCES

K. Asano

- [1] 'Quotient bildung und schiefringe'. J. Math. Soc. Japan, 1 (1949), 73-78.

H. Bass

- [1] 'K-theory and stable algebra'. Institut des Hautes Scientifiques; Publ. Math. No. 22, (1964).
- [2] 'Algebraic K-theory'. W. A. Benjamin, Inc., New York. 1968.

R. A. Beauregard

- [1] 'Infinite primes and unique factorization in a principal right ideal domain'. Trans. Amer. Math. Soc., (to appear).

E. Cartan and S. Eilenberg

- [1] 'Homological Algebra'. Princeton Univ. Press, Princeton, New Jersey. 1956.

S. U. Chase

- [1] 'Direct products of modules'. Trans. Amer. Math. Soc. 97 (1960), 457-473.

W. E. Clark

- [1] 'Murase's quasi-matrix rings and generalizations'. Sci. paper of the College of Education, Univ. Tokyo, 18 (1968), 99-109.

P. M. Cohn

- [1] 'On a generalization of the Euclidean algorithm'. Proc. Cambridge Phil. Soc., 57 (1961), 18-30.
- [2] 'On the embedding of rings in skew fields'. Proc. London Math. Soc., (3) 11 (1961), 511-530.
- [3] 'Quadratic extension of skew fields'. Proc. London Math. Soc., (3) 11 (1961), 531-556.
- [4] 'A remark on matrix rings over free ideal rings'. Proc. Cambridge Phil. Soc., 62 (1966), 1-4.

J. H. Cozzens

- [1] 'Homological properties of the ring of differential polynomial'. (to appear).

C. W. Curtis

- [1] 'A note on non-commutative polynomials'. Proc. Amer. Math. Soc., 3 (1952), 965-969.

N. J. Divinsky

- [1] 'Rings and radicals'. Math. expositions No. 14, University of Toronto Press, 1965.

C. Faith

- [1] 'Lectures on injective modules and quotient rings'. Lecture notes in Math.; Springer-Verlag, Berlin, Heidelberg, New York.

C. Faith and Y. Utumi

- [1] 'On Noetherian prime rings'. Trans. Amer. Math. Soc. 114 (1965), 53-60.

A. W. Goldie

- [1] 'The structure of prime rings under ascending chain condition'. Proc. London Math. Soc., 8 (1958), 589-608.
- [2] 'Semi-prime rings with maximum condition'. Proc. London Math. Soc., 10 (1960), 201-220.
- [3] 'Non-commutative principal ideal rings'. Arch. Math., 13 (1962), 214-221.
- [4] 'Rings with maximum condition'. Lecture notes, Yale University Press. 1961.
- [5] 'Some aspects of ring theory'. Bull. London Math. Soc. 1 (1969), 129-154.

R. N. Gupta

- [1] 'Characterizations of rings whose classical quotient rings are perfect rings'.
- [2] 'Self-injective quotient rings and injective quotient modules'. Osaka J. Math. 5 (1968), 69-87.

R. N. Gupta and F. Saha

- [1] 'A remark on a paper of Small'. *J. Math. Sci.*, 2 (1967), 7-16.

R. Hart

- [1] 'Simple rings with uniform right ideals'. *J. London Math. Soc.*, 42 (1967), 614-617.

I. N. Herstein

- [1] 'Topics in ring theory'. *Math. lecture Notes; Univ. Chicago Press.* 1965.

I. N. Herstein and L. Small

- [1] 'Nil rings satisfying certain chain conditions'. *Canad. J. Math.*, 16 (1964), 771-776.
- [2] 'Nil rings satisfying certain chain conditions: An addendum'. *Canad. J. Math.* 18 (1966), 300-303.

N. Jacobson

- [1] 'A note on non-commutative polynomials'. *Ann. of Math.* (2) 35 (1934), 209-210.
- [2] 'Theory of rings'. *Amer. Math. Soc., Math Surveys*, vol. 1, Providence, R. I., 1943.
- [3] 'Structure of rings'. *Amer. Math. Soc. Coll. publ.*, vol. 37 (revised edition), Providence, R. I., 1964.

J. P. Jans

- [1] 'Rings and homology'. *Holt, Rinehart and Winston*, New York. 1964.
- [2] 'On orders in Quasi-Frobenius rings'. *J. Algebra*, 7 (1967), 35-43.

A. V. Jategaonkar

- [1] 'Left principal ideal rings'. *Thesis (unpublished)*. Univ. of Rochester, Rochester, N. Y.; 1968.
- [2] 'An example in principal left ideal rings' (unpublished).

- [3] 'Left principal ideal domains'. J. Algebra, 8 (1968), 148-155.
- [4] 'A counter-example in ring theory and homological algebra'. J. Algebra 12 (1969), 418-440.
- [5] 'Structure of left principal ideal rings'. Notices Amer. Math. Soc. 15 (1968), 217.
- [6] 'Local left Noetherian ipli-rings'. Bull. Amer. Math. Soc., 75 (1969), 514-516.
- [7] 'Rings with transfinite left division algorithm'. Bull. Amer. Math. Soc., 75 (1969), 559-561.
- [8] 'Orders in Artinian rings'. (To appear).
- [9] 'Non-isomorphic Noetherian rings with isomorphic matrix rings'. (To appear).
- [10] 'Ore domains and free algebras'. Bull. London Math. Soc., 1 (1969), 45-46.

R. E. Johnson

- [1] 'Principal right ideal rings'. Canad. J. Math. 15 (1963), 297-301.
- [2] 'Unique factorization in principal right ideal domains'. Proc. Amer. Math. Soc., 16 (1965), 526-528.

J. Lambek

- [1] 'Lectures on rings and modules'. Blaisdell Publ. Co. Waltham, Mass., 1966.

C. Lanski

- [1] 'Nil subrings of Goldie rings are nilpotent'. (To appear)

L. Levy

- [1] 'Torsion-free and divisible modules over non-integral domains'. Canad. J. Math., 15 (1963), 132-151.

J. Levitski

- [1] 'On nil subrings'. Israel J. Math., 1 (1963), 215-216.

L. Lesieur and R. Croisot

- [1] 'Sur les anneaux premiers Noetheriens a gauche', Ann. Sci. Ec. Norm. sup. 76 (1959), 161-183.

N. H. McCoy

- [1] 'The Theory of rings'. MacMillan, New York. 1964.

A. C. Mewborn and C. N. Winton

- [1] 'Orders in self-injective semi-perfect rings'. J. Algebra 13 (1969), 5-9.

G. Mitchler

- [1] 'On maximal nilpotent subrings of right Noetherian rings'. Glasgow Math. J., 8 (1967), 89-101.

O. Ore

- [1] 'Theory of non-commutative polynomials'. Ann. of Math., 34 (1933), 480-508.

B. L. Osofsky

- [1] 'Global dimension of valuation rings'. Trans. Amer. Math. Soc., 127 (1967), 136-149.

C. Procesi and L. Small

- [1] 'On a theorem of Goldie'. J. Algebra, 2 (1965), 80-84.

J. C. Robson

- [1] 'Artinian quotient rings'. Proc. London Math. Soc., (3) 17 (1967), 600-616.
- [2] 'Rings in which finitely generated right ideals are principal'. Proc. London Math. Soc., (3) 17 (1967), 617-628.
- [3] 'Pri-rings and ipri-rings'. Quart. J. Math. Oxford (2) 18 (1967), 125-145.
- [4] 'Non-commutative Dedekind rings'. J. Algebra, 9 (1968), 249-265.

W. Sierpinski

- [1] 'Cardinal and ordinal numbers'. Polska Acad. Nauk, Monografie Math., vol. 34. Warszawa, 1958.

L. W. Small

- [1] 'Orders in Artinian rings'. J. Algebra, 4 (1966), 13-41.
- [2] 'Orders in Artinian rings: Corrections and addendum'. J. Algebra, 4 (1966), 505-507.
- [3] 'Orders in Artinian rings, II'. J. Algebra, 9 (1968), 266-273.
- [4] 'On some questions in Noetherian rings'. Bull. Amer. Math. Soc. 72 (1966), 853-857.

R. C. Shock

- [1] to appear.
- [2] 'Nil ideals of rings satisfying maximum condition on right annihilators'. Notices AMS, 16 (1969), 806.

D. A. Smith

- [1] 'On semi-groups, semi-rings and rings of quotients'. J. Sci. Hiroshima Univ. Ser A-I, 30 (1966), 123-130.

T. D. Talintyre

- [1] 'Quotient rings of rings with maximum condition for right ideals'. J. London Math. Soc. 38 (1963), 439-450.
- [2] 'Quotient rings with minimum condition on right ideals'. J. London Math. Soc. 41 (1966), 141-144.

Y. Utumi

- [1] 'A theorem of Levitski'. Amer. Math. Monthly, 70 (1963), 286.

K. G. Wolfson

- [1] 'Isomorphisms of the endomorphism ring of a free module over a principal left ideal domain'. Michigan Math. J., 9 (1962), 69-75.

O. Zariski and P. Samuel

- [1] 'Commutative algebra'. vol. 1. Von Nostrand Co., Inc., New York; 1958.

INDEX

Admissible n-tuples of prime ideals	104	_____ Artinian ring	3
Annihilator ideal	2	_____ associates	105
Annihilator left ideal	2	_____ common multiple property	5
Centralizer	36	_____ divisor set	5
Classical left quotient ring l.q. ring	5	_____ Goldie ring	3
Common denominator Theorem	6	_____ Noetherian ring	3
Completely primary ring	60	_____ order	5
Divisible module	79	_____ Ore domain	10
Domain	3	_____ quotient ring (l.q. ring)	5
Essential left ideal	15	_____ regular element	2
Fully left Goldie ring	4	_____ T-Goldie ring	26
Full regularity condition	26	Local ring	111
$G_A(R)$	69	Matrix units	36
Goldie's first Theorem	12	matrixally reduced module (ring)	41
Goldie's second Theorem	12	Normal form of ordinal	111
Goldie's Third Theorem	45	Ore's Theorem	7
Goldie's Fourth Theorem	88	$P(R)$	3
Graded ring	69	$P(I)$	94
Ideal	2	PF_n -ring	40
Index of nilpotency	2	Permissible rearrangement	104
initial prime component of an ordinal	111	Pli-ring	3
ipli-ring	3	Primary ideal	94
Johnson's first Theorem	60	Primary ring	60
Johnson's second Theorem	60	Prime component	111
Johnson's Third Theorem	60	_____ element	105
$t_R(A)$	2	_____ ideal	2
Large prime radical (rings with)	70	_____ radical of a ring	3
Left Annihilator	2	_____ radical of an ideal	94

_____ ring	2
$R[X, \rho]$	54
$r_R(A)$	2
Regularity condition	26
regular element	2
Robson's Theorem	46
Semi-linear map	37
Semi-pri-ring	3
semi-prime ring	3
semi-simple ring	3
Shock's Theorem	21
skew polynomial ring	54
_____ power series ring	54
Small prime radical (rings with)	69
Small's Theorem	26
$T(R)$	69
Torsion-free modules	79
Torsion module	79
Total left quotient ring	5
Transfinite powers of an ideal	94
Uniform module	46