APPENDIX

§1. Limits.

We recall some facts about limits. Since the results are well known and/or routine, we will omit most of the proofs. We will work in a fixed universe U ([3], VI), so that all categories are assumed to be U-categories, but not necessarily U-small, i.e., a set of U ([3], VI, 2.1).

Let I be an "index" category (the word index has the empty meaning), and

(1.1) $$X: I \to C$$

a functor, where C is another category. For $i \in I$ denote by $X_i \in C$ the value of X on i . A direct limit

$$\varinjlim X = \varinjlim_i X_i$$

is an object of C together with maps $X_i \to \varinjlim X$ for each $i \in I$ such that for every map $i \to i'$ in I the resulting diagram

$$\begin{array}{ccc} X_i & \longrightarrow & X_{i'} \\ & \searrow \quad \swarrow & \\ & \varinjlim X & \end{array}$$

commutes, and such that $\varinjlim X$ is universal with respect to these properties. Clearly $\varinjlim X$ is unique up to unique isomorphism, if it exists. It does always exist if C is for instance the category of sets, groups, or abelian groups and if I is a U-small category.

The inverse limit $\varprojlim X$ is defined in an analogous way ([3], I).

A category I will be called <u>filtering</u> if

(1.2)(a) Every pair i, i' of objects of I can be embedded in a diagram

$$\begin{matrix} i & & \\ & \searrow & \\ & & i'' \\ & \nearrow & \\ i' & & \end{matrix} .$$

(b) (essential uniqueness of maps) If $i \rightrightarrows i'$ is a pair of maps in I, there is a map $i' \to i''$ such that the two composed maps $i \rightrightarrows i''$ are equal.

When I is a U-small filtering category, and $C = (\text{sets})$ in (1.2), then $\varinjlim X$ can be described in the familiar way as the set of equivalence classes in $\cup X_i$ for the following equivalence relation:

(1.3) Let $\alpha \in X_i$, $\alpha' \in X_{i'}$. Then $\alpha \sim \alpha'$ iff there is a diagram

$$\begin{matrix} i & & \\ & \searrow & \\ & & i'' \\ & \nearrow & \\ i' & & \end{matrix}$$

such that the images of α, α' in $X_{i''}$ under the induced maps are equal.

For a U-small I , the functor

$$(1.4) \qquad \varinjlim : \mathrm{Hom}(I, (\mathrm{sets})) \to (\mathrm{sets})$$

always commutes with arbitrary direct limits. If I is filtering, it is also left exact in the sense that it commutes with finite inverse limits, i.e., it preserves monomorphisms and commutes with finite fibred products.

If X: I -> (grps) is a U-small filtering system of groups (or of abelian groups), then the set-theoretic limit has an (abelian) group structure making it into the direct limit in the category of (ab) groups. The functor $\varinjlim$ is an exact functor on abelian groups.

Now let $I \xrightarrow{\phi} J$ be a functor, and <u>suppose</u> I <u>filtering</u>. We will call ϕ <u>cofinal</u> if it has the following properties:

(1.5)(a) For every $j \in J$, there is an $i \in I$ and a map

$$j \to \phi(i) \ .$$

(b) (essential uniqueness) If $j \in J$, and $i \in I$, and if $j \rightrightarrows \phi(i)$ are two maps in J , there is a map $i \to i'$ in I such that the composed maps $j \rightrightarrows \phi(i')$ are equal.

The condition to be cofinal is transitive. Moreover, one verifies easily that ϕ cofinal implies J filtering (where we always assume as above that I is filtering in any case). Condition (b) can be restated in several equivalent ways, especially if J is filtering. But note that (b) does not follow from (a) even if both I and J are filtering:

Example (1.6): Let J be the category consisting of one object j, with one map $f: j \to j$ other than the identity. Suppose that $ff = f$. Then J is filtering. The one point subcategory I consisting of the object j and the identity map is also filtering, and the inclusion satisfies (a) but not (b).

A functor $X: J \to$ (sets) is just a set X together with a projection map f from X to a subset $Y \subset X$. Clearly

$$\varinjlim_{J} X = Y$$

while

$$\varinjlim_{I} X|I = X .$$

(1.7) One case in which (b) follows from (a) is that J is filtering and that I is a full subcategory of J. The category I is then automatically filtering.

Proposition (1.8): Let $I \xrightarrow{\phi} J$ be cofinal, with I filtering, and let $X: J \to$ (sets) be a functor. Suppose I,J U-small. Denote by X_ϕ the composed functor $i \rightsquigarrow X_{\phi(i)}$. Then the canonical map

$$\varinjlim_{I} X_\phi \to \varinjlim_{J} X$$

is bijective.

Proof: Apply (1.3). The surjectivity of the map follows immediately from (1.5)(a). To prove injectivity, let $\alpha \in X_{\phi(i)}$, $\alpha' \in X_{\phi(i')}$ and suppose their images are equal in $\varinjlim X$. This means that there is a diagram in J

(1.9)
$$\begin{array}{ccc} \phi(i) & \searrow & \\ & & j \\ \phi(i') & \nearrow & \end{array}$$

such that $\alpha = \alpha'$ in X_j. Choose $i'' \in I$ and a map $j \to \phi(i'')$ (cf. (1.5)(a)). Changing i'' if necessary, we may assume that there are maps $i \to i''$ and $i' \to i''$. Then we obtain two maps $\phi(i) \rightrightarrows \phi(i'')$, one factoring through j. By (1.5)(b), there is a map $i'' \to i'''$ such that the composed maps $\phi(i) \rightrightarrows \phi(i''')$ are equal. Replacing i'' by i''' and applying the same reasoning to i', it follows that there exists a diagram

$$\begin{array}{ccc} i & \searrow & \\ & & i'' \\ i' & \nearrow & \end{array}$$

in I whose image in J fits into a commutative diagram

$$\begin{array}{ccccc} \phi(i) & & \longrightarrow & & \\ & \searrow & & \searrow & \\ & & j & \longrightarrow & \phi(i'') \\ & \nearrow & & \nearrow & \\ \phi(i') & & \longrightarrow & & \end{array}$$

with (1.9). Hence the images of α, α' are equal in $X_{\phi(i'')}$, hence equal in $\varinjlim X_\phi$.

<u>Proposition (1.10)</u>: Let I be a filtering category, and let J be the category of maps $i \xrightarrow{f} i'$ in I, where a morphism $f \to g$ in J is a commutative diagram

$$\begin{array}{ccc} i & \xrightarrow{f} & i' \\ \downarrow & & \downarrow \\ i'' & \longrightarrow & i''' \end{array}$$

in I. Then

(a) J is filtering.

(b) The functors "domain" and "range" from J to I, taking

$$i \xrightarrow{f} i' \quad \begin{array}{l} \rightsquigarrow i \\ \rightsquigarrow i' \end{array},$$

are cofinal.

(c) The functor "identity map" $I \to J$ taking

$$i \rightsquigarrow i = i$$

is cofinal.

<u>Proposition (1.11)</u>: Let I be a U-small filtering category and let J be the category of maps of I above.

(a) For fixed $i \in I$, the sub-category J_i of J consisting of maps from i to varying $i' \in I$ is filtering.

(b) Let $X: J \to$ (sets) be a functor. There is a natural functor $I \to$ (sets) sending

$$i \rightsquigarrow \varinjlim_{J_i} X|J_i \quad ,$$

and a canonical isomorphism

$$\varinjlim_{i} \left(\varinjlim_{J_i} X|J_i\right) \xrightarrow{\sim} \varinjlim_{J} X \ .$$

§2. Pro-objects and pro-representable functors.

Let C be a category. A pro-object [15] in C is a (contravariant) functor

$$X: I^{\circ} \to C$$

from some U-small filtering index category I to C . We will often use the notation

$$X = \{X_i\}_{i \in I} \quad .$$

One thinks of a pro-object X as an inverse system of objects of C , and the point is that the pro-objects of C can be made in a good way into a category

$$\text{pro-}C$$

by the rule

$$\text{(2.1)} \qquad \operatorname{Hom}(X,Y) = \varprojlim_j (\varinjlim_i \operatorname{Hom}(X_i, Y_j))$$

when $X = \{X_i\}_{i \in I}$ and $Y = \{Y_j\}_{j \in J}$, where we leave it to the reader to elucidate the maps involved in the directed systems. Note that the index categories are not assumed equal. Also, it is important to understand right away that the pro-object contains much more information than the inverse limit $\varprojlim X$ even if the latter exists in C , which we do not assume.

Clearly, a functor $F: C \to C'$ induces in the obvious way a functor

$$\text{pro-}F: \text{pro-}C \to \text{pro-}C' \quad . \tag{2.2}$$

The objects of C themselves are pro-objects if the index category is taken to be the one point category, and by (2.1), C forms a full subcategory of pro-C . If $X = \{X_i\} \in$ pro-C and $Y \in C$, then

$$\mathrm{Hom}(X,Y) = \varinjlim_{i} \mathrm{Hom}(X_i,Y) \quad . \tag{2.3}$$

In this way every $X \in$ pro-C gives rise to a <u>functor</u>

$$\mathrm{Hom}(X,.): C \to (\text{sets}) \quad . \tag{2.4}$$

It can be shown ([25]) that morphisms between the functors associated to pro-objects X,Y are in 1-1 correspondence with elements of Hom(X,Y) defined as in (2.1). Thus pro-C is equivalent via (2.4) to a full subcategory of Hom(C,(sets)) . The functors which are isomorphic to Hom(X,.) for some pro-object X are called <u>pro-representable functors</u>.

<u>Corollary (2.5)</u>: Let $X = \{X_j\}_{j \in J}$ be a pro-object and let $I \xrightarrow{\phi} J$ be a cofinal functor, with I filtering and U-small. Then the pro-object $X_\phi = \{X_{\phi(i)}\}_{i \in I}$ is isomorphic with X .

In fact, it follows from (2.1) that the functors represented by X and X_ϕ are isomorphic. Of course, the isomorphism $X_\phi \to X$ is given by the obvious element of $\varprojlim_j(\varinjlim_i \operatorname{Hom}(X_{\phi(i)}, X_j))$. We will refer to X_ϕ as obtained from X by "re-indexing" with the index category I via ϕ.

The conditions on a functor to be pro-representable are easily understood. Recall that if $F: C \to$ (sets) is a functor, then the morphisms of functors

$$\operatorname{Hom}(Z,.) \to F$$

are in 1-1 correspondence with elements of $F(Z)$. This is easily seen. Suppose we denote by J the category of pairs

$$J = \{(Z,\xi) \mid Z \in C \text{ and } \xi \in F(Z)\} \quad , \tag{2.6}$$

where a morphism

$$(Z,\xi) \to (Z',\xi')$$

is a map $Z \leftarrow Z'$ such that the induced image of ξ' in $F(Z)$ is ξ. The functor

$$J^\circ \to C$$

sending

$$(Z,\xi) \rightsquigarrow Z$$

gives rise to a functor $\mathcal{J}: C \to (\text{sets})$ by

$$\mathcal{J}(Y) = \varinjlim_{(Z,\xi) \in J} \operatorname{Hom}(Z,Y) .$$

The index category J is in general not a U-small category, so this limit is a priori in the next universe U^+. But since ξ gives a map $\operatorname{Hom}(Z;.) \to F$ as above, we obtain a map $\mathcal{J} \to F$, and it is well known that this morphism is bijective (hence in particular that the limit can be taken in U).

<u>Proposition (2.7)</u>: (i) Let $X = \{X_i\}_{i \in I}$ be a pro-object, and let J be the category of pairs $\{(Z,\xi) | \xi \in \operatorname{Hom}(X,Z)\}$ as above. There is a functor $I \xrightarrow{\phi} J$ given by

$$i \rightsquigarrow (X_i, \xi_i)$$

where ξ_i is the image in $\operatorname{Hom}(X,X_i)$ of $\mathrm{id}_{X_i} \in \operatorname{Hom}(X_i,X_i)$. This ϕ is cofinal (hence J is filtering).

(ii) Conversely, let $F: C \to (\text{sets})$ be a functor and let J be the category of pairs defined as in (2.6). Then F is pro-representable iff there is a filtering U-small category I and a cofinal functor $I \xrightarrow{\phi} J$. Equivalently (cf. [15]), F is pro-representable iff J is filtering and contains a U-small cofinal sub-category.

We omit this routine verification.

<u>Corollary (2.8)</u>: Suppose C is itself a U-small category

which is closed under finite inverse limits, i.e., under finite products and finite fibred products. Then a functor F: C -> (sets) is pro-representable iff it is left exact, i.e., commutes with finite inverse limits.

§3. Morphisms of pro-objects.

Let $X = \{X_i\}_{i \in I}$, $Y = \{Y_j\}_{j \in J}$ be in pro-C . If $I = J$ and if

$$f_i \colon X_i \to Y_i \qquad i \in I$$

is a compatible system of maps, i.e., a morphism of functors $f \colon X \to Y$, then f determines in an obvious way a morphism of pro-objects $X \to Y$ which we denote by the same letter f . Of course, a general morphism of pro-objects will not be of this form even if the index categories are equal. However, one can put it into such a form by re-indexing as follows:

A morphism $f \colon X \to Y$ is by (2.1) a compatible collection of maps $\{f_j \colon X \to Y_j\}_{j \in J}$, and each f_j is an element of $\varinjlim_i \operatorname{Hom}(X_i, Y_j)$. We will say that a map

$$\phi \colon X_i \to Y_j$$

<u>represents</u> f if the image of ϕ in $\varinjlim_i \operatorname{Hom}(X_i, Y_j)$ is f_j . This is the same as saying that the diagram in pro-C

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \downarrow & & \downarrow \\ X_i & \xrightarrow{\phi} & Y_j \end{array}$$

commutes. A morphism $\phi \to \phi'$ between maps representing f

consists of a map $i \to i'$ in I and a map $j \to j'$ in J such that the diagram

$$\begin{array}{ccc} X_{i'} & \xrightarrow{\phi'} & Y_{j'} \\ \downarrow & & \downarrow \\ X_i & \xrightarrow{\phi} & Y_j \end{array}$$

commutes.

Proposition (3.1): The category M of maps ϕ representing a map $f\colon X \to Y$ of pro-objects is U-small and filtering, and the functors

$$M \longrightarrow I \quad \text{sending} \quad \phi\colon X_i \longrightarrow Y_j \rightsquigarrow i$$

and

$$M \longrightarrow J \quad \text{sending} \quad \phi\colon X_i \longrightarrow Y_j \rightsquigarrow j$$

are cofinal.

Corollary (3.2): A map $f\colon X \to Y$ of pro-objects of C can be represented, up to isomorphism, by a (U-small) filtering inverse system of maps $\{f_i\colon X_i \to Y_i\}_{i \in I}$, i.e., by a pro-object in the category of maps of C.

The above results can be generalized as follows.

Proposition (3.3): ("uniform approximation") Let Δ be a finite diagram with commutation relations, and suppose that

Δ has no loops, i.e., that the beginning and end of a chain of arrows are always distinct. Let D be a diagram in pro-C of the type of Δ , i.e., a morphism of Δ to pro-C. There is a filtering inverse system $\{D_i\}_{i \in I}$ of diagrams of C such that the diagram in pro-C determined by $\{D_i\}$ is isomorphic to D .

Remark (3.4): One can also make related assertions of the following type: Let $F: C \to C'$ be a functor, $X \in$ pro-C and $Y \in$ pro-C' . Let $f: F(X) \to Y$ be a morphism. Then we may re-index X,Y by a single index category I so that f is represented by a compatible system of maps $f_i: F(X_i) \to Y_i$. This follows immediately from (3.3).

To prove (3.3) consider the category I of diagrams D_i of type Δ in C which represent D , i.e., each of whose maps represents the corresponding map of D . The problem is essentially to show that I is filtering and has a U-small cofinal subcategory, and that for every object X of D , $X = \{X_i\}_{i \in I}$, i.e., that the X_i are cofinal among all maps of X to objects of C .

Induction on the number of vertices of Δ . Suppose it true for (n-1) vertices, and that Δ has n vertices. Choose an "initial" vertex v of Δ , i.e., one having no arrows leading to it. This is possible since Δ has no loops. Let Δ' be the diagram obtained from Δ by removing v and all arrows leading out of v . Let D' be the corresponding diagram in pro-C . By induction, D' can be approximated uniformly, say by $\{D'_j\}_{j \in J}$. Let $X = \{X_i\}_{i \in I}$ be the object of D corresponding to v . Let K be the category of objects k consisting of a pair of indices (i,j) and a diagram D_k representing D and made up out of X_i, D'_j and some maps, where a morphism k -> k' is a map (i,j) -> (i',j') such that the induced maps give a morphism of diagrams $D_{k'} \to D_k$. It remains to show that K is filtering, and that the obvious functors K -> I , K -> J are cofinal. We leave this as an exercise for the reader.

Scholie (3.5): Suppose given a U-small filtering inverse system of pairs of maps

$$X_i \overset{f_i}{\underset{g_i}{\rightrightarrows}} Y_i \quad .$$

Then by (2.1) the induced maps of pro-objects f,g: X -> Y are equal iff for each i the maps $X \to Y_i$ induced by f,g are equal, i.e., iff for each i there is an i' such that the composed maps

$$X_{i'} \longrightarrow X_i \overset{f_i}{\underset{g_i}{\rightrightarrows}} Y_i$$

are equal. In particular, suppose C has a zero object 0 . Then the maps f: X -> Y is the zero map iff for every i there is an i -> i' such that the composed map $X_{i'} \to X_i \to Y_i$ is the zero map. If we set $X_i = Y_i$, $f_i = id$, we find that an object is the zero object (which is equivalent with the assertion that the identity map is the zero map) iff for every $i \in I$ there is an i -> i' such that the structure map

$$X_{i'} \to X_i$$

is the zero map.

§4. Exactness properties of the pro-category.

Proposition (4.1): Let I be a U-small filtering category and C any category. Suppose that C has finite direct (inverse) limits. Then the functor

$$\mathrm{Hom}(I^\circ, C) \to \text{pro-}C$$

associating with an $X\colon I^\circ \to C$ the corresponding pro-object commutes with finite direct (inverse) limits.

Proof: Let $D = \{D_i\}_{i \in I}$ be a finite diagram of functors $I^\circ \to C$. We will denote by $X_\alpha = \{X_{\alpha i}\}_{i \in I}$ the various objects making up the diagram D (we do not bother to label the maps). Let $\underline{X} = \{\underline{X}_i\}_{i \in I}$ be the object $\varinjlim D = \{\varinjlim D_i\}_{i \in I}$. Then in pro-$C$,

$$\mathrm{Hom}(\underline{X}, Y) = \varprojlim_j \varinjlim_i \mathrm{Hom}(\underline{X}_i, Y_j)$$

$$= \varinjlim_j \varinjlim_i \varprojlim_\alpha \mathrm{Hom}(X_{\alpha i}, Y_j)$$

$$= \varprojlim_j \varprojlim_\alpha \varinjlim_i \mathrm{Hom}(X_{\alpha i}, Y_j) \quad \text{because } \varinjlim_i$$

commutes with finite inverse limits ([3], I, 2.8)

$$= \varprojlim_\alpha \varprojlim_j \varinjlim_i \mathrm{Hom}(X_{\alpha i}, Y_j) \quad \text{because } \varprojlim_j$$

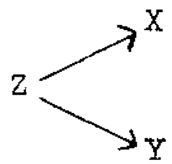
commutes with inverse limits

$$= \varprojlim_\alpha \mathrm{Hom}(X_\alpha, Y) \quad .$$

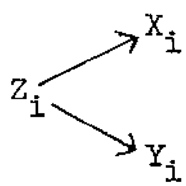
This shows that $\underline{X}$ has the property required of a direct limit in the category pro-C . The proof for an inverse limit goes the same way.

<u>Proposition (4.2)</u>: If C is closed under finite direct (inverse) limits, so is pro-C .

<u>Proof</u>: To show pro-C closed under arbitrary finite direct limits, it suffices to show it closed under coproducts and amalgamated coproducts ([3], I, 2) $X \amalg Y$ and $X \amalg_Z Y$. To show for instance the second, we suppose given a diagram



in pro-C. By (App. 3.3) we may represent it up to isomorphism by a filtering system



$i \in I$.

Now apply (4.1).

<u>Proposition (4.3)</u>: If C is a U-small category which is closed under finite inverse limits, then pro-C is closed under arbitrary (U-small) direct limits.

<u>Proof</u>: If $\{X^\alpha\}$ is a system of objects of pro-C , the

functor

$$\varprojlim_{\alpha} \mathrm{Hom}(X^{\alpha},Y) \qquad Y \in C$$

is a left exact functor from C to sets, since each $\mathrm{Hom}(X^{\alpha},Y)$ is left exact. Hence the functor is pro-representable (2.8).

<u>Proposition (4.4)</u>: For any C , pro-C is closed under (U-small) filtering inverse limits.

<u>Proof</u>: Let $\{X^j\}_{j \in J}$ be a filtering inverse system of pro-objects of C , and say $X^j = \{X^j_i\}$ where $i \in I_j$. Let K be the category whose objects are pairs of indices (j,i) with $j \in J$ and $i \in I_j$, and where a map $(j,i) \to (j',i')$ consists of a map $j \to j'$ and a map $X^{j'}_{i'} \to X^j_i$ representing $X^{j'} \to X^j$. One verifies easily that K is filtering (and U-small), and we claim that the pro-object

$$\{X^j_i\}_{(j,i) \in K}$$

is an inverse limit of the filtering system $\{X^j\}$. In fact, it is clear that if $Z \in$ pro-C , then an element $\varprojlim_{j} \mathrm{Hom}(Z,X^j)$ determines a unique element of $\varprojlim_{(j,i)} \mathrm{Hom}(Z,X^j_i)$, and conversely.

<u>Proposition (4.5)</u>: Let A be an additive (abelian) category. Then pro-A is again additive (abelian).

<u>Proof</u>: The axioms of ([11]) for pro-A follow immediately from those of A using (4.1), (4.2). Note that if C is a U-small category, then pro-A is equivalent by (2.7)

with the category $S_{ex}(A,(ab))$ considered by Gabriel ([11]).

Proposition (4.6): Let A be an abelian category. A monomorphism (epimorphism) in pro-A can be represented by an inverse system of monomorphisms (epimorphisms).

Proof: In an abelian category, a monomorphism is a kernel of a map, and an epimorphism is a cokernel. Since these are finite limits, we can apply (4.1). We leave the assertion for pro-(grps) as an exercise.

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