Solutions

Chapter 1

1.1
(a) 10^{-6}
(b) 10^{-10}

1.2 Theory

1.3 Uncertainties of numerical parameters and model intrinsic uncertainties

Chapter 2

2.1 Rotational velocity of a wheel

2.2 Revolutions per time

2.3 Distance per revolution

2.4
(a) Analog
(b) Digital
(c) Digital
(d) Analog
(e) Digital
(f) Digital
(g) Analog
(h) Digital
(i) Analog (because time is analog!)

2.5 2.252 – 2.3 – 2.25 – 2.3

2.6

(a) 34.
(b) 4.85
(c) 4.8
(d) 5.
(e) 0.06

2.7

(a) $8. \times 10^1$
(b) $7. \times 10^1$
(c) $4. \times 10^2$
(d) $4. \times 10^4$

2.8

(a) 32.35
(b) 32.7
(c) 32.4

Chapter 3

3.1 All best estimates have an uncertainty!

3.2 22.

3.3 46.2

3.4 2.1×10^7

3.5

	Implicit	Explicit
(a)	$10^{1.}$	$+21.62/-6.84$
(b)	$1. \times 10^1$	$\pm 5.$
(c)	1.0×10^1	± 0.5
(d)	10.	± 0.5
(e)	10.0	± 0.05
(f)	10.000	± 0.0005
(g)	$1/4 (= 2^{-2.} \neq 0.25)$	$+0.10/-0.07$
(h)	$(12.)_8$	10 ± 0.5

3.6 74 times

3.7 1.04%

3.8

(a) $(5.4 \pm 4.1)\,\mathrm{m\Omega/m}$
(b) Nonlinearity of the ohmmeter
(c) Using a milliohmmeter the uncertainty could be reduced by about a factor of 10, or perform an optimized current–voltage measurement.

Chapter 4

4.1
(a) No
(b) Zero dead time

4.2 A good approach might be a drawing.

4.3 Statistical fluctuations

4.4
(a) 7.6
(b) 1.44

4.5
(a) 4
(b) 4
(c) 0
(d) 0
(e) 0%

4.6
(a) 0.101
(b) −0.055
(c) 1.094
(d) 0.346
(e) 343.%

4.7
(a) 6.36
(b) 0.78
(c) 11.8
(d) 3.7
(e) 58.%

4.8
(a) A data set with only one data point, a set of data where at least one value has a different weight
(b) A data set without a best estimate by way of a function

4.9
(a) No, only 3 (or at most 4) significant figures
(b) $9910. \pm 29.$

4.10 Graphic solution

4.11 From (4.25) one gets $a_0 = \sum y_i/n$ with $x_i = 0$.

4.12 $y = 95.$

4.13 For $x_m = 6.5$ one gets $y = -13.458 \cdot 6.5 + 9997.6 = 9910.1 = y_m$.

Chapter 5

5.1 Divide all data values by $n = 100$

5.2 2.77

5.3

(a) 9913.3
(b) 9910.1
(c) Loss of resolution because of the grouping into classes

5.4

(a) 171.4 cm
(b) 171.43 cm (assuming equidistant height distribution in the bin)
(c) 170.5 cm
(d) 30.0 cm
(e) 2.6 cm
(f) 1.5%

5.5

(a) $0.77 \neq 3 \cdot (-0.23)$
(b) $-29.9 \approx 3 \cdot (-6.9)$
(c) $0.9 \neq 3 \cdot (-0.03)$
(d) Too few data points, wide bins

5.6

(a) Mode
(b) Median
(c) Mean

5.7 $1. - 0.9^5 = 0.41$

5.8 37.%

5.9 $1! = 1 \times 0! = 1$

5.10 5.0%

5.11 A line parallel to the y-axis with a height of 5168 at the position $x = 16$

5.12

(a) 0.135×10^7
(b) $(1. - 0.14 - 0.27 - 0.27 - 0.18) \cdot 10^7 = 1.4 \times 10^6$

5.13

(a) 1.00σ
(b) 2.00σ
(c) 3.00σ

5.14 $0.5 \cdot (0.8740 - 0.5878) = 0.1431$

5.15

(a) 683
(b) 253
(c) 23
(d) 136

5.16 15.9%

5.17 1/2

5.18 3/4

5.19 1/2

5.20 11/36

5.21 1/9

5.22

(a) 1/169
(b) 1/221

5.23 16/52

5.24 15.2%

5.25

(a) 0.25
(b) 0.75

5.26 $p_S = 2/11$

5.27 $p_1 = 3/13$, $p_2 = 3/13$, $p_3 = 4/13$, $p_4 = 2/13$, $p_5 = 1/13$

5.28 7.6%

5.29 No, answer is within statistical uncertainties.

5.30 For example, the ratio of female to male shoppers

5.31 The ratio of the average length of time one has to wait for either train.

5.32

(a) 0.0063%
(b) 39.3%

5.33

(a) $(175.0000 \pm 0.0043)\,\text{cm}$
(b) It is the best estimate of what a Chinese *thinks* that the height of an American is.
(c) The data values are not normally distributed.

Chapter 6

6.1

(a) $y_m = 9910.1$
(b) $\sigma = \pm 28.7$

6.2

(a) No
(b) The data are not normally distributed.

6.3

(a) 100.1 vs. 99.5
(b) 28.7 vs. 28.9

6.4 (b) and (d)

6.5 $\Delta y_{mw} = 1/\sqrt{(1/\Delta y_1)^2 + (1/\Delta y_2)^2}$

6.6 $\Delta y_m = \Delta y/\sqrt{2}$, with $\Delta y = \Delta y_1 = \Delta y_2$

6.7 0.58

6.8 6.0

6.9 Nondominant components of weighted means may not be disregarded.

6.10 $y_m = (9910.1 \pm 28.7)$ events per minute

6.11 $a_1 = -13.458$, $a_0 = 9990.9$

Chapter 7

7.1 Age

7.2

(a) $\Delta V_m/V_m = \pm 0.022\%$
(b) $\Delta V_m/V_m \approx \pm\sqrt{3} \cdot 0.02\%$

7.3

(a) Ratio $R_s = 1.951 \pm 0.086$
(b) Because the uncertainties of the foreground counts are dominant

7.4 See the *transmission line* example

7.5

(a) $N = 11,111$ events, $ER = 1111$ events/s
(b) $\Delta ER/ER \approx \Delta N/N = \pm 0.95\%$. The uncertainty of the time measurement can be disregarded.

7.6

(a) $N = N_t - 0.5 \cdot (A_1 + A_2) \cdot [X_2 - (X_1 - 1)] = 14{,}512{,}310$
(b) $\Delta N = \sqrt{}(N_t + 0.25 \cdot 104^2 \cdot [A_1 + A_2]) = \pm\, 17{,}863$.
(c) $\Delta N/N = \pm\, 0.12\%$ vs. $\Delta N_t/N_t = \pm\, 0.02\%$, i.e., the uncertainty of the background is dominant.

7.7

(a) $N = N_t - N_{bg} = 18{,}582{,}987$
(b) $\Delta N = \sqrt{N_t + (0.2 \cdot N_{bg})^2} = \pm 335{,}199$
(c) $\Delta N/N = \pm 1.8\%$ vs. $\Delta N_t/N_t = \pm 0.02\%$, i.e., the uncertainty of the background is dominant.
(d) 7.6, uncertainty of the background is ±0.3%; however, the more sophisticated background of 7.7. is 71% smaller!
(e) 7.7, despite the fact that the uncertainty of the background is assumed to be ± 20%, the best estimate is expected to be much closer to the true value than in Problem 7.6.

7.8 See Sects. 7.4.1 and 7.4.2.

7.9 Yes

7.10

(a) Yes, $r_{xy} = 0.934 \approx 1$.
(b) $a_0 = 73.72$, $a_1 = 1.327$

7.11 1.34%

7.12 0.89% < 1%, highly significant

Chapter 8

8.1

(a) $\Delta F = x^{m-1} \cdot y^{n-1} \cdot \sqrt{(m \cdot y)^2 \cdot (\Delta x)^2 + (n \cdot x)^2 \cdot (\Delta y)^2}$
(b) $\Delta F/F = \sqrt{m^2 \cdot (\Delta x/x)^2 + n^2 \cdot (\Delta y/y)^2}$

8.2

(a) Averaging over slightly different diameters
(b) 65.5 mm^3
(c) 2.0 mm^3

8.3 $\Delta\varphi = \tan(\varphi) \cdot \sqrt{(\Delta x/x)^2 + (\Delta X/X)^2}$

8.4

(a) Graphic solution
(b) The line intersects five uncertainty bars instead of the expected four
(c) $chi^2 = 0.69$
(d) The correlated component is 65.8

(e) $chi^2 = 1.00$
(f) Trustworthiness of the chi-squared test requires at least ten degrees of freedom.

Chapter 9

9.1 $\pm 0.012\,\mathrm{V}$

9.2 Seven times shorter; compare ± 0.150 with ± 0.055

9.3 See Sect. 9.3

9.4

(a) 10^{-6}
(b) Temperature variations

9.5 Example 3 in Sect. 7.3.2

Chapter 10

10.1 $\pm 1.7\%$

10.2

(a) 0.27
(b) 0.73
(c) The data are overfitted.
(d) Fewer independent data, less than ten degrees of freedom

10.3

(a) (9.29 ± 0.38) m
(b) If the short circuit has a resistance > 0.
(c) Negligible. The scale uncertainties cancel and the interpolation uncertainties nearly cancel because both measured values R_1 and R_2 will be very close to each other.

10.4 (11.02 ± 0.09) m because scale uncertainties cancel.

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