Appendix A
Bessel Functions

We are collecting some definitions and, mostly well-known, relevant material for Bessel functions. A main reference for those properties of the Bessel functions K_ν and I_ν listed here is Gradshteyn and Ryzhik (1996) and the number codes given in square brackets "[...]" below refer to that work. Adhering to standard notation, throughout the following ν denotes a positive real number, used as an index.

For $\nu > 0$, the modified Bessel function of the third kind, denoted K_ν, may be defined by the relations

$$\int_0^\infty z^{\nu-1}(z+u)^{\nu-1}e^{-2\lambda z}dz = \frac{\Gamma(\nu)}{\Gamma\left(\frac{1}{2}\right)}2^{-\nu+\frac{1}{2}}\lambda^{-2\nu+1}e^{\lambda u}\overline{K}_{\nu-\frac{1}{2}}(\lambda u), \tag{A.1}$$

where

$$\overline{K}_\nu(x) = x^\nu K_\nu(x).$$

Formula (A.1) follows from [3.383.8].[1]

From the elementary exact properties of the Bessel functions K_ν [8.486.16; 8.486.10; 8.486.13] we have that :

$$K_\nu(x) = K_{-\nu}(x),$$
$$K_{\nu+1}(x) = 2\nu x^{-1}K_\nu(x) + K_{\nu-1}(x),$$
$$K_\nu'(x) = -K_{\nu-1}(x) - \nu x^{-1}K_\nu(x).$$

[1]There is a misprint in the latter formula: The factor $e^{\beta\mu}$ on the right hand side should be replaced by $e^{\beta\mu/2}$.

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From this follows the recursive expression for $\overline{K}$,

$$\overline{K}_{\nu+1}(x) = 2\nu\overline{K}_{\nu}(x) + x^2\overline{K}_{\nu-1}(x),$$

and, moreover, the derivative becomes

$$\overline{K}'_{\nu}(x) = -x\overline{K}_{\nu-1}(x). \tag{A.2}$$

Introduce the Bessel functions I_ν by,

$$I_\nu\left(x\right) = \left(\frac{x}{2}\right)^\nu \sum_{n=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2n}}{n!\Gamma\left(n+\nu+1\right)}. \tag{A.3}$$

This definition is quoted from Watson (1966). The Bessel functions K_ν and I_ν are, provided that ν is not an integer, connected by [8.485] as follows

$$K_\nu\left(x\right) = \frac{1}{2}\frac{\pi}{\sin\left(\pi\nu\right)}\left(I_{-\nu}\left(x\right) - I_\nu\left(x\right)\right),$$

which, due to the well-known formula [8.334.3] for Γ-functions,

$$\Gamma\left(1-\nu\right)\Gamma\left(\nu\right) = \frac{\pi}{\sin\left(\pi\nu\right)},$$

may be rewritten as

$$K_\nu\left(x\right) = \frac{1}{2}\Gamma\left(1-\nu\right)\Gamma\left(\nu\right)\left(I_{-\nu}\left(x\right) - I_\nu\left(x\right)\right). \tag{A.4}$$

Further, the Bessel functions I_ν satisfy the recursion relation [8.486.1]:

$$xI_{\nu-1}(x) - xI_{\nu+1}(x) = 2\nu I_\nu(x). \tag{A.5}$$

Also, the Bessel functions of the first kind J_ν and the second kind N_ν are defined as

$$J_\nu(x) = \frac{x^\nu}{2^\nu}\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{2^{2k}k!\Gamma(\nu+k+1)},$$

$$N_\nu(x) = \frac{1}{\sin(\nu\pi)}\left(\cos(\nu\pi J_\nu(x) - J_{-\nu}(x))\right).$$

A graphical illustration of the modified Bessel function of the third kind $K_\nu(x)$ and of $\overline{K}_\nu(x)$ is given in Figs. A.1, A.2, A.3, and A.4.

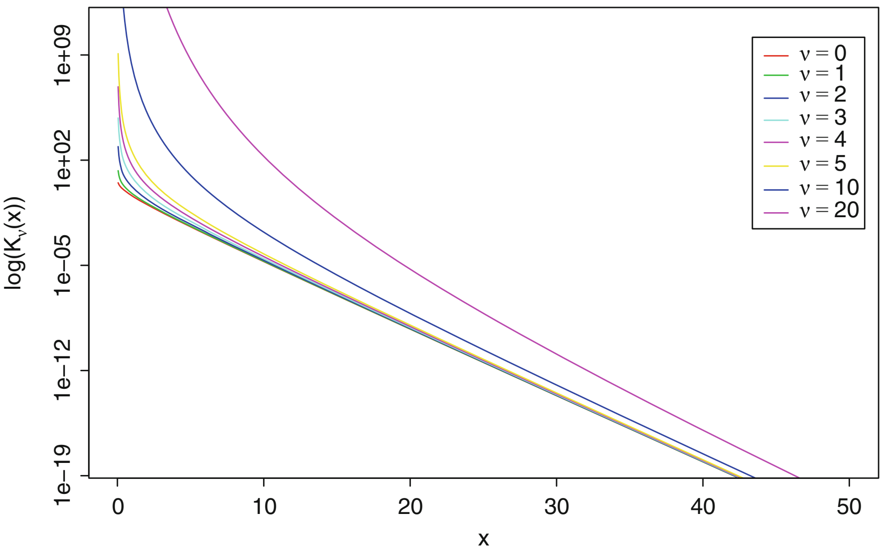

Fig. A.1 Plots of the logarithmic transform of the Bessel function $K_\nu(x)$ for various choices of ν

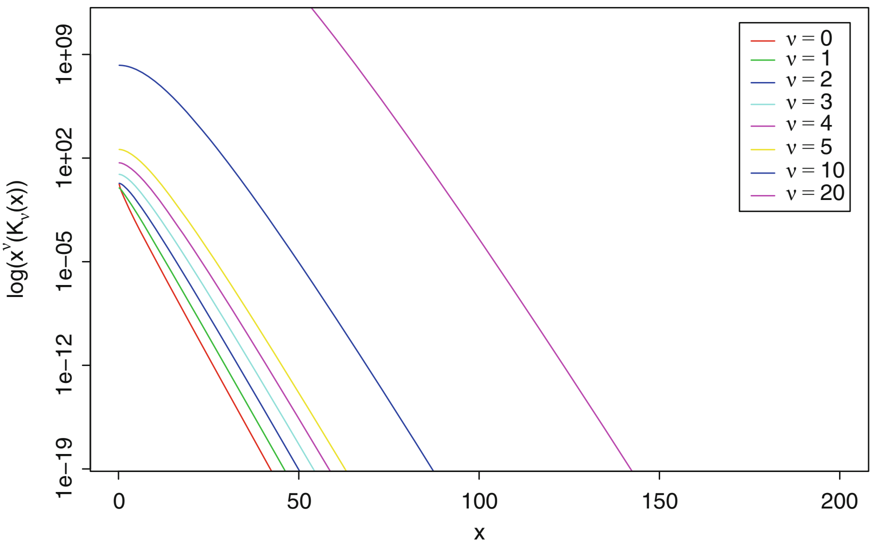

Fig. A.2 Plots of the logarithmic transform of the function $\overline{K}_\nu(x)$ for various choices of ν

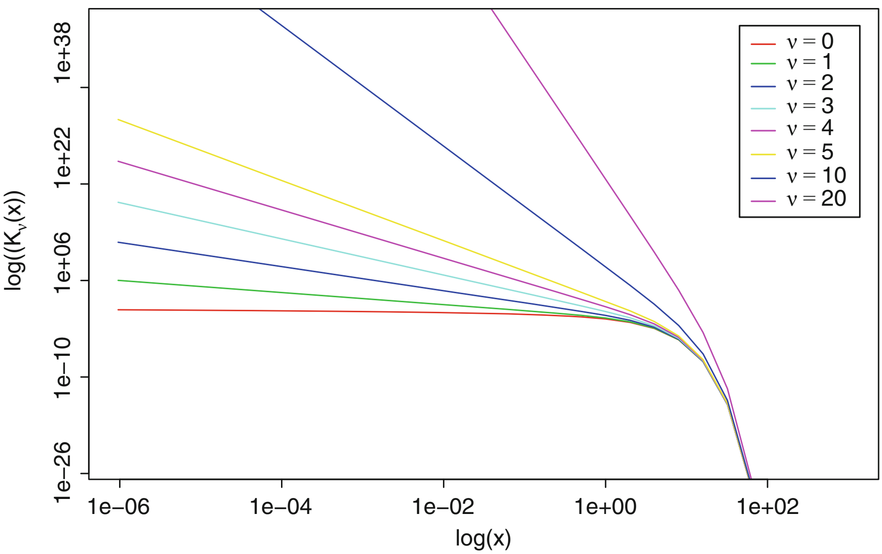

Fig. A.3 Plots of the Bessel function $K_\nu(x)$ for various choices of ν near zero. The plot is presented on a log-log scale

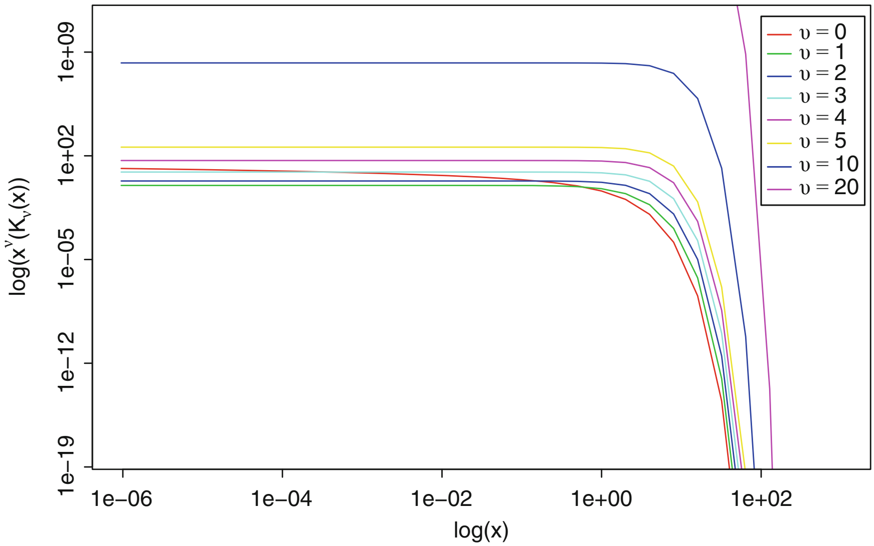

Fig. A.4 Plots of the function $\overline{K}_\nu(x)$ for various choices of ν near zero. The plot is presented on a log-log scale

Appendix B
Generalised Hyperbolic Distribution

In the following we are summarising basic definitions of the generalised inverse Gaussian (GIG) and the generalised hyperbolic (GH) distribution.

B.1 Generalised Inverse Gaussian Distribution

The generalised inverse Gaussian $\mathrm{GIG}(\nu, \delta, \gamma)$ distribution is a three-parameter probability distribution with support on $(0, \infty)$ whose density is given by

$$f(x; \nu, \delta, \gamma) = \frac{(\gamma/\delta)^\nu}{2K_\nu(\delta\gamma)} x^{\nu-1} \exp\left(-\frac{1}{2}(\delta^2 x^{-1} + \gamma^2 x)\right), \quad \text{for } x > 0, \tag{B.1}$$

where $\nu \in \mathbb{R}$ and γ and δ are both nonnegative and not simultaneously equal to zero; also recall that $K_\nu(\cdot)$ is the modified Bessel function of the third kind, see Appendix A.

If we set $\overline{\gamma} = \delta\gamma$, (B.1) can be rewritten as

$$f(x; \nu, \delta, \gamma) = \frac{\overline{\gamma}^\nu}{2K_\nu(\overline{\gamma})} \delta^{-2\nu} x^{\nu-1} \exp\left(-\frac{1}{2}\left(\delta^2 x^{-1} + \frac{\overline{\gamma}^2}{\delta^2} x\right)\right), \quad \text{for } x > 0. \tag{B.2}$$

From the latter representation we see that δ^2 plays the role of a scale parameter, whereas $\overline{\gamma}$ is invariant under scale transformations. The case when $\overline{\gamma} = \delta\gamma = 0$ in (B.1) should be interpreted as a limiting case. The Lévy density associated with the $\mathrm{GIG}(\nu, \delta, \gamma)$ distribution is given by

$$u(x) = x^{-1}\left(\frac{1}{2}\int_0^\infty e^{-\frac{1}{2}\delta^{-2}x\xi} g_\nu(\xi) d\xi + \max(0, \nu)\nu\right) \exp(-\gamma^2 x/2),$$

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where $g_\nu(x) = \frac{2}{x\pi^2}\left(J^2_{|\nu|}(\sqrt{x}) + N^2_{|\nu|}(\sqrt{x})\right)^{-1}$. Recall that $J_{|\nu|}$ and $N_{|\nu|}$ denote the Bessel functions of the first and second kind, respectively, see Appendix A.

Special cases of the GIG(ν, δ, γ) include the:

Inverse Gaussian distribution IG(δ, γ) = GIG$(-\frac{1}{2}, \delta, \gamma)$ for $\delta > 0, \gamma \geq 0$, which has probability density

$$f(x) = \frac{\delta}{\sqrt{2\pi}} e^{\delta\gamma} x^{-3/2} \exp\left(-\frac{1}{2}(\delta^2 x^{-1} + \gamma^2 x)\right), \qquad \text{for } x > 0,$$

and Lévy density

$$u(x) = \frac{\delta}{\sqrt{2\pi}} x^{-3/2} \exp\left(-\frac{1}{2}\gamma^2 x\right).$$

Gamma Distribution $\Gamma(\nu, \alpha)$ = GIG$(\nu, 0, \gamma)$ for $\nu > 0$ and $\alpha = \gamma^2/2$, which has probability density

$$f(x) = \frac{\alpha^\nu}{\Gamma(\nu)} x^{\nu-1} \exp(-\alpha x), \qquad \text{for } x > 0,$$

and Lévy density

$$u(x) = \nu x^{-1} \exp(-\alpha x).$$

Other important sub-classes of the GIG distribution include the reciprocal inverse Gaussian distribution RIG$(\delta > 0, \gamma \geq 0)$ = GIG$(\frac{1}{2}, \delta, \gamma)$, the reciprocal gamma distribution R$\Gamma(\overline{\nu} > 0, \alpha = \delta^2/2)$ = GIG$(-\nu, \delta, 0)$ (for $\overline{\nu} = -\nu$), the positive hyperbolic distribution PH$(\delta > 0, \gamma \geq 0)$ = GIG$(1, \delta, \gamma)$, and the reciprocal positive hyperbolic distribution RPH(δ, γ) = GIG$(-1, \delta, \gamma)$.

B.2 Generalised Hyperbolic Distribution

The generalised hyperbolic distribution can be constructed via a mean-variance mixture of a standard normally distributed random variable, where the mixing distribution is chosen to be a GIG distribution. We briefly summarise the univariate construction of a random variable with generalised hyperbolic distribution, see in particular Barndorff-Nielsen (1977) for the hyperbolic distribution and Barndorff-Nielsen (1997) for the normal inverse Gaussian distribution.

Let $U \sim \mathrm{N}(0,1)$ and $\sigma^2 \sim \mathrm{GIG}(\nu,\delta,\gamma)$ and suppose that U and σ are independent. Let $\mu,\beta \in \mathbb{R}$ and set

$$Y = \mu + \beta\sigma^2 + \sigma U.$$

Then

$$Y|\sigma^2 \sim \mathrm{N}(\mu + \beta\sigma^2, \sigma^2).$$

The random variable Y follows a generalised hyperbolic distribution $\mathrm{GH}(\nu,\alpha,\beta,\mu,\delta)$, where $\alpha = \sqrt{\gamma^2+\beta^2}$. Its probability density function is given by

$$f(x;\nu,\alpha,\beta,\mu,\delta) = \frac{\gamma^{2\nu}\alpha^{1-2\nu}}{\sqrt{2\pi}\,\overline{K}_\nu(\delta\gamma)}\overline{K}_{\nu-\frac{1}{2}}\left(\alpha\sqrt{\delta^2+(y-\mu)^2}\right)\exp(\beta(y-\mu)),$$

for $x \in \mathbb{R}$. One can show that the the generalised hyperbolic laws have semi-heavy tails; more specifically, as $x \to \pm\infty$, we have that

$$f(x;\nu,\alpha,\beta,0,\delta) = const.|x|^{\nu-1}\exp(-\alpha|x|+\beta).$$

The following sub-classes of the generalised hyperbolic distribution are of particular interest in applications, see e.g. Chap. 10:

Gaussian Distribution $\mathrm{N}(\mu,\sigma^2) = \lim_{\gamma\to\infty}\mathrm{GH}(\nu,\gamma,0,\mu,\sigma^2\gamma)$ with density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right).$$

Normal Inverse Gaussian Distribution $\mathrm{NIG}(\alpha,\beta,\mu,\delta) = \mathrm{GH}(-\frac{1}{2},\alpha,\beta,\mu,\delta)$, which corresponds to choosing $\sigma^2 \sim \mathrm{IG}(\delta,\gamma)$, with density function

$$f(x) = \pi^{-1}\alpha\exp\left(\delta\sqrt{\alpha^2-\beta^2}\right)(q((x-\mu)/\delta))^{-1}K_1(\delta\alpha q((x-\mu)/\delta))e^{\beta(x-\mu)},$$

where $q(x) = \sqrt{1+x^2}$.

Variance Gamma Distribution $\mathrm{VG}(\nu,\alpha,\beta,\mu) = \mathrm{GH}(\nu,\alpha,\beta,\mu,0)$, which corresponds to choosing $\sigma^2 \sim \Gamma(\nu,\gamma^2/2)$, with density function

$$f(x) = \frac{(\alpha^2-\beta^2)^\nu\alpha^{1-2\nu}}{\sqrt{2\pi}\,\Gamma(\nu)2^{\nu-1}}\overline{K}_{\nu-1/2}(\alpha|x-\mu|)e^{\beta(x-\mu)}.$$

Student t Distribution $\mathrm{T}(\nu, \beta, \delta, \mu) = \mathrm{GH}(-\nu, \beta, \beta, \mu, \delta)$, which corresponds to choosing $\sigma^2 \sim \mathrm{R}\Gamma(\nu, \delta^2/2)$, with density function

$$f(x) = \frac{1}{\sqrt{2\pi}\delta\Gamma(\nu)2^{\nu-1}}(q((x-\mu)/\delta))^{-2\nu-1}\overline{K}_{\nu+1/2}(\delta|\beta|q((x-\mu)/\delta))\,e^{\beta(x-\mu)},$$

where $q(x) = \sqrt{1+x^2}$.

Hyperbolic Distribution $\mathrm{H}(\alpha, \beta, \mu, \delta) = \mathrm{GH}(1, \alpha, \beta, \mu, \delta)$, which corresponds to choosing $\sigma^2 \sim \mathrm{PH}(\delta, \gamma)$, with density function

$$f(x) = \frac{\sqrt{\alpha^2-\beta^2}}{2\alpha\delta K_1(\delta\sqrt{\alpha^2-\beta^2})}\exp\left(-\alpha\sqrt{\delta^2+(x-\mu)^2}+\beta(x-\mu)\right).$$

If $\beta = 0$ in the GH, NIG, VG, T and H distributions, then they are symmetric.

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