Appendix A
Potential Function Solutions for Elasticity Problems

Elastic contact problems are greatly facilitated by representing the stress and displacement fields in terms of scalar potential functions. These can be tailored so as to give relatively simple expressions for the tractions and displacements at a plane surface, starting from the Papkovitch–Neuber solution in terms of harmonic potentials.[1]

A.1 Frictionless Problems

For problems in which the elastic half-space $z > 0$ is subjected to purely normal tractions, the elastic fields can conveniently be expressed in terms of a potential function φ where

$$\nabla^2\varphi \equiv \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0. \tag{A.1}$$

The stress and displacement components in Cartesian coordinates (x, y, y) are given by

$$2Gu_x = z\frac{\partial^2\varphi}{\partial x\partial z} + (1-2\nu)\frac{\partial\varphi}{\partial x}; \quad 2Gu_y = z\frac{\partial^2\varphi}{\partial y\partial z} + (1-2\nu)\frac{\partial\varphi}{\partial y}$$

$$2Gu_z = z\frac{\partial^2\varphi}{\partial z^2} - 2(1-\nu)\frac{\partial\varphi}{\partial z}$$

$$\sigma_{xx} = z\frac{\partial^3\varphi}{\partial x^2\partial z} + \frac{\partial^2\varphi}{\partial x^2} + 2\nu\frac{\partial^2\varphi}{\partial y^2}; \quad \sigma_{xy} = z\frac{\partial^3\varphi}{\partial x\partial y\partial z} + (1-2\nu)\frac{\partial^2\varphi}{\partial x\partial y} \tag{A.2}$$

$$\sigma_{yy} = z\frac{\partial^3\varphi}{\partial y^2\partial z} + \frac{\partial^2\varphi}{\partial y^2} + 2\nu\frac{\partial^2\varphi}{\partial x^2}$$

[1] For more details of this procedure, see Barber (2010), Chaps. 21,22.

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$$\sigma_{xz} = z\frac{\partial^3\varphi}{\partial x\partial z^2}; \quad \sigma_{yz} = z\frac{\partial^3\varphi}{\partial y\partial z^2}; \quad \sigma_{zz} = z\frac{\partial^3\varphi}{\partial z^3} - \frac{\partial^2\varphi}{\partial z^2},$$

where G, ν are the shear modulus and Poisson's ratio respectively for the material (Green and Zerna 1954; Barber 2010, pp. 339–341).

Corresponding expressions in cylindrical polar coordinates (r, θ, z) are

$$2Gu_r = z\frac{\partial^2\varphi}{\partial r\partial z} + (1-2\nu)\frac{\partial\varphi}{\partial r}; \quad 2Gu_\theta = \frac{z}{r}\frac{\partial^2\varphi}{\partial\theta\partial z} + \frac{(1-2\nu)}{r}\frac{\partial\varphi}{\partial\theta}$$

$$2Gu_z = z\frac{\partial^2\varphi}{\partial z^2} - 2(1-\nu)\frac{\partial\varphi}{\partial z}$$

$$\sigma_{rr} = z\frac{\partial^3\varphi}{\partial r^2\partial z} + \frac{\partial^2\varphi}{\partial r^2} - 2\nu\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{\partial^2\varphi}{\partial z^2}\right) \tag{A.3}$$

$$\sigma_{r\theta} = \frac{z}{r}\frac{\partial^3\varphi}{\partial r\partial\theta\partial z} - \frac{z}{r^2}\frac{\partial^2\varphi}{\partial\theta\partial z} + \frac{(1-2\nu)}{r}\left(\frac{\partial^2\varphi}{\partial r\partial\theta} - \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right)$$

$$\sigma_{\theta\theta} = -(1-2\nu)\frac{\partial^2\varphi}{\partial r^2} - \frac{\partial^2\varphi}{\partial z^2} - z\frac{\partial^3\varphi}{\partial r^2\partial z} - z\frac{\partial^3\varphi}{\partial z^3}$$

$$\sigma_{rz} = z\frac{\partial^3\varphi}{\partial r\partial z^2}; \quad \sigma_{z\theta} = \frac{z}{r}\frac{\partial^3\varphi}{\partial\theta\partial z^2}; \quad \sigma_{zz} = z\frac{\partial^3\varphi}{\partial z^3} - \frac{\partial^2\varphi}{\partial z^2}.$$

These expressions satisfy the condition that the shear tractions σ_{zx}, σ_{zy} or $\sigma_{z\theta}, \sigma_{zr}$ be zero on the surface $z = 0$ for all harmonic functions φ and on this surface, the normal traction and the normal displacement are given by the simpler expressions

$$\sigma_{zz}(r, \theta, 0) = -\frac{\partial^2\varphi}{\partial z^2}; \quad u_z(r, \theta, 0) = -\frac{(1-\nu)}{G}\frac{\partial\varphi}{\partial z} \tag{A.4}$$

respectively.

A.2 Problems with Tangential Tractions

If tangential tractions are also applied to the surface $z = 0$, the preceding solution should be supplemented by two additional harmonic potential functions χ, ψ, defining the additional stress and displacement components

$$2Gu_x = 2(1-\nu)\frac{\partial\chi}{\partial x} + z\frac{\partial^2\chi}{\partial x\partial z} + 2\frac{\partial\psi}{\partial y}; \quad 2Gu_y = 2(1-\nu)\frac{\partial\chi}{\partial y} + z\frac{\partial^2\chi}{\partial y\partial z} - 2\frac{\partial\psi}{\partial x}$$

$$2Gu_z = -(1-2\nu)\frac{\partial\chi}{\partial z} + z\frac{\partial^2\chi}{\partial z^2}$$

$$\sigma_{xx} = 2(1-\nu)\frac{\partial^2\chi}{\partial x^2} + z\frac{\partial^3\chi}{\partial x^2\partial z} - 2\nu\frac{\partial^2\chi}{\partial z^2} + 2\frac{\partial^2\psi}{\partial x\partial y} \tag{A.5}$$

$$\sigma_{xy} = 2(1-\nu)\frac{\partial^2\chi}{\partial x\partial y} + z\frac{\partial^3\chi}{\partial x\partial y\partial z} + \frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial x^2}$$

$$\sigma_{yy} = 2(1-\nu)\frac{\partial^2\chi}{\partial y^2} + z\frac{\partial^3\chi}{\partial y^2\partial z} - 2\nu\frac{\partial^2\chi}{\partial z^2} - 2\frac{\partial^2\psi}{\partial x\partial y}$$

$$\sigma_{xz} = \frac{\partial^2\chi}{\partial x\partial z} + z\frac{\partial^3\chi}{\partial x\partial z^2} + \frac{\partial^2\psi}{\partial y\partial z}; \quad \sigma_{yz} = \frac{\partial^2\chi}{\partial y\partial z} + z\frac{\partial^3\chi}{\partial y\partial z^2} - \frac{\partial^2\psi}{\partial x\partial z}; \quad \sigma_{zz} = z\frac{\partial^3\chi}{\partial z^3},$$

or in cylindrical polar coordinates (r, θ, z),

$$2Gu_r = 2(1-\nu)\frac{\partial\chi}{\partial r} + z\frac{\partial^2\chi}{\partial r\partial z} + \frac{2}{r}\frac{\partial\psi}{\partial\theta}; \quad 2Gu_\theta = \frac{2(1-\nu)}{r}\frac{\partial\chi}{\partial\theta} + \frac{z}{r}\frac{\partial^2\chi}{\partial\theta\partial z} - 2\frac{\partial\psi}{\partial r}$$

$$2Gu_z = -(1-2\nu)\frac{\partial\chi}{\partial z} + z\frac{\partial^2\chi}{\partial z^2}$$

$$\sigma_{rr} = 2(1-\nu)\frac{\partial^2\chi}{\partial r^2} + z\frac{\partial^3\chi}{\partial r^2\partial z} - 2\nu\frac{\partial^2\chi}{\partial z^2} + \frac{2}{r}\frac{\partial^2\psi}{\partial r\partial\theta} - \frac{2}{r^2}\frac{\partial\psi}{\partial\theta} \tag{A.6}$$

$$\sigma_{r\theta} = \frac{2(1-\nu)}{r}\left(\frac{\partial^2\chi}{\partial r\partial\theta} - \frac{1}{r}\frac{\partial\chi}{\partial\theta}\right) + \frac{z}{r}\frac{\partial^3\chi}{\partial z\partial r\partial\theta} - \frac{z}{r^2}\frac{\partial^2\chi}{\partial z\partial\theta} + \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}$$

$$\sigma_{\theta\theta} = -2(1-\nu)\frac{\partial^2\chi}{\partial r^2} - 2\frac{\partial^2\chi}{\partial z^2} + \frac{z}{r}\frac{\partial^2\chi}{\partial z\partial r} + \frac{z}{r^2}\frac{\partial^3\chi}{\partial z\partial\theta^2} - \frac{2}{r}\frac{\partial^2\psi}{\partial r\partial\theta} + \frac{2}{r^2}\frac{\partial\psi}{\partial\theta}$$

$$\sigma_{rz} = \frac{\partial^2\chi}{\partial r\partial z} + z\frac{\partial^3\chi}{\partial r\partial z^2} + \frac{1}{r}\frac{\partial^2\psi}{\partial\theta\partial z}; \quad \sigma_{z\theta} = \frac{1}{r}\frac{\partial^2\chi}{\partial\theta\partial z} + \frac{z}{r}\frac{\partial^3\chi}{\partial\theta\partial z^2} - \frac{\partial^2\psi}{\partial r\partial z}$$

$$\sigma_{zz} = z\frac{\partial^3\chi}{\partial z^3}.$$

For axisymmetric problems, only the function χ is required.

A.3 Two-Dimensional Problems

Two-dimensional plane strain solutions can be expressed in terms of two harmonic potential functions ϕ, ψ through the relations

$$2Gu_x = \frac{\partial\phi}{\partial x} + z\frac{\partial\psi}{\partial x}; \quad 2Gu_z = \frac{\partial\phi}{\partial z} + z\frac{\partial\psi}{\partial z} - (3-4\nu)\psi \tag{A.7}$$

$$\sigma_{xx} = \frac{\partial^2\phi}{\partial x^2} + z\frac{\partial^2\psi}{\partial x^2} - 2\nu\frac{\partial\psi}{\partial z}; \quad \sigma_{xz} = \frac{\partial^2\phi}{\partial x\partial z} + z\frac{\partial^2\psi}{\partial x\partial z} - (1-2\nu)\frac{\partial\psi}{\partial x}$$

$$\sigma_{zz} = \frac{\partial^2\phi}{\partial z^2} + z\frac{\partial^2\psi}{\partial z^2} - 2(1-\nu)\frac{\partial\psi}{\partial z}. \tag{A.8}$$

Appendix B
Integrals over Elliptical Domains

In the elastic contact of bodies with quadratic surfaces, the contact area $\mathcal{A}$ is an ellipse defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1. \tag{B.1}$$

Problems of this class often require the evaluation of integrals of the form

$$\begin{aligned} J_n(x, y) &= \iint_{\mathcal{A}} \left(1 - \frac{\xi^2}{a^2} - \frac{\eta^2}{b^2}\right)^{n-1/2} \frac{H(\theta) d\xi d\eta}{r} \\ &= \int_0^{\pi} \int_{S_1}^{S_2} \left(1 - \frac{\xi^2}{a^2} - \frac{\eta^2}{b^2}\right)^{n-1/2} H(\theta) dr d\theta, \end{aligned} \tag{B.2}$$

where r, θ is a set of polar coordinates centred on the field point $P(x, y)$, and S_1, S_2 are defined in Fig. B.1,

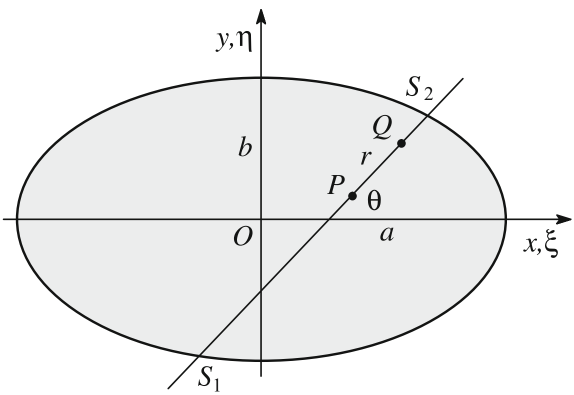

Fig. B.1 Elliptical contact area

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The function $H(\theta)$ might arise from anisotropy of the material, as in Sect. 2.2.2, or from the Green's function for tangential loading, as in Eq. (7.6) and Sect. 7.6.2. In both cases, $H(\theta)$ satisfies the condition $H(\theta + \pi) = H(\theta)$ and hence can be expanded in the Fourier series

$$H(\theta) = \sum_{m=0}^{\infty} a_m \cos(2m\theta) + \sum_{m=1}^{\infty} b_m \sin(2m\theta). \tag{B.3}$$

Some results of this kind were given in Chaps. 2 and 3 in connection with Galin's theorem and the Hertzian theory of contact. Here, we give a more unified treatment of the procedure and show how it can be extended to other problems such as the contact of anisotropic materials, or tangential loading of Hertzian contacts.

B.1 Mathematical Preliminaries

We note from Fig. B.1 or Eq. (2.24) that $\xi = x + r\cos\theta$; $\eta = y + r\sin\theta$, and hence

$$1 - \frac{\xi^2}{a^2} - \frac{\eta^2}{b^2} = C_0 - C_1(\theta)r - C_2(\theta)r^2, \tag{B.4}$$

where

$$C_0 = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}; \quad C_1(\theta) = 2\left(\frac{x\cos\theta}{a^2} + \frac{y\sin\theta}{b^2}\right) \tag{B.5}$$

$$C_2(\theta) = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{(1 - e^2\cos^2\theta)}{b^2}, \tag{B.6}$$

where the eccentricity e is defined by

$$e^2 = 1 - \frac{b^2}{a^2}. \tag{B.7}$$

The integral (B.2) can then be written

$$J_n(x, y) = \int_0^{\pi} H(\theta) \int_{S_1}^{S_2} \left\{C_0 - C_1(\theta)r - C_2(\theta)r^2\right\}^{n-1/2} dr d\theta, \tag{B.8}$$

where S_1, S_2 are the two points at which the quadratic function $\{C_0 - C_1 B(\theta)r - C_2(\theta)r^2\} = 0$, from (B.4), (B.1).

Writing

$$r = t - \frac{C_1}{2C_2}; \quad D^2 = \frac{C_0}{C_2} + \frac{C_1^2}{4C_2^2}, \tag{B.9}$$

we can evaluate the inner integral as

$$\int_{S_1}^{S_2} \left\{C_0 - C_1(\theta)r - C_2(\theta)r^2\right\}^{n-1/2} dr = C_2^{n-1/2} \int_{-D}^{D} \left(D^2 - t^2\right)^{n-1/2} dt = \frac{(2n-1)!!\,\pi C_2^{n-1/2} D^{2n}}{(2n)!!}, \tag{B.10}$$

where $(2n-1)!! = 1.3.5...(2n-1)$, $(2n)!! = 2.4.6...(2n)$. We then have

$$J_n(x,y) = \frac{(2n-1)!!\pi}{(2n)!!} \int_0^{\pi} H(\theta) C_2^{n-1/2} D^{2n} d\theta. \tag{B.11}$$

B.1.1 The Singular Field $n = 0$

If $n = 0$, Eq. (B.2) corresponds to a traction distribution that is singular at the boundaries of $\mathcal{A}$. Setting $n = 0$ in (B.11) and using (B.6) for C_2, we obtain

$$J_0(x,y) = \pi b \int_0^{\pi} \frac{H(\theta) d\theta}{\sqrt{1 - e^2 \cos^2\theta}}. \tag{B.12}$$

If the function $H(\theta)$ is defined by (B.3), we obtain

$$J_0(x,y) = \pi b \sum_{m=0}^{\infty} a_m I_0(m,e), \tag{B.13}$$

where $I_0(m,e)$ is defined in Eq. (B.31). Notice that the sine terms in (B.3) are antisymmetric about $\theta = \pi/2$ and hence make no contribution to the integral.

B.1.2 The Hertzian Field $n = 1$

If $n = 1$,

$$J_1(x,y) = \frac{\pi}{2} \int_0^{\pi} H(\theta) D^2 \sqrt{C_2}\, d\theta \tag{B.14}$$

and substituting for C_2, D from Eqs. (B.6), (B.9), (B.5), we obtain

$$J_1(x,y) = \frac{\pi b}{2} \left[\int_0^{\pi} \frac{H(\theta) d\theta}{\sqrt{1 - e^2\cos^2\theta}} - \frac{x^2}{a^2} \int_0^{\pi} \frac{H(\theta)\sin^2\theta d\theta}{(1 - e^2\cos^2\theta)^{3/2}} - \frac{y^2}{a^2} \int_0^{\pi} \frac{H(\theta)\cos^2\theta d\theta}{(1 - e^2\cos^2\theta)^{3/2}} + \frac{2xy}{a^2} \int_0^{\pi} \frac{H(\theta)\sin\theta\cos\theta d\theta}{(1 - e^2\cos^2\theta)^{3/2}} \right]. \tag{B.15}$$

If $H(\theta)$ is defined by (B.3), we have

$$J_1(x,y) = \frac{\pi b}{2}\left[\sum_{m=0}^{\infty} a_m \left\{ I_0(m,e) - \frac{x^2}{a^2} I_1(m,e) - \frac{y^2}{a^2} I_2(m,e) \right\} + \frac{xy}{a^2}\sum_{m=1}^{\infty} b_m I_3(m,e)\right], \tag{B.16}$$

where $I_1(m,e)$, $I_2(m,e)$, $I_3(m,e)$ are defined in (B.31, B.32).

B.2 Applications

We now apply these results to several traction distributions arising in three-dimensional contact problems.

B.2.1 Normal Loading of an Isotropic Half-Space

We showed in Sect. 2.3 and Eq. (2.17) that the normal surface displacement $u_z(x,y)$ of an isotropic half-space due to a contact pressure distribution $p(x,y)$ is

$$u_z(x,y) = \frac{1}{\pi E^*}\iint_{\mathcal{A}} \frac{p(\xi,\eta)d\xi d\eta}{r}. \tag{B.17}$$

It follows that the displacement due to the distribution

$$p(x,y) = p_0\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{n-1/2} \tag{B.18}$$

is given by (B.2) with $H(\theta) = p_0/\pi E^*$. Thus, $a_0 = p_0/\pi E^*$ and the remaining coefficients in (B.3) are zero.

For the flat punch ($n=0$), Eq. (B.13) then gives

$$u_z(x,y) = \frac{p_0 b I_0(0,e)}{E^*} = \frac{2p_0 b K(e)}{E^*} \qquad (x,y) \in \mathcal{A}. \tag{B.19}$$

For the Hertzian pressure distribution

$$p(x,y) = p_0\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{1/2}, \tag{B.20}$$

$n = 1$ and (B.16) yields

$$u_z(x, y) = \frac{p_0 b}{2E^*}\left[I_0(0, e) - \frac{x^2}{a^2} I_1(0, e) - \frac{y^2}{a^2} I_2(0, e)\right] \quad (x, y) \in \mathcal{A}. \tag{B.21}$$

B.2.2 The Anisotropic Half-Space

If the elastic material is anisotropic, the Green's function will generally depend on θ as defined in Eq. (2.9) and hence the normal surface displacements due to the pressure distribution $p(x, y)$ are

$$u_z(x, y) = \iint_{\mathcal{A}} \frac{h(\theta) p(\xi, \eta) d\xi d\eta}{r}, \tag{B.22}$$

where $h(\theta)$ has the form of Eq. (2.11). It follows that the displacement due to the singular pressure distribution

$$p(x, y) = p_0 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{-1/2} \tag{B.23}$$

is

$$u_z(x, y) = \pi p_0 b \sum_{m=0}^{\infty} A_m I_0(m, e), \tag{B.24}$$

where the constants A_m are defined in (2.11). Also, the displacements due to the Hertzian distribution (B.20) are

$$u_z(x, y) = \frac{\pi p_0 b}{2}\left[\sum_{m=0}^{\infty} A_m \left\{I_0(m, e) - \frac{x^2}{a^2} I_1(m, e) - \frac{y^2}{a^2} I_2(m, e)\right\} + \frac{xy}{a^2} \sum_{m=1}^{\infty} B_m I_3(m, e)\right]. \tag{B.25}$$

Notice that if the coefficients $B_m \neq 0$, the axes of the ellipse will be inclined to the principal axes of the initial gap function $g_0(x, y)$.

B.2.3 Tangential Loading of an Isotropic Half-Space

If tangential tractions

$$q_x(x,y) = q_1\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^{-1/2}; \quad q_y(x,y) = q_2\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^{-1/2} \tag{B.26}$$

are applied to the surface of an isotropic half-space, the tangential surface displacements are given by Eqs. (7.59), (7.60) and (B.12) as

$$u_x = \frac{(1+\nu)(2-\nu)q_1 b}{2E} I_0(0,e) + \frac{\nu(1+\nu)q_1 b}{2E} I_0(1,e) \tag{B.27}$$

$$u_y = \frac{(1+\nu)(2-\nu)q_2 b}{2E} I_0(0,e) - \frac{\nu(1+\nu)q_2 b}{2E} I_0(1,e), \tag{B.28}$$

and are independent of x, y.

Alternatively, if

$$q_x(x,y) = q_1\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^{1/2}; \quad q_y(x,y) = q_2\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^{1/2} \tag{B.29}$$

we obtain

$$u_x(x,y) = L_0 - L_1 x^2 - L_2 y^2 + L_3 xy; \quad u_y(x,y) = M_0 - M_1 x^2 - M_2 y^2 + M_3 xy, \tag{B.30}$$

where

$$\begin{aligned}
L_0 &= \frac{(1+\nu)q_1 b}{4E}\left[(2-\nu)I_0(0,e) + \nu I_0(1,e)\right]\\
L_1 &= \frac{(1+\nu)q_1 b}{4Ea^2}\left[(2-\nu)I_1(0,e) + \nu I_1(1,e)\right]\\
L_2 &= \frac{(1+\nu)q_1 b}{4Ea^2}\left[(2-\nu)I_2(0,e) + \nu I_2(1,e)\right]\\
L_3 &= \frac{\nu(1+\nu)q_2 b}{4Ea^2} I_3(1,e)\\
M_0 &= \frac{(1+\nu)q_2 b}{4E}\left[(2-\nu)I_0(0,e) - \nu I_0(1,e)\right]\\
M_1 &= \frac{(1+\nu)q_2 b}{4Ea^2}\left[(2-\nu)I_1(0,e) - \nu I_1(1,e)\right]\\
M_2 &= \frac{(1+\nu)q_2 b}{4Ea^2}\left[(2-\nu)I_2(0,e) - \nu I_2(1,e)\right]\\
M_3 &= \frac{\nu(1+\nu)q_1 b}{4Ea^2} I_3(1,e).
\end{aligned}$$

B.3 Evaluation of Integrals

Following Barber and Ciavarella (2014), we define the integrals

$$I_0(m,e)=\int_0^\pi \frac{\cos(2m\theta)d\theta}{\sqrt{1-e^2\cos^2\theta}};\quad I_1(m,e)=\int_0^\pi \frac{\sin^2\theta\cos(2m\theta)d\theta}{(1-e^2\cos^2\theta)^{3/2}} \tag{B.31}$$

$$I_2(m,e)=\int_0^\pi \frac{\cos^2\theta\cos(2m\theta)d\theta}{(1-e^2\cos^2\theta)^{3/2}};\quad I_3(m,e)=\int_0^\pi \frac{\sin(2\theta)\sin(2m\theta)d\theta}{(1-e^2\cos^2\theta)^{3/2}}. \tag{B.32}$$

The integral $I_0(m,e)$ can be performed in Maple or Mathematica for any given m, the first few being

$$\begin{aligned}
I_0(0,e)&=2K(e)\\
I_0(1,e)&=\frac{4\,[K(e)-E(e)]}{e^2}-2K(e)\\
I_0(2,e)&=\frac{32\,[K(e)-E(e)]}{3e^4}+\frac{16\,[E(e)-2K(e)]}{3e^2}+2K(e)\\
I_0(3,e)&=\frac{512\,[K(e)-E(e)]}{15e^6}-\frac{256\,[3K(e)-2E(e)]}{15e^4}\\
&\qquad+\frac{4\,[79K(e)-23E(e)]}{15e^2}-2K(e).
\end{aligned} \tag{B.33}$$

Higher order terms can also be obtained from the recurrence relation

$$I_0(m+1,e)=\frac{4m}{(2m+1)}\left(\frac{2}{e^2}-1\right)I_0(m,e)-\left(\frac{2m-1}{2m+1}\right)I_0(m-1,e). \tag{B.34}$$

The remaining integrals can then be found by elementary algebraic operations as

$$\begin{aligned}
I_1(m,e)&=I_0(m,e)-\frac{(1-e^2)}{e}\frac{d}{de}I_0(m,e)=I_0(m,e)-(1-e^2)I_2(m,e)\\
I_2(m,e)&=\frac{1}{e}\frac{d}{de}I_0(m,e)\\
I_3(m,e)&=\frac{1}{2}\left[I_1(m-1,e)+I_2(m-1,e)-I_1(m+1,e)-I_2(m+1,e)\right].
\end{aligned} \tag{B.35}$$

Appendix C
Cauchy Singular Integral Equations

The Green's function for two-dimensional elastic contact problems generates singular integral equations with Cauchy kernels. Here, we shall collect results for the various forms of these equations and their general solutions. These results are all presented in the normalized form, where the range of integration is $-1 < t < 1$. Equations involving dimensional contact boundaries such as $b < x < a$ can be normalized by making the linear coordinate transformation

$$x = \frac{a+b}{2} + \frac{(a-b)t}{2}. \tag{C.1}$$

C.1 Integral Equations of the First Kind

The normalized form of the Cauchy singular integral equation of the first kind is

$$\frac{1}{\pi}\int_{-1}^{1} \frac{F(t)dt}{(s-t)} = f(s) \qquad -1 < s < 1, \tag{C.2}$$

where $f(s)$ is a known function. The form of the solution depends on the asymptotic behaviour of the function $F(t)$ near the end points, as discussed in Chap. 10. For example, for a frictionless normal contact problem, the contact pressure will be square-root singular at the end point if the indenting body has a sharp corner, but will be square-root bounded if the indenter is smooth and the contact boundary is determined by the Signorini inequalities.

If $F(t)$ is singular at $t = \pm 1$, the solution of Eq. (C.2) is

$$F(t) = \frac{w(t)}{\pi}\left[P - \int_{-1}^{1} \frac{f(s)ds}{w(s)(t-s)}\right] \qquad -1 < t < -1, \tag{C.3}$$

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where

$$w(t) = \frac{1}{\sqrt{1-t^2}} \tag{C.4}$$

is the characteristic function, and

$$P = \int_{-1}^{1} F(t)dt \tag{C.5}$$

can take any value. Notice that the term involving P remains, even if $f(s) = 0$, so it also defines the general solution of the corresponding homogeneous integral equation. In most of the applications considered in this book, $F(t)$ will represent a traction distribution, so P will represent the corresponding normalized total force.

For all other cases, the solution of (C.2) can be written

$$F(t) = -\frac{w(t)}{\pi} \int_{-1}^{1} \frac{f(s)ds}{w(s)(t-s)} \qquad -1 < t < -1, \tag{C.6}$$

where

- $F(t)$ singular at $t=-1$ and bounded at $t=1$

$$w(t) = \sqrt{\frac{1-t}{1+t}}; \quad P = \int_{-1}^{1} \sqrt{\frac{1+s}{1-s}}\, f(s)ds. \tag{C.7}$$

- $F(t)$ bounded at $t=-1$ and singular at $t=1$

$$w(t) = \sqrt{\frac{1+t}{1-t}}; \quad P = -\int_{-1}^{1} \sqrt{\frac{1-s}{1+s}}\, f(s)ds. \tag{C.8}$$

- $F(t)$ bounded at $t=\pm 1$

$$w(t) = \sqrt{1-t^2}; \quad P = \int_{-1}^{1} \frac{sf(s)ds}{\sqrt{1-s^2}}, \tag{C.9}$$

  and in this case, we must also satisfy the consistency condition

$$\int_{-1}^{1} \frac{f(s)ds}{\sqrt{1-s^2}} = 0, \tag{C.10}$$

  if a solution is to be possible.

C.2 Integral Equations of the Second Kind

The Cauchy singular integral equation of the second kind is

$$F(s) + \frac{\lambda}{\pi} \int_{-1}^{1} \frac{F(t)dt}{(s-t)} = f(s) \qquad -1 < s < 1. \tag{C.11}$$

We define a parameter γ such that if λ is real,

$$\cot(\pi\gamma) = \lambda \qquad -\frac{1}{2} < \gamma < \frac{1}{2}, \tag{C.12}$$

and if it is complex,

$$\gamma = \frac{1}{2\pi\imath} \ln\left(\frac{\lambda+\imath}{\lambda-\imath}\right) \qquad -\frac{1}{2} < \Re(\gamma) < \frac{1}{2}. \tag{C.13}$$

If $F(t)$ is singular at $t = \pm 1$, the solution is

$$F(t) = \frac{f(t)}{(1+\lambda^2)} + \frac{w(t)}{\pi}\left[P\cos(\pi\gamma) - \frac{\lambda}{(1+\lambda^2)} \int_{-1}^{1} \frac{f(s)ds}{w(s)(t-s)}\right], \tag{C.14}$$

where

$$w(t) = \frac{1}{(1-t)^{1/2+\gamma}(1+t)^{1/2-\gamma}} \tag{C.15}$$

and

$$P = \int_{-1}^{1} F(t)dt \tag{C.16}$$

can take any value.

For all other cases

$$F(t) = \frac{f(t)}{(1+\lambda^2)} - \frac{\lambda w(t)}{\pi(1+\lambda^2)} \int_{-1}^{1} \frac{f(s)ds}{w(s)(t-s)}, \tag{C.17}$$

and $w(t)$, P are defined as follows:

- $F(t)$ singular at $t = -1$ and bounded at $t = 1$

$$w(t) = \left(\frac{1-t}{1+t}\right)^{1/2-\gamma}; \quad P = \sin(\pi\gamma) \int_{-1}^{1} \left(\frac{1+s}{1-s}\right)^{1/2-\gamma} f(s)ds \tag{C.18}$$

- $F(t)$ bounded at $t = -1$ and singular at $t = 1$

$$w(t) = \left(\frac{1+t}{1-t}\right)^{1/2+\gamma} \quad ; \quad P = -\sin(\pi\gamma)\int_{-1}^{1}\left(\frac{1-s}{1+s}\right)^{1/2+\gamma} f(s)ds \qquad \text{(C.19)}$$

- $F(t)$ bounded at $t = \pm 1$

$$\begin{aligned} w(t) &= (1-t)^{1/2-\gamma}(1+t)^{1/2+\gamma} \\ P &= \sin(\pi\gamma)\int_{-1}^{1}\frac{sf(s)ds}{(1-s)^{1/2-\gamma}(1+s)^{1/2+\gamma}} \end{aligned} \qquad \text{(C.20)}$$

and we must also satisfy the consistency condition

$$\int_{-1}^{1}\frac{f(s)ds}{(1-s)^{1/2-\gamma}(1+s)^{1/2+\gamma}} = 0. \qquad \text{(C.21)}$$

Appendix D
Dundurs' Bimaterial Constants

If an isotropic elastic body is subjected to prescribed surface tractions, it follows from dimensional analysis that the resulting stress field is independent of Young's modulus, since the elasticity problem can be formulated in terms of normalized stress components of the form σ/E. However, if the body is simply connected and the geometry and loading are two-dimensional, it also follows that the stresses are independent of Poisson's ratio ν, since the terms containing ν cancel when Hooke's law is used to express the compatibility equation in terms of stresses (Barber 2010). This result can also be extended to multiply connected two-dimensional bodies subject to the restriction that the resultant of the tractions acting on each separate hole of the body be zero.

A similar reduction in dependence on material properties applies to two-dimensional problems involving two different isotropic elastic materials with properties E_1, ν_1 and E_2, ν_2 respectively. Dimensional analysis again shows that the resulting stress field can at most depend on the three dimensionless parameters $E_1/E_2, \nu_1, \nu_2$, but Dundurs (1969) has shown that the dependence can be further reduced to the two independent bimaterial parameters

$$\alpha = \left(\frac{\kappa_1 + 1}{G_1} - \frac{\kappa_2 + 1}{G_2}\right) \bigg/ \left(\frac{\kappa_1 + 1}{G_1} + \frac{\kappa_2 + 1}{G_2}\right) \tag{D.1}$$

$$\beta = \left(\frac{\kappa_1 - 1}{G_1} - \frac{\kappa_2 - 1}{G_2}\right) \bigg/ \left(\frac{\kappa_1 + 1}{G_1} + \frac{\kappa_2 + 1}{G_2}\right) \tag{D.2}$$

where G is the modulus of rigidity (shear modulus) and $\kappa = 3 - 4\nu$ for plane strain

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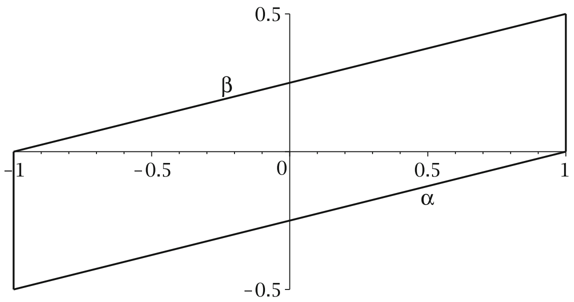

Fig. D.1 Range of values of α, β if ν is restricted to the range $0 \leq \nu \leq 0.5$ for both materials

and $\kappa = (3-\nu)/(1-\nu)$ for plane stress. In the plane strain case, these expressions can be written in the terms of E_1, E_2, ν_1, ν_2 as

$$\alpha = E^* \left(\frac{1-\nu_1^2}{E_1} - \frac{1-\nu_2^2}{E_2} \right) \tag{D.3}$$

$$\beta = E^* \left\{ \frac{(1-2\nu_1)(1+\nu_1)}{2E_1} - \frac{(1-2\nu_2)(1+\nu_2)}{2E_2} \right\} \tag{D.4}$$

where the composite modulus E^* is defined by

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}. \tag{D.5}$$

If Poisson's ratio is restricted to the range $0 \leq \nu_1, \nu_2 \leq 0.5$, the constants α, β must lie within the parallelogram shown in Fig. D.1.

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