Appendix A

A.1 Solutions for the infinite domains

Asymptotic technique based on a small parameter characterizing certain dimensions of the domain is one of the most frequently used approaches in mechanics. As a rule, application of this technique implies that the sufficiently large domain is substituted by an infinite or semi-infinite region, which substantially simplifies the solution of the problem at hand. However, the following difficulty occurs in this approach; namely, the uniqueness of the solution may disappear if the finiteness of the integral of energy cannot be ensured, see [1]. Therefore, the problem of selection of the required solution arises. In such situation, the procedure of the regularization of the solution can be used. This approach is described below by an example of bending of a circular plate with a central circular hole. The plate is subject to the uniformly distributed load, and the boundary of the hole is free of load.

After obtaining the particular solution of the non-homogeneous equation, with the nonzero right side, it is possible to turn to the boundary value problem for the homogeneous equation with non-homogeneous boundary conditions. If the radius R of the hole is small in comparison with the dimensions of the plate, then in the first approximation, it is possible to consider a problem of bending of the infinite plane with a circular hole instead of the original problem for a plate. This problem can be formulated in the polar coordinate system as follows:

$$\Delta_{\rho}^{2} w = 0, \tag{A.1}$$

$$\begin{aligned} L_1[w] \equiv\ & w_{,\rho\rho\rho} + \rho^{-2} w_{,\rho\varphi\varphi} + \rho^{-1} w_{,\rho\rho} - 2\rho^{-3} w_{,\varphi\varphi} - \rho^{-2} w_{,\rho} \\ & + (1-\nu)\left(\rho^{-2} w_{,\rho\varphi\varphi} - \rho^{-3} w_{,\varphi\varphi}\right) = F_1(\varphi) \quad \text{on} \quad \rho = R, \end{aligned} \tag{A.2}$$

$$L_2[w] \equiv w_{,\rho\rho} + \nu\left(\rho^{-2} w_{,\varphi\varphi} - \rho^{-1} w_{,\rho}\right) = F_2(\varphi) \quad on \quad \rho = R. \tag{A.3}$$

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Let us represent the right sides of boundary conditions (A.2) and (A.3) in the form of Fourier series

$$\begin{aligned} F_1(\varphi) &= f_0 + \sum_{n=1}^{\infty}(f_n \cos n\varphi + f_n' \sin n\varphi), \\ F_2(\varphi) &= \psi_0 + \sum_{n=1}^{\infty}(\psi_n \cos n\varphi + \psi_n' \sin n\varphi). \end{aligned} \tag{A.4}$$

Then, the solution of boundary value problem (A.1)–(A.3) can be obtained as the sum $w = w_0 + w_1$ of the solutions of the following two boundary value problems:

$$\Delta_\rho^2 w_0 = 0, \tag{A.5}$$

$$L_1\,[w_0] = f_0 \quad \text{on} \quad \rho = R, \tag{A.6}$$

$$L_2\,[w_0] = \psi_0 \quad \text{on} \quad \rho = R, \tag{A.7}$$

$$\Delta_\rho^2 w_1 = 0, \tag{A.8}$$

$$L_1\,[w_1] = \sum_{n=1} \left(f_n \cos n\varphi + f'_n \sin n\varphi\right) \quad \text{on} \quad \rho = R, \tag{A.9}$$

$$L_2\,[w_1] = \sum_{n=1} \left(\psi_n \cos n\varphi + \psi'_n \sin n\varphi\right) \quad \text{on} \quad \rho = R. \tag{A.10}$$

Solution of boundary value problem (A.5)–(A.7) is not unique. In order to select the required unique solution, a fictitious elastic foundation with the coefficient k is introduced and the following problem of symmetrical bending of a plane with the circular hole on this foundation is considered:

$$D\Delta_1^2 w_0 = -k w_1, \tag{A.11}$$

$$L_1\,[w_0] = f_0 \quad \text{on} \quad \rho = R, \tag{A.12}$$

$$L_2\,[w_0] = \psi_0 \quad \text{on} \quad \rho = R. \tag{A.13}$$

Solution of the problem (A.11)–(A.13) can be written as follows [2]:

$$w_0 = C_1 Ber Z + C_2 Bei Z + C_3 Ker Z + C_4 Kei Z, \tag{A.14}$$

where $Z = \rho\sqrt{k/D}$, and $Ber Z,\ \ Bei Z,\ \ Ker Z,\ \ Kei Z$ are the Kelvin functions.

Functions $Ber Z$ and $Bei Z$ grow unlimitedly with an increase of argument [3]. Therefore, taking into account decaying condition at infinity, we obtain $C_1 = C_2 = 0$, and therefore, Eq. (A.14) yields

$$w_0 = C_3 Ker Z + C_4 Kei Z. \tag{A.15}$$

Using asymptotic expressions for the functions $Ker Z$ and $Kei Z$ for the small values of argument Z [3], and retaining only first terms for $k \to 0$, we obtain the following solution of boundary value problem (A.5)–(A.7):

$$w_0 = C_{11} \ln \rho + C_{22} \rho^2 \ln \rho, \quad \text{where} \quad C_{11}, C_{22} = const. \tag{A.16}$$

The solution of boundary value problem (A.8)–(A.10) can be obtained easily. Note that solution of the problem of bending of plane with the hole leaves the discrepancy in the boundary conditions on the outer boundary of the plate, which can be compensated by the solution of boundary value problem for the original domain without hole.

A.2 Bending plate. Formulae for the coefficients of the Eqs. 7.68, 7.71, 7.72, 7.73

Coefficients of Eqs. 7.68 and 7.72

$$\begin{aligned}
&C_{200} = \left(w_{0,xx} + w_{0,yy}\right) \overline{C}_{200}, \quad B_{202} = \left(w_{0,xx} - w_{0,yy}\right) \overline{B}_{202},\\
&D_{202} = \left(w_{0,xx} - w_{0,yy}\right) \overline{D}_{202}, \quad B'_{214} = 0, \quad B'_{202} = w_{0,xy} \overline{B}'_{202},\\
&D'_{202} = w_{0,xy} \overline{D}'_{202}, \quad B_{212} = \left(w_{0,xx} - w_{0,yy}\right) \overline{B}_{212},\\
&D_{212} = \left(w_{0,xx} - w_{0,yy}\right) \overline{D}_{212}, \quad B'_{212} = w_{0,xy} \overline{B}'_{212},\\
&B_{214} = \left(w_{0,xx} + w_{0,yy}\right) \overline{B}_{214}, \quad D'_{212} = w_{0,xy} \overline{D}'_{212},\\
&D_{214} = \left(w_{0,xx} + w_{0,yy}\right) \overline{D}_{214}, \quad D'_{214} = 0,\\
&B_{216} = \left(w_{0,xx} - w_{0,yy}\right) \overline{B}_{216}, \quad D_{216} = \left(w_{0,xx} - w_{0,yy}\right) \overline{D}_{216},\\
&B'_{216} = w_{0,xy} \overline{B}'_{216}, \quad D'_{216} = w_{0,xy} \overline{D}'_{216}.
\end{aligned} \tag{A.17}$$

Coefficients of Eq. (A.17)

$$\begin{aligned}
&\overline{C}_{200} = (1+\nu)\,[2(1-\nu)]^{-1}R^2, \quad \overline{B}_{202} = (3-\nu)\,[12(3+\nu)]^{-1}R^4,\\
&\overline{D}_{202} = (1-\nu)\,[4(3+\nu)]^{-1}R^2\\
&\overline{D}_{212} = -\,(6+17\nu)\,(3+\nu)^{-1}\overline{B}_{202}\\
&+\left(5\nu^2-3\nu-4\right)[(1-\nu)(3+\nu)]^{-1}R^2\overline{D}_{202} + (3-\nu)[3(3+\nu)]^{-1}R^4,\\
&\overline{B}_{212} = -45\,(1-\nu)\left[2(3+\nu)R^2\right]^{-1}\overline{B}_{202}\\
&-(11--13\nu)\,[2(3+\nu)]^{-1}\overline{D}_{202} - (1-\nu)(3+\nu)^{-1}R^2,\\
&\overline{B}'_{202} = -2\overline{B}_{212}, \quad \overline{B}'_{212} = -2\overline{B}_{212}, \quad \overline{D}'_{202} = -2\overline{D}_{212},\\
&\overline{B}_{214} = (2-\nu)\,[5(1--13\nu)]^{-1}R^4\overline{C}_{200}, \quad \overline{B}'_{216} = 2\overline{B}_{216},\\
&\overline{D}_{214} = (1-\nu)\,[3(1--13\nu)]^{-1}R^2\overline{C}_{200},\\
&\overline{D}'_{212} = -2\overline{D}_{212}, \quad \overline{D}'_{216} = 2\overline{D}_{216},\\
&\overline{B}_{216} = (61+4\nu)\,[21(3+\nu)]^{-1}R^4\overline{B}_{202}\\
&-\left(10\nu^2+17\nu-53\right)[63(1-\nu)(3+\nu)]^{-1}R^6\overline{D}_{202}\\
&-(5-3\nu)\,[21(3+\nu)]^{-1}R^8,\\
&\overline{D}_{216} = -7\,(1-\nu)\,[10(3+\nu)]^{-1}R^2\overline{B}_{202}\\
&+S(1+\nu)[30(3+\nu)]^{-1}R^4\overline{D}_{202} - (1-\nu)\,[5(3+\nu)]^{-1}R^6.
\end{aligned} \tag{A.18}$$

Coefficients of Eq. 7.71 and 7.72, see also Eq. A.22

$$\begin{aligned}
&A_n^{(21)} = w_{0xy}\overline{A}_n^{(21)}, \quad B_n^{(21)} = w_{0,xy}\overline{B}_n^{(21)}, \quad C_n^{(21)} = w_{0,xy}\overline{C}_n^{(21)},\\
&D_n^{(21)} = w_{0,xy}\overline{D}_n^{(21)}, \quad A_n^{(22)} = w_{0,xy}\overline{A}_n^{(22)}, \quad B_n^{(22)} = w_{0,xy}\overline{B}_n^{(22)},\\
&C_n^{(22)} = w_{0,xy}\overline{C}_n^{(22)}, \quad D_n^{(22)} = w_{0,xy}\overline{D}_n^{(22)},\\
&A_n^{(31)} = (w_{0,xxx}+w_{0,xyy})\overline{A}_n^{(31)}, \quad B_n^{(31)} = (w_{0,yyy}+w_{0,yxx})\overline{B}_n^{(31)},\\
&C_n^{(31)} = (w_{0,yyy}+w_{0,yxx})\overline{C}_n^{(31)}, \quad D_n^{(31)} = (w_{0,xxx}+w_{0,xyy})\overline{D}_n^{(31)},\\
&A_n^{(32)} = (w_{0,xxx}+w_{0,xyy})\overline{A}_n^{(32)}, \quad B_n^{(32)} = (w_{0,yyy}+w_{0,yxx})\overline{B}_n^{(32)},\\
&C_n^{(32)} = (w_{0,yyy}+w_{0,yxx})\overline{C}_n^{(32)}, \quad D_n^{(32)} = (w_{0,xxx}+w_{0,xyy})\overline{D}_n^{(32)}.
\end{aligned} \tag{A.19}$$

Coefficients of Eq. A.19, see also Eq. A.24

$$
\begin{aligned}
\overline{A}_n^{(21)} &= \left[(2\sinh n\pi + n\pi \cosh n\pi)\, K_n^{(11)} - \frac{a^2}{n\pi} \cosh n\pi\, K_n^{(13)} \right] \left(4\sinh^2 n\pi\right)^{-1}, \\
\overline{B}_n^{(21)} &= \left[\left(\frac{3a}{n\pi} \sinh n\pi + a\cosh n\pi \right) K_n^{(12)} \right. \\
&\left. - \left(\frac{a}{n\pi} \sinh n\pi + a\cosh n\pi \right) a^2 K_n^{(14)} \right] \left(4\sinh^2 n\pi\right)^{-1}, \\
\overline{C}_n^{(21)} &= \left[a^2 K_n^{(14)} - K_n^{(12)} \right] \left(4\sinh^2 n\pi\right)^{-1}, \\
\overline{D}_n^{(21)} &= \left(\frac{a}{n\pi} K_n^{(13)} - \frac{n\pi}{a} K_n^{(11)} \right) (4\sinh n\pi)^{-1}, \\
\bar{A}_n^{(22)} &= \overline{A}_n^{(21)}, \quad \bar{B}_n^{(22)} = \overline{B}_n^{(21)}, \quad \bar{C}_n^{(22)} = \overline{C}_n^{(21)}, \quad \bar{D}_n^{(22)} = \overline{D}_n^{(21)}.
\end{aligned}
\tag{A.20}
$$

Coefficients of Eq. (A.20)

$$
\begin{aligned}
K_n^{(11)} &= -4\overline{\overline{D}} \int_{-a}^{a} \xi (a^2 + \xi^2)^{-1} \cos \frac{n\pi\xi}{a} d\xi, \\
K_n^{(12)} &= 4\overline{\overline{D}} a^{-1} \int_{-a}^{a} \xi (a^2 - \xi^2)(a^2 + \xi^2)^{-2} \cos \frac{n\pi\xi}{a} d\xi, \\
K_n^{(13)} &= -8\overline{\overline{D}} \int_{-a}^{a} \xi (a^2 - 3\xi^2)(a^2 + \xi^2)^{-3} \cos \frac{n\pi\xi}{a} d\xi, \\
K_n^{(14)} &= 24\overline{\overline{D}} a^{-1} \int_{-a}^{a} \xi (a^2 - 6a^2\xi^2 + \xi^4)(a^2 + \xi^2)^{-4} \cos \frac{n\pi\xi}{a} d\xi, \\
\overline{\overline{D}} &= \overline{D}'_{202} + \varepsilon_1 \overline{D}'_{212}.
\end{aligned}
\tag{A.21}
$$

Remaining coefficients of Eq. 7.72

$$
\begin{aligned}
C_{301} &= \left(w_{0,xxx} + w_{0,xyy}\right) \overline{C}_{301}, \quad D_{301} = \left(w_{0,xxx} + w_{0,xyy}\right) \overline{D}_{301}, \\
C'_{301} &= \left(w_{0,yyy} + w_{0,yxx}\right) \overline{C}'_{301}, \quad D'_{301} = \left(w_{0,yyy} + w_{0,yxx}\right) \overline{D}'_{301}, \\
B_{303} &= \left(w_{0,xxx} - 3w_{0,xyy}\right) \overline{B}_{303}, \quad D_{303} = \left(w_{0,xxx} - 3w_{0,xyy}\right) \overline{D}_{303}, \\
B'_{303} &= \left(w_{0,yyy} - 3w_{0,yxx}\right) \overline{B}'_{303}, \quad D'_{303} = \left(w_{0,yyy} - 3w_{0,yxx}\right) \overline{D}'_{303}, \\
C_{311} &= \left(w_{0,xxx} - 3w_{0,xyy}\right) \overline{C}_{311}, \quad D_{311} = \left(w_{0,xxx} - 3w_{0,xyy}\right) \overline{D}_{311}, \\
C'_{311} &= \left(w_{0,yyy} - 3w_{0,yxx}\right) \overline{C}'_{311}, \quad D'_{311} = \left(w_{0,yyy} - 3w_{0,yxx}\right) \overline{D}'_{311}, \\
B_{313} &= \left(w_{0,xxx} + w_{0,xyy}\right) \overline{B}_{313}, \quad D_{313} = \left(w_{0,xxx} + w_{0,xyy}\right) \overline{D}_{313}, \\
B'_{313} &= \left(w_{0,xxx} + w_{0,xyy}\right) \overline{B}'_{313}, \quad D'_{313} = \left(w_{0,yyy} + w_{0,yxx}\right) \overline{D}'_{313}.
\end{aligned}
\tag{A.22}
$$

Coefficients of Eq. (A.22)

$$
\begin{aligned}
&\overline{C}_{301} = (\nu^2 + 4\nu - 5)[8(1-\nu)]^{-1} R^2 \overline{C}_{200} \\
&+ (1+\nu)(6-\nu)[4(1-\nu)]^{-1} R^4 \ln R \overline{D}_{200} \\
&+ (13\nu^2 - 32\nu - 109)[24(1-\nu)]^{-1} R^4 \overline{D}_{200} \\
&+ (15 + 7\nu)8^{-1} \overline{B}_{202} - (\nu^2 + \nu - 4)[4(1-\nu)]^{-1} R^2 \overline{D}_{202} \\
&- (5-\nu)(1+\nu)[32(1-\nu)]^{-1} R^4, \\
&\overline{D}_{301} = -(3+\nu)4^{-1} \overline{C}_{200} - (2-\nu)2^{-1} R^2 \ln R \overline{D}_{200} \\
&+ 13(1-\nu)12^{-1} R^2 \overline{D}_{200} - 7(1-\nu)(4R^2)^{-1} \overline{B}_{202} \\
&- (4-\nu)2^{-1} \overline{D}_{202} + (5-\nu)16^{-1} R^2, \\
&\overline{B}_{303} = -(\nu^2 + 18\nu - 19)[24(1-\nu)(3+\nu)]^{-1} R^2 \overline{B}_{202} \\
&- (\nu^2 - \nu + 4)[36(1-\nu)(3+\nu)]^{-1} R^4 \overline{D}_{202} - (1 - -5\nu)[288(3+\nu)]^{-1} R^6 \\
&\overline{D}_{303} = -(7+\nu)[4(3+\nu)]^{-1} \overline{B}_{202} \\
&+ (4+\nu)[6(3+\nu)]^{-1} R^2 \overline{D}_{202} + (1-\nu)[48(3+\nu)]^{-1} R^4, \\
&\overline{C}_{311} = -9(65 + 49\nu)(16R^2)^{-1} \overline{B}_{303} \\
&+ 3(71\nu^2 + 32\nu - 129)[16(1-\nu)]^{-1} \overline{D}_{303} \\
&- 3(1+\nu)(3-\nu)[8(1-\nu)]^{-1} \overline{B}_{202} - (13\nu^2 - \nu - 20)[8(1-\nu)]^{-1} R^2 \overline{D}_{202} \\
&+ (15 + 7\nu)8^{-1} \overline{B}_{212} - (\nu^2 + \nu - 4)[4(1-\nu)]^{-1} R^2 \overline{D}_{212} - 3(1+\nu)16^{-1} R^4, \\
&\overline{D}_{311} = 441(1-\nu)(8R^4)^{-1} \overline{B}_{303} + 3(63\nu^2 - 71\nu)(8R^2)^{-1} \overline{D}_{303} \\
&- 0.75(1+\nu) R^{-2} \overline{B}_{202} - 0.25(20 - 13\nu) \overline{D}_{202} \\
&- (1-\nu) R^{-2} \overline{B}_{212} - 0.5(4-\nu) \overline{D}_{212} + 0.375(1-\nu) R^2, \\
&\overline{B}_{313} = -(49 + 75\nu)[48(3+\nu)]^{-1} R^2{}_{301} - \left[2(11\nu^2 + 53\nu - 84) - (1-\nu)(1 - -5\nu) \ln R\right] \\
&\times [144(1-\nu)(3+\nu)]^{-1} R^4 \overline{D}_{301} + (19\nu^2 - 53\nu + 92)[72(1-\nu)(3+\nu)]^{-1} R^4 \overline{C}_{200} \\
&+ \left[590\nu^2 + 1835\nu + 819 - 18(3\nu^2 - 12\nu + 1) \ln R\right] [48(1-\nu)(3+\nu)]^{-1} R^6 \overline{D}_{200} \\
&+ 10(7 - 3\nu)[48(3+\nu)]^{-1} R^2 \overline{B}_{202} - (53\nu^2 - 17\nu - 64)[72(1-\nu)(3+\nu)]^{-1} R^4 \overline{D}_{202} \\
&- (\nu^2 + 18\nu - 19)[24(1-\nu)(3+\nu)]^{-1} R^2 \overline{B}_{212} - (\nu^2 - \nu + 4)[36(1-\nu)(3+\nu)]^{-1} \\
&\times R^4 \overline{D}_{212} + (9 + 19\nu)[12(3+\nu)]^{-1} \overline{B}_{214} - (23\nu^2 - 28\nu - 3)[72(1-\nu)(3+\nu)]^{-1} \\
&\times R^2 \overline{D}_{214} - (5-\nu)(1 - -5\nu)[144(1-\nu)(3+\nu)]^{-1} R^6, \\
&\overline{D}_{313} = -33(1-\nu)[24(3+\nu)]^{-1}{}_{301} - [6 - 4\nu + (1-\nu) \ln R] [24(3+\nu)]^{-1} R^2 \overline{D}_{301} \\
&- (2-\nu)[12(3+\nu)]^{-1} R^2 \overline{C}_{200} + \left[639\nu^2 + 226 + 18(1-\nu) \ln R\right] [2(3+\nu)]^{-1} R^4 \overline{D}_{200} \\
&- 5(1-\nu)[4(3+\nu)]^{-1} \overline{B}_{202} + (8 - 7\nu)[12(3+\nu)]^{-1} R^2 \overline{D}_{202} \\
&- (7+\nu)[4(3+\nu)]^{-1} \overline{B}_{212} + (4+\nu)[6(3+\nu)]^{-1} R^2 \overline{D}_{212} \\
&+ 3(1-\nu)\left[2(3+\nu)R^2\right]^{-1} \overline{B}_{214} + 3(3-\nu)[4(3+\nu)]^{-1} \overline{D}_{214} + (5-\nu)[24(3+\nu)]^{-1} R^4, \\
&\overline{B}'_{303} = -\overline{B}_{303}, \quad \overline{B}'_{313} = -\overline{B}_{313}, \quad \overline{C}'_{301} = \overline{C}_{301}, \quad \overline{C}'_{311} = \overline{C}_{311}, \\
&\overline{D}'_{301} = \overline{D}_{301}, \quad \overline{D}'_{303} = -\overline{D}_{303}, \quad \overline{D}'_{311} = \overline{D}_{311}, \quad \overline{D}'_{313} = -\overline{D}_{313}.
\end{aligned}
\tag{A.23}
$$

Remaining coefficients of Eq. (A.19)

$$\bar{A}_n^{(31)} = \overline{A}_n^{(21)}, \quad \bar{B}_n^{(31)} = \overline{B}_n^{(21)}, \quad \bar{C}_n^{(31)} = \overline{C}_n^{(21)}, \quad \bar{D}_n^{(31)} = \overline{D}_n^{(21)},$$

$$\left(K_n^{(11)} \leftrightarrow L_n^{(11)}, K_n^{(12)} \leftrightarrow L_n^{(12)}, K_n^{(13)} \leftrightarrow L_n^{(13)}, K_n^{(14)} \leftrightarrow L_n^{(14)}\right),$$

$$\bar{A}_n^{(32)} = \overline{A}_n^{(31)}, \quad \bar{B}_n^{(32)} = \overline{B}_n^{(31)}, \quad \bar{C}_n^{(32)} = \overline{C}_n^{(31)}, \quad \bar{D}_n^{(32)} = \overline{D}_n^{(31)},$$

$$L_n^{(11)} = \int_{-a}^{a} \left\{\left(a^2+\xi^2\right)\left[\ln\left(a^2+\xi^2\right) - 11/3\right]\overline{D}_{200}\right.$$

$$\left. -2\left(a^2+\xi^2\right)^{-1}\overline{C}_{301} - \ln\left(a^2+\xi^2\right)\overline{D}_{301}\right\}\cos\frac{n\pi\xi}{a}d\xi,$$

$$L_n^{(12)} = \int_{-a}^{a} \left\{2\xi\left[\ln\left(a^2+\xi^2\right) - 8/3\right]\overline{D}_{200} + 4\xi\left(a^2+\xi^2\right)^{-2}\overline{C}'_{301}\right.$$

$$\left. -2\xi\left(a^2+\xi^2\right)^{-1}\overline{D}'_{301}\right\}\cos\frac{n\pi\xi}{a}d\xi,$$

$$L_n^{(13)} = \int_{-a}^{a} \left\{2\left[2a^2\left(a^2+\xi^2\right)^{-1} + 3\ln\left(a^2+\xi^2\right) - 8\right]\overline{D}_{200} + 4\left(3\xi^2 - a^2\right)\right.$$

$$\left. \times\left(a^2+\xi^2\right)^{-3}\overline{C}_{301} - 2\left(3\xi^2+a^2\right)\left(a^2+\xi^2\right)^{-2}\overline{D}_{301}\right\}\cos\frac{n\pi\xi}{a}d\xi,$$

$$L_n^{(14)} = 4\int_{-a}^{a} \left[\xi\left(3\xi^2 - a^2\right)\left(a^2+\xi^2\right)^{-2}\overline{D}_{200}\right.$$

$$\left. - 12\xi\left(\xi^2 - a^2\right)\left(a^2+\xi^2\right)^{-4}\overline{C}'_{301} + 2\xi^3\left(a^2+\xi^2\right)^{-3}\overline{D}'_{301}\right]\cos\frac{n\pi\xi}{a}d\xi. \tag{A.24}$$

Coefficients of Eq. 7.73

$$A = 1 + \frac{1}{\left|\Omega_k^*\right|}\iint_{\Omega_k^*}\left[3F_{1,\eta\eta} + \nu F_{1,\xi\xi} + F_{2,\xi\xi\xi} - (2 - -3\nu)F_{2,\xi\eta\eta}\right]d\xi d\eta,$$

$$B = 1 + \frac{1}{\left|\Omega_k^*\right|}\iint_{\Omega_k^*}\left\{F_{3,\xi\xi} + F_{4,\eta\eta} + 4\nu F_{5,\xi\eta} + \frac{1}{2}\left[F_{6,\xi\xi\xi} + F_{7,\eta\eta\eta}\right.\right.$$

$$\left.\left. - (2 - -3\nu)\left(F_{6,\xi\eta\eta} + F_{7,\eta\xi\xi}\right)\right]\right\}d\xi d\eta,$$

$$F_1 = \frac{1}{2}\left(\overline{C}_{200} + N\overline{D}_{200}\right)\ln N + MN^{-1}\left(\overline{B}_{202}N^{-1} + \overline{D}_{202}\right)$$

$$+ \varepsilon_1\left[MN^{-1}\left(\overline{B}_{212}N^{-1} + \overline{D}_{212}\right) + LN^{-3}\left(\overline{B}_{214}N^{-1} + \overline{D}_{214}\right)\right.$$

$$\left. + MKN^{-5}\left(\overline{B}_{216}N^{-1} + \overline{D}_{216}\right)\right],$$

$$F_2 = \frac{1}{2}\left\{-\xi N\left(\ln N - 11/3\right)\overline{D}_{200} - \varepsilon_1 \xi N^{-3}\left(\overline{D}_{214}T + \overline{D}_{216}HN^{-2}\right)\right.$$
$$+ 2\xi N^{-1}\left(\overline{C}_{301} + \varepsilon_1\overline{C}_{311}\right) + \xi \ln N\left(\overline{D}_{301} + \varepsilon_1\overline{D}_{311}\right)$$
$$+ \xi S N^{-1}\left[-\left(\overline{D}_{202} + \varepsilon_1\overline{D}_{212}\right) + 2N^{-2}\left(\overline{B}_{303} + \varepsilon_1\overline{B}_{313}\right)\right.$$
$$\left.\left. + 2N^{-1}\left(\overline{D}_{303} + \varepsilon_1\overline{D}_{313}\right)\right]\right\}$$
$$+ \sum_{n=1}\left[\left(A_n^{(31)}\sinh\frac{n\pi\eta}{a} + D_n^{(31)}\eta\cosh\frac{n\pi\eta}{a}\right)\cos\frac{n\pi\xi}{a}\right.$$
$$\left. + \left(A_n^{(32)}\sinh\frac{n\pi\xi}{a} + D_n^{(32)}\xi\cosh\frac{n\pi\xi}{a}\right)\cos\frac{n\pi\eta}{a}\right],$$
$$F_3 = \frac{1}{2}(3+\nu)\left[\frac{1}{2}\left({}_{200} + N\overline{D}_{200}\right)\ln N + \varepsilon_1 L N^{-3}\left(\overline{B}_{214}N^{-1} + \overline{D}_{214}\right)\right]$$
$$- \frac{1}{2}(3-\nu)\left\{MN^{-1}\left(\overline{B}_{202}N^{-1} + \overline{D}_{202}\right) + \varepsilon_1\left[MN^{-1}\left(\overline{B}_{212}N^{-1} + \overline{D}_{212}\right)\right.\right.$$
$$\left.\left. + MKN^{-5}\left(\overline{B}_{216}N^{-1} + \overline{D}_{216}\right)\right]\right\},$$
$$F_4 = \frac{1}{2}(3+\nu)\left[\frac{1}{2}\left({}_{200} + N\overline{D}_{200}\right)\ln N + \varepsilon_1 L N^{-3}\left(\overline{B}_{214}N^{-1} + \overline{D}_{214}\right)\right]$$
$$+ \frac{1}{2}(3-\nu)\left\{MN^{-1}\left(\overline{B}_{202}N^{-1} + \overline{D}_{202}\right) + \varepsilon_1\left[MN^{-1}\left(\overline{B}_{212}N^{-1} + \overline{D}_{212}\right)\right.\right.$$
$$\left.\left. + MKN^{-5}\left(\overline{B}_{216}N^{-1} + \overline{D}_{216}\right)\right]\right\},$$
$$F_5 = QN^{-1}\left\{\overline{B}'_{202}N^{-1} + \overline{D}'_{202} + \varepsilon_1\left[\overline{B}'_{212}N^{-1} + \overline{D}'_{212}\right.\right.$$
$$\left.\left. + 2MN^{-2}\left(\overline{B}'_{214}N^{-1} + \overline{D}'_{214}\right) + \quad PN^{-1}\left(\overline{B}'_{216}N^{-1} + \overline{D}'_{216}\right)\right]\right\}$$
$$+ \frac{1}{2}\sum_{n=1}\left[\left(A_n^{(21)}\sinh\frac{n\pi\eta}{a} + B_n^{(21)}\cosh\frac{n\pi\eta}{a}\right.\right.$$
$$\left. + C_n^{(21)}\eta\sinh\frac{n\pi\eta}{a} + D_n^{(21)}\eta\cosh\frac{n\pi\eta}{a}\right)\cos\frac{n\pi\xi}{a} + \left(A_n^{(22)}\sinh\frac{n\pi\xi}{a}\right.$$
$$\left.\left. + B_n^{(22)}\cosh\frac{n\pi\xi}{a} + C_n^{(22)}\xi\sinh\frac{n\pi\xi}{a} + D_n^{(22)}\xi\cosh\frac{n\pi\xi}{a}\right)\cos\frac{n\pi\eta}{a}\right],$$
$$F_6 = \frac{1}{2}\left\{-\xi N\left(\ln N - 11/3\right)\overline{D}_{200} - \varepsilon_1\xi N^{-3}\left(\overline{D}_{214}T - 3\overline{D}_{216}HN^{-2}\right)\right.$$
$$+ 2\xi N^{-1}\left(\overline{C}_{301} - 3\varepsilon_1\overline{C}_{311}\right) + \xi\ln N\left(\overline{D}_{301} - 3\varepsilon_1\overline{D}_{311}\right)$$
$$\left. + \xi S N^{-1}\left[3\overline{D}_{202} - \varepsilon_1\overline{D}_{212} - 2N^{-2}\left(3\overline{B}_{303} - \varepsilon_1\overline{B}_{313}\right) - 2N^{-1}\left(3\overline{D}_{303} - \varepsilon_1\overline{D}_{313}\right)\right]\right\}$$
$$+ \sum_{n=1}\left[\left(A_n^{(31)}\sinh\frac{n\pi\eta}{a} + D_n^{(31)}\eta\cosh\frac{n\pi\eta}{a}\right)\cos\frac{n\pi\xi}{a}\right.$$
$$\left. + \left(A_n^{(32)}\sinh\frac{n\pi\xi}{a} + D_n^{(32)}\xi\cosh\frac{n\pi\xi}{a}\right)\cos\frac{n\pi\eta}{a}\right];$$

$$F_7 = \frac{1}{2}\left\{-\eta N\left(\ln N - 11/3\right)\overline{D}_{200} - \varepsilon_1 \xi N^{-3}\left(\overline{D}_{214}T_1 - 3\overline{D}_{216}H_1 N^{-2}\right)\right.$$
$$+ 2\eta N^{-1}\left(\overline{C}_{301} - 3\varepsilon_1\overline{C}_{311}\right) + \eta \ln N\left(\overline{D}'_{301} - 3\varepsilon_1\overline{D}'_{311}\right)$$
$$+ \eta S_1 N^{-1}\left[3\overline{D}_{202} - \varepsilon_1\overline{D}_{212} + 2N^{-2}\left(3\overline{B}'_{303} - \varepsilon_1\overline{B}'_{313}\right)\right.$$
$$\left.\left.+2N^{-1}\left(3\overline{D}'_{303} - \varepsilon_1\overline{D}'_{313}\right)\right]\right\} + \sum_{n=1}\left[\left(B_n^{(31)}\cosh\frac{n\pi\eta}{a} + C_n^{(31)}\eta\sinh\frac{n\pi\eta}{a}\right)\right.$$
$$\left.\cos\frac{n\pi\xi}{a} + \left(B_n^{(32)}\cosh\frac{n\pi\xi}{a} + C_n^{(32)}\xi\sinh\frac{n\pi\xi}{a}\right)\cos\frac{n\pi\eta}{a}\right]. \tag{A.25}$$

A.3 Plane problem. Formulae for the coefficients entering Eqs. 7.172, 7.173, 7.180, 7.186

$$A_{200} = A_{200}(x, y), \quad C_{200} = \left(\Phi_{0,xx} + \Phi_{0,yy}\right)\overline{C}_{200}, \quad \overline{C}_{200} = -0.5R^2,$$
$$B_{202} = \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{B}_{202}, \quad \overline{B}_{202} = -0.25R^4,$$
$$D_{202} = \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{D}_{202}, \quad \overline{D}_{202} = 0.5R^2, \quad B'_{202} = \Phi_{0,xy}\overline{B}'_{202}$$
$$\overline{B}'_{202} = 0.5R^4, \quad D'_{202} = \Phi_{0,xy}\overline{D}'_{202}, \quad \overline{D}'_{202} = -R^2,$$
$$B_{212} = \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{B}_{212}, \quad \overline{B}_{212} = -0.5R^4, \quad D_{212} = \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{D}_{212},$$
$$\overline{D}_{212} = 0.5R^2, \quad B'_{212} = \Phi_{0,xy}\overline{B}'_{212}, \quad \overline{B}'_{212} = -R^4,$$
$$D'_{212} = \Phi_{0,xy}\overline{D}'_{212}, \quad \overline{D}'_{212} = R^2, \quad B_{214} = \left(\Phi_{0,xx} + \Phi_{0,yy}\right)\overline{B}_{214},$$
$$\overline{B}_{214} = 0.5R^6, \quad D_{214} = \left(\Phi_{0,xx} + \Phi_{0,yy}\right)\overline{D}_{214}, \quad \overline{D}_{214} = -0.5R^4,$$
$$B'_{214} = D'_{214} = 0, \quad B_{216} = \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{B}_{216}, \quad \overline{B}_{216} = -0.5R^8,$$
$$D_{216} = \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{D}_{216}, \quad \overline{D}_{216} = 0.5R^6, \quad B'_{216} = \Phi_{0,xy}\overline{B}'_{216},$$
$$\overline{B}'_{216} = R^8, \quad D'_{212} = \Phi_{0,xy}\overline{D}'_{216}, \quad \overline{D}'_{216} = -R^6,$$
$$C_{301} = \left(\Phi_{0,xxx} + \Phi_{0,xyy}\right)\overline{C}_{301}, \quad \overline{C}_{301} = -0.25R^4,$$
$$C'_{301} = \left(\Phi_{0,yyy} + \Phi_{0,xxy}\right)\overline{C}'_{301},$$
$$\overline{C}'_{301} = -0.25R^4, \quad D_{301} = D'_{301} = 0, \quad B_{303} = \left(\Phi_{0,xxx} - 3\Phi_{0,xyy}\right)\overline{B}_{303},$$
$$\overline{B}_{303} = {}^1/_{12}R^6, \quad D_{303} = \left(\Phi_{0,xxx} - 3\Phi_{0,xyy}\right)\overline{D}_{303}, \quad \overline{D}_{303} = -0.25R^4,$$
$$B'_{303} = \left(\Phi_{0,yyy} - 3\Phi_{0,xxy}\right)\overline{B}'_{303}, \quad \overline{B}'_{303} = -{}^1/_{12}R^6,$$
$$D'_{303} = \left(\Phi_{0,yyy} - 3\Phi_{0,xxy}\right)\overline{D}'_{303}, \quad \overline{D}'_{303} = 0.25R^4,$$
$$C_{311} = D_{311} = C'_{311} = D'_{311} = 0,$$
$$B_{313} = \left(\Phi_{0,xxx} + \Phi_{0,xyy}\right)\overline{B}_{313}, \quad \overline{B}_{313} = -0.25R^6,$$
$$B'_{313} = \left(\Phi_{0,yyy} + \Phi_{0,xxy}\right)\overline{B}'_{313},$$
$$\overline{B}'_{313} = 0.25R^6, \quad D_{313} = D'_{313} = 0, \quad A_n^{(21)} = \Phi_{0,xy}\overline{A}_{2n}^{(1)},$$
$$B_n^{(21)} = \left(\Phi_{0,xx} + \Phi_{0,yy}\right)\overline{B}_{2n}^{(1)} + \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{\overline{B}}_{2n}^{(1)},$$
$$C_n^{(21)} = \left(\Phi_{0,xx} + \Phi_{0,yy}\right)\overline{C}_{2n}^{(1)} + \left(\Phi_{0,yy} - \Phi_{0,xx}\right)\overline{\overline{C}}_{2n}^{(1)}, \quad D_n^{(21)} = \Phi_{0,xy}\overline{D}_{2n}^{(1)},$$
$$\overline{A}_{2n}^{(1)} = \left[(2\sinh(n\pi) + n\pi\cosh(n\pi))\,\overline{K}_{2n}^{(11)} - \frac{a^2}{n\pi}\cosh(n\pi)\overline{K}_{2n}^{(13)}\right]\left(4\sinh^2(n\pi)\right)^{-1},$$
$$\overline{B}_{2n}^{(1)} = \left[\frac{a}{n\pi}\,(3\sinh(n\pi) + n\pi\cosh(n\pi))\,\overline{K}_{2n}^{(12)} - \frac{a^3}{n\pi}\,(\sinh(n\pi)\right.$$

$$+n\pi\cosh(n\pi))\ \overline{K}^{(14)}_{2n}\Big]\left(4\mathrm{sh}^2(n\pi)\right)^{-1},$$

$$\overline{C}^{(1)}_{2n} = \left(a^2\overline{K}^{(14)}_{2n} - \overline{K}^{(12)}_{2n}\right)(4\sinh(n\pi))^{-1},$$

$$\overline{D}^{(1)}_{2n} = \left(\frac{a}{n\pi}\overline{K}^{(13)}_{2n} - \frac{n\pi}{a}\overline{K}^{(11)}_{2n}\right)(4\sinh(n\pi))^{-1},$$

$$\overline{\overline{B}}^{(1)}_{2n} = \overline{B}^{(1)}_{2n},\quad {}^{(1)}_{2n} = {}^{(1)}_{2n},\quad \left(\overline{K}^{(12)}_{2n} \leftrightarrow \overline{\overline{K}}^{(12)}_{2n};\overline{K}^{(14)}_{2n} \leftrightarrow \overline{\overline{K}}^{(14)}_{2n}\right),$$

$$\overline{K}^{(11)}_{2n} = -4aR^2\left[\frac{1}{2}R^2(1--2\varepsilon_1)\int_{-a}^{a}\xi(a^2+\xi^2)^{-2}\cos\frac{n\pi\xi}{a}d\xi\right.$$

$$\left.-\ (1-\varepsilon_1)\int_{-a}^{a}\xi(a^2+\xi^2)^{-1}\cos\frac{n\pi\xi}{a}d\xi\right],$$

$$\overline{K}^{(13)}_{2n} = 8aR^2\left[3R^2(1--2\varepsilon_1)\int_{-a}^{a}\xi\left(\xi^2-a^2\right)(a^2+\xi^2)^{-4}\cos\frac{n\pi\xi}{a}d\xi\right.$$

$$\left.(1-\varepsilon_1)\int_{-a}^{a}\xi\left(3\xi^2-a^2\right)(a^2+\xi^2)^{-3}\cos\frac{n\pi\xi}{a}d\xi\right],$$

$$\overline{K}^{(12)}_{2n} = aR^2\int_{-a}^{a}\xi(a^2+\xi^2)^{-1}\cos\frac{n\pi\xi}{a}d\xi,$$

$$\overline{\overline{K}}^{(12)}_{2n} = -aR^2\left[R^2(1+2\varepsilon_1)\int_{-a}^{a}\xi\left(3\xi^2-a^2\right)(a^2+\xi^2)^{3}\cos\frac{n\pi\xi}{a}d\xi\right.$$

$$\left.-4(1+\varepsilon_1)\int_{-a}^{a}\xi^2(a^2+\xi^2)^{-2}\cos\frac{n\pi\xi}{a}d\xi\right],$$

$$\overline{K}^{(14)}_{2n} = -2aR^2\int_{-a}^{a}\left(3\xi^2-a^2\right)(a^2+\xi^2)^{-3}\cos\frac{n\pi\xi}{a}d\xi,$$

$$\overline{\overline{K}}^{(14)}_{2n} = 12aR^2\left[R^2(1+\varepsilon_1)\int_{-a}^{a}\left(a^4-10a^2\xi^2+5\xi^4\right)(a^2+\xi^2)^{-5}\cos\frac{n\pi\xi}{a}d\xi\right.$$

$$\left.-4(1+\varepsilon_1)\int_{-a}^{a}\xi^2\left(\xi^2-a^2\right)(a^2+\xi^2)^{-4}\cos\frac{n\pi\xi}{a}d\xi\right],$$

$$A^{(22)}_n = \Phi_{0,xy}\overline{A}^{(2)}_{2n},\quad B^{(22)}_n = \left(\Phi_{0,xx}+\Phi_{0,yy}\right)\overline{B}^{(2)}_{2n} + \left(\Phi_{0,yy}-\Phi_{0,xx}\right)\overline{\overline{B}}^{(2)}_{2n},$$

$$C^{(22)}_n = \left(\Phi_{0,xx}+\Phi_{0,yy}\right)\overline{C}^{(2)}_{2n} + \left(\Phi_{0,yy}-\Phi_{0,xx}\right)\overline{\overline{C}}^{(2)}_{2n},\quad D^{(22)}_n = \Phi_{0,xy}\overline{D}^{(2)}_{2n},$$

$$\overline{A}_{2n}^{(2)} = \overline{A}_{2n}^{(1)}, \quad \overline{B}_{2n}^{(2)} = \overline{B}_{2n}^{(1)}, \quad \overline{\overline{B}}_{2n}^{(2)} = \overline{\overline{B}}_{2n}^{(1)}, \quad \overline{C}_{2n}^{(2)} = {}_{2n}^{(1)}, \quad \overline{\overline{C}}_{2n}^{(2)} = \overline{\overline{C}}_{2n}^{(1)}, \quad \overline{D}_{2n}^{(2)} = \overline{D}_{2n}^{(1)},$$

$$A_n^{(31)} = \left(\Phi_{0,yyy} + \Phi_{0,xxy}\right) \overline{A}_{3n}^{(1)} + \left(\Phi_{0,yyy} - 3\Phi_{0,xxy}\right) \overline{\overline{A}}_{3n}^{(1)},$$

$$B_n^{(31)} = \left(\Phi_{0,xxx} + \Phi_{0,xyy}\right) \overline{B}_{3n}^{(1)} + \left(\Phi_{0,xxx} - 3\Phi_{0,xyy}\right) \overline{\overline{B}}_{3n}^{(1)},$$

$$C_n^{(31)} = \left(\Phi_{0,xxx} + \Phi_{0,xyy}\right) \overline{C}_{3n}^{(1)} + \left(\Phi_{0,xxx} - 3\Phi_{0,xyy}\right) \overline{\overline{C}}_{3n}^{(1)},$$

$$D_n^{(31)} = \left(\Phi_{0,yyy} + \Phi_{0,xxy}\right) \overline{D}_{3n}^{(1)} + \left(\Phi_{0,yyy} - 3\Phi_{0,xxy}\right) \overline{\overline{D}}_{3n}^{(1)},$$

$$\overline{A}_{3n}^{(1)} = \overline{A}_{2n}^{(1)}, \quad \overline{B}_{3n}^{(1)} = \overline{B}_{2n}^{(1)}, \quad \overline{C}_{3n}^{(1)} = {}_{2n}^{(1)}, \quad \overline{\overline{A}}_{3n}^{(1)} = \overline{A}_{3n}^{(1)},$$

$$\overline{\overline{B}}_{3n}^{(1)} = \overline{B}_{3n}^{(1)}, \quad \overline{\overline{C}}_{3n}^{(1)} = \overline{C}_{3n}^{(1)}, \quad \overline{\overline{D}}_{3n}^{(1)} = \overline{D}_{3n}^{(1)}, \quad \overline{D}_{3n}^{(1)} = \overline{D}_{2n}^{(1)},$$

$$\left(\overline{K}_{2n}^{(11)} \leftrightarrow \overline{K}_{3n}^{(11)}, \overline{K}_{2n}^{(12)} \leftrightarrow \overline{K}_{3n}^{(12)}, \overline{K}_{2n}^{(13)} \leftrightarrow \overline{K}_{3n}^{(13)}, \overline{K}_{2n}^{(14)} \leftrightarrow \overline{K}_{3n}^{(14)}\right),$$

$$\left(\overline{K}_{3n}^{(11)} \leftrightarrow \overline{\overline{K}}_{3n}^{(11)}, \overline{K}_{3n}^{(12)} \leftrightarrow \overline{\overline{K}}_{3n}^{(12)}, \overline{K}_{3n}^{(13)} \leftrightarrow \overline{\overline{K}}_{3n}^{(13)}, \overline{K}_{3n}^{(14)} \leftrightarrow \overline{\overline{K}}_{3n}^{(14)}\right),$$

$$\overline{K}_{3n}^{(11)} = 0.5aR^4 \left[\int_{-a}^{a} (a^2+\xi^2)^{-1} \cos\frac{n\pi\xi}{a} d\xi + \varepsilon_1 R^2 \int_{-a}^{a} \left(a^2 - 3\xi^2\right)(a^2+\xi^2)^{-3} \cos\frac{n\pi\xi}{a} d\xi\right],$$

$$\overline{\overline{K}}_{3n}^{(11)} = 0.5aR^4 \left[\int_{-a}^{a} \left(a^2 - 3\xi^2\right)(a^2+\xi^2)^{-2} \cos\frac{n\pi\xi}{a} d\xi\right.$$

$$\left. - \frac{1}{3} R^2 \int_{-a}^{a} \left(a^2 - 3\xi^2\right)(a^2+\xi^2)^{-3} \cos\frac{n\pi\xi}{a} d\xi\right],$$

$$\overline{K}_{3n}^{(12)} = -aR^4 \left[\int_{-a}^{a} \xi (a^2+\xi^2)^{-2} \cos\frac{n\pi\xi}{a} d\xi - 6\varepsilon_1 R^2 \int_{-a}^{a} \xi \left(a^2 - \xi^2\right)(a^2+\xi^2)^{-4} \cos\frac{n\pi\xi}{a} d\xi\right],$$

$$\overline{\overline{K}}_{3n}^{(12)} = aR^4 \left[\int_{-a}^{a} \xi \left(3a^2 - 5\xi^2\right)(a^2+\xi^2)^{-3} \cos\frac{n\pi\xi}{a} d\xi\right.$$

$$\left. -2R^2 \int_{-a}^{a} \xi \left(a^2 - \xi^2\right)(a^2+\xi^2)^{-4} \cos\frac{n\pi\xi}{a} d\xi\right],$$

$$\overline{K}_{3n}^{(13)} = aR^4 \left[\int_{-a}^{a} (a^2 - 3\xi^2)(a^2+\xi^2)^{-3} \cos\frac{n\pi\xi}{a} d\xi\right.$$

$$\left. +6\varepsilon_1 R^2 \int_{-a}^{a} \left(a^4 - 10a^2\xi^2 + 5\xi^4\right)(a^2+\xi^2)^{-5} \cos\frac{n\pi\xi}{a} d\xi\right],$$

$$\overline{\overline{K}}_{3n}^{(13)} = aR^4 \left[\int_{-a}^{a} \left(a^4 - 26a^2\xi^2 + 21\xi^4\right)(a^2+\xi^2)^{-4} \cos\frac{n\pi\xi}{a} d\xi\right.$$

$$\left. -2R^2 \int_{-a}^{a} \left(a^4 - 10a^2\xi^2 + 5\xi^4\right)(a^2+\xi^2)^{-5} \cos\frac{n\pi\xi}{a} d\xi\right],$$

$$\overline{K}_{3n}^{(14)} = -12aR^4 \left[\int_{-a}^{a} \xi (a^2 - \xi^2)(a^2+\xi^2)^{-4} \cos\frac{n\pi\xi}{a} d\xi\right.$$

$$-5\varepsilon_1 R^2 \int_{-a}^{a} \xi \left(3a^4 - 10a^2\xi^2 + 3\xi^4\right)\left(a^2+\xi^2\right)^{-6} \cos\frac{n\pi\xi}{a} d\xi \Bigg],$$

$$\overline{\overline{K}}{}_{3n}^{(14)} = 12aR^4 \Bigg[\int_{-a}^{a} \xi \left(3a^4 - 20a^2\xi^2 + 9\xi^4\right)\left(a^2+\xi^2\right)^{-5} \cos\frac{n\pi\xi}{a} d\xi$$

$$-\frac{5}{3}R^2 \int_{-a}^{a} \xi \left(3a^4 - 10a^2\xi^2 + 3\xi^4\right)\left(a^2+\xi^2\right)^{-6} \cos\frac{n\pi\xi}{a} d\xi \Bigg],$$

$$A_n^{(32)} = \left(\Phi_{0,yyy} + \Phi_{0,xxy}\right)\overline{A}_{3n}^{(2)} + \left(\Phi_{0,xxx} - 3\Phi_{0,xyy}\right)\overline{\overline{A}}{}_{3n}^{(2)}, \quad \overline{A}_{3n}^{(2)} = \overline{A}_{3n}^{(1)}, \quad \overline{\overline{A}}{}_{3n}^{(2)} = \overline{\overline{A}}{}_{3n}^{(1)},$$

$$B_n^{(32)} = \left(\Phi_{0,yyy} + \Phi_{0,xxy}\right)\overline{B}_{3n}^{(2)} + \left(\Phi_{0,yyy} - 3\Phi_{0,xxy}\right)\overline{\overline{B}}{}_{3n}^{(2)}, \quad \overline{B}_{3n}^{(2)} = \overline{B}_{3n}^{(1)}, \quad \overline{\overline{B}}{}_{3n}^{(2)} = \overline{\overline{B}}{}_{3n}^{(1)},$$

$$C_n^{(32)} = \left(\Phi_{0,yyy} + \Phi_{0,xxy}\right)\overline{C}_{3n}^{(2)} + \left(\Phi_{0,yyy} - 3\Phi_{0,xxy}\right)\overline{\overline{C}}{}_{3n}^{(2)}, \quad \overline{C}_{3n}^{(2)} = {}_{3n}^{(1)}, \quad \overline{\overline{C}}{}_{3n}^{(2)} = \overline{\overline{C}}{}_{3n}^{(1)},$$

$$D_n^{(32)} = \left(\Phi_{0,xxx} + \Phi_{0,xyy}\right)\overline{D}_{3n}^{(2)} + \left(\Phi_{0,xxx} - 3\Phi_{0,xyy}\right)\overline{\overline{D}}{}_{3n}^{(2)}, \quad \overline{D}_{3n}^{(2)} = \overline{D}_{3n}^{(1)}, \quad \overline{\overline{D}}{}_{3n}^{(2)} = \overline{\overline{D}}{}_{3n}^{(1)}.$$

Coefficients of Eqs. 7.180 and 7.186

$$C = 1 + \frac{2}{\left|\Omega_k^*\right|}\iint_{\Omega_k^*}\left[3F_{1\xi\xi} + F_{1\eta\eta} + 2(F_{2\xi\xi\xi} + F_{2\xi\eta\eta})\right] d\xi d\eta,$$

$$F = 1 + \frac{2}{\left|\Omega_k^*\right|}\iint_{\Omega_k^*}\left(F_{3\xi\xi} + F_{4\eta\eta} + 4F_{5\xi\eta} + F_{6\xi\xi\xi} + F_{6\xi\eta\eta} + F_{7\eta\eta\eta} + F_{7\eta\xi\xi}\right) d\xi d\eta,$$

$$F_1 = 0.5\overline{C}_{200}\ln N - MN^{-1}\left(\overline{B}_{202}N^{-1} + \overline{D}_{202}\right) + \varepsilon_1\left[-MN^{-1}\left(\overline{B}_{212}N^{-1} + \overline{D}_{212}\right)\right.$$

$$\left. + LN^{-3}\left(\overline{B}_{214}N^{-1} + \overline{D}_{214}\right) - MKN^{-5}\left(\overline{B}_{216}N^{-1} + \overline{D}_{216}\right)\right]$$

$$+\sum_{n=1}^{\infty}\left\{\left[\left(\overline{B}_{2n}^{(1)} - \overline{\overline{B}}{}_{2n}^{(1)}\right)\cosh\frac{n\pi\eta}{a} + \left(\overline{C}_{2n}^{(1)} - \overline{\overline{C}}{}_{2n}^{(1)}\right)\eta\sinh\frac{n\pi\eta}{a}\right]\cos\frac{n\pi\xi}{a}\right.$$

$$\left. + \left[\left(\overline{B}_{2n}^{(2)} - \overline{\overline{B}}{}_{2n}^{(2)}\right)\cosh\frac{n\pi\xi}{a} + \left(\overline{C}_{2n}^{(2)} - \overline{\overline{C}}{}_{2n}^{(2)}\right)\xi\sinh\frac{n\pi\xi}{a}\right]\cos\frac{n\pi\eta}{a}\right\},$$

$$F_2 = 0.5\xi SN^{-1}\left(\overline{D}_{202} + 2\overline{B}_{303}N^{-2} + 2\overline{D}_{303}N^{-1}\right) + \xi\overline{C}_{301}N^{-1}$$

$$+\varepsilon_1\left[0.5\xi SN^{-1}\left(\overline{D}_{212} + 2\overline{B}_{313}N^{-2}\right) - 0.5\xi N^{-3}\left(\overline{D}_{214}T - \overline{D}_{216}HN^{-2}\right)\right]$$

$$+\sum_{n=1}^{\infty}\left\{\left[\left(\overline{B}_{3n}^{(1)} + \overline{\overline{B}}{}_{3n}^{(1)}\right)\cosh\frac{n\pi\eta}{a} + \left(\overline{C}_{3n}^{(1)} + \overline{\overline{C}}{}_{3n}^{(1)}\right)\eta\sinh\frac{n\pi\eta}{a}\right]\cos\frac{n\pi\xi}{a}\right.$$

$$\left. + \left[\left(\overline{A}_{3n}^{(2)} + \overline{\overline{A}}{}_{3n}^{(2)}\right)\sinh\frac{n\pi\xi}{a} + \left(\overline{D}_{3n}^{(2)} + \overline{\overline{D}}{}_{3n}^{(2)}\right)\xi\cosh\frac{n\pi\xi}{a}\right]\cos\frac{n\pi\eta}{a}\right\},$$

$$F_3 = \overline{C}_{200}\ln N + MN^{-1}\left(\overline{B}_{202}N^{-1} + \overline{D}_{202}\right) + \varepsilon_2\left[MN^{-1}\left(\overline{B}_{212}N^{-1} + \overline{D}_{212}\right)\right.$$

$$\left. + 2LN^{-3}\left(\overline{B}_{214}N^{-1} + \overline{D}_{214}\right) + MKN^{-5}\left(\overline{B}_{216}N^{-1} + \overline{D}_{216}\right)\right]$$

$$+\sum_{n=1}^{\infty}\left\{\left[\left(2\overline{B}_{2n}^{(1)} + \overline{\overline{B}}{}_{2n}^{(1)}\right)\cosh\frac{n\pi\eta}{a} + \left(2\overline{C}_{2n}^{(1)} + \overline{\overline{C}}{}_{2n}^{(1)}\right)\eta\sinh\frac{n\pi\eta}{a}\right]\cos\frac{n\pi\xi}{a}\right.$$

$$+\left[\left(2\overline{B}_{2n}^{(2)}+\overline{\overline{B}}_{2n}^{(2)}\right)\cosh\frac{n\pi\xi}{a}+\left(2\overline{C}_{2n}^{(2)}+\overline{\overline{C}}_{2n}^{(2)}\right)\xi\sinh\frac{n\pi\xi}{a}\right]\cos\frac{n\pi\eta}{a}\Bigg\},$$

$$F_4=\overline{C}_{200}\ln N-MN^{-1}\left(\overline{B}_{202}N^{-1}+\overline{D}_{202}\right)+\varepsilon_2\left[-MN^{-1}\left(\overline{B}_{212}N^{-1}+\overline{D}_{212}\right)\right.$$

$$\left.+2LN^{-3}\left(\overline{B}_{214}N^{-1}+\overline{D}_{214}\right)-MKN^{-5}\left(\overline{B}_{216}N^{-1}+\overline{D}_{216}\right)\right]$$

$$+\sum_{n=1}^{\infty}\left\{\left[\left(2\overline{B}_{2n}^{(1)}-\overline{\overline{B}}_{2n}^{(1)}\right)\cosh\frac{n\pi\eta}{a}+\left(2\overline{C}_{2n}^{(1)}-\overline{\overline{C}}_{2n}^{(1)}\right)\eta\sinh\frac{n\pi\eta}{a}\right]\cos\frac{n\pi\xi}{a}\right.$$

$$+\left[\left(2\overline{B}_{2n}^{(2)}-\overline{\overline{B}}_{2n}^{(2)}\right)\cosh\frac{n\pi\xi}{a}+\left(2\overline{C}_{2n}^{(2)}-\overline{\overline{C}}_{2n}^{(2)}\right)\xi\sinh\frac{n\pi\xi}{a}\right]\cos\frac{n\pi\eta}{a}\Bigg\},$$

$$F_5=QN^{-1}\left(\overline{B}'_{202}N^{-1}+\overline{D}'_{202}\right)+\varepsilon_2\left[QN^{-1}\left(\overline{B}'_{212}N^{-1}+\overline{D}'_{212}\right)\right.$$

$$\left.+QPN^{-5}\left(\overline{B}'_{216}N^{-1}+\overline{D}'_{216}\right)\right]+0.5\sum_{n=1}^{\infty}\left[\left(\overline{A}_{2n}^{(1)}\sinh\frac{n\pi\eta}{a}+\overline{D}_{2n}^{(1)}\eta\cosh\frac{n\pi\eta}{a}\right)\cos\frac{n\pi\xi}{a}\right.$$

$$\left.+\left(\overline{A}_{2n}^{(2)}\sinh\frac{n\pi\xi}{a}+\overline{D}_{2n}^{(2)}\xi\cosh\frac{n\pi\xi}{a}\right)\cos\frac{n\pi\eta}{a}\right],$$

$$F_6=-0.5\xi SN^{-1}\left[\overline{D}_{202}-\overline{D}'_{202}+6N^{-1}\left(\overline{B}_{303}N^{-1}+\overline{D}_{303}\right)\right]+\xi\overline{C}_{301}N^{-1}$$

$$+\varepsilon_1\left\{-0.5\xi SN^{-1}\left(\overline{D}_{212}-\overline{D}'_{212}-2\overline{B}_{313}N^{-2}\right)\right.$$

$$\left.-0.5\xi N^{-3}\left[\overline{D}_{214}T+\left(\overline{D}_{212}-\overline{D}'_{216}\right)HN^{-2}\right]\right\}$$

$$+\sum_{n=1}^{\infty}\left\{\left[\left(\overline{B}_{3n}^{(1)}-3\overline{\overline{B}}_{3n}^{(1)}\right)\cosh\frac{n\pi\eta}{a}+\left(\overline{C}_{3n}^{(1)}-\overline{\overline{C}}_{3n}^{(1)}\right)\eta\sinh\frac{n\pi\eta}{a}\right]\cos\frac{n\pi\xi}{a}\right.$$

$$+\left[\left(\overline{A}_{3n}^{(2)}-3\overline{\overline{A}}_{3n}^{(2)}\right)\sinh\frac{n\pi\xi}{a}+\left(\overline{D}_{3n}^{(2)}-3\overline{\overline{D}}_{3n}^{(2)}\right)\xi\cosh\frac{n\pi\xi}{a}\right]\cos\frac{n\pi\eta}{a}\Bigg\},$$

$$F_7=-0.5\eta S_1N^{-1}\left[\overline{D}_{202}-\overline{D}'_{202}-6N^{-1}\left(\overline{B}'_{303}N^{-1}+\overline{D}'_{303}\right)\right]+\eta\overline{C}'_{301}N^{-1}$$

$$+\varepsilon_1\left\{-0.5\eta S_1N^{-1}\left(\overline{D}_{212}-\overline{D}'_{212}+2\overline{B}'_{313}N^{-2}\right)\right.$$

$$\left.-0.5\eta N^{-3}\left[\overline{D}_{214}T_1+\left(\overline{D}_{212}-\overline{D}'_{216}\right)H_1N^{-2}\right]\right\}$$

$$+\sum_{n=1}^{\infty}\left\{\left[\left(\overline{A}_{3n}^{(1)}-3\overline{\overline{A}}_{3n}^{(1)}\right)\sinh\frac{n\pi\eta}{a}+\left(\overline{D}_{3n}^{(1)}-3\overline{\overline{D}}_{3n}^{(1)}\right)\eta\cosh\frac{n\pi\eta}{a}\right]\cos\frac{n\pi\xi}{a}\right.$$

$$+\left[\left(\overline{B}_{3n}^{(2)}-3\overline{\overline{B}}_{3n}^{(2)}\right)\cosh\frac{n\pi\xi}{a}+\left(\overline{C}_{3n}^{(2)}-3\overline{\overline{C}}_{3n}^{(2)}\right)\xi\sinh\frac{n\pi\xi}{a}\right]\cos\frac{n\pi\eta}{a}\Bigg\}. \tag{A.26}$$

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