Solutions to Selected Problems

Problems of Chap. 1

1.1 Physical constants

(a) $c = 2.99792458 \times 10^8\,\mathrm{m\,s^{-1}}$ (exact).
(b) $h = 6.6261 \times 10^{-34}\,\mathrm{J\,s}$.
(c) $\hbar = 1.0545 \times 10^{-34}\,\mathrm{J\,s}$.
(d) $e = 1.6022 \times 10^{-19}\,\mathrm{C}$.
(e) $m_0 = 0.9109 \times 10^{-30}\,\mathrm{kg}$.
(f) $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{V\,s\,A^{-1}\,m^{-1}}$.
(g) $\varepsilon_0 = 1/(\mu_0 c^2) = 0.8854 \times 10^{-11}\,\mathrm{A\,s\,V^{-1}\,m^{-1}}$.
(h) $k = 1.3807 \times 10^{-23}\,\mathrm{J\,K^{-1}}$.
(i) $N_A = 6.022 \times 10^{26}$ molecules per Mole.
(j) $R = kN_A = 8.315 \times 10^3\,\mathrm{J\,K^{-1}}$ per Mole.
(k) $L_0 = 2.687 \times 10^{25}$ molecules per $\mathrm{m^3}$ at $0\,^\circ\mathrm{C}$ and normal pressure.

1.2 Frequency, wavelength, wavenumber and energy scale

(a) $1\,\mu\mathrm{m}$; 300 THz; $10^4\,\mathrm{cm^{-1}}$; 1.9878×10^{-20} J; 1.2407 eV.
(b) $300\,\mu\mathrm{m}$; 1 THz; $3300\,\mathrm{m^{-1}} = 33.33\,\mathrm{cm^{-1}}$; 6.626×10^{-23} J; 4.136 meV.
(c) 1 nm; 300 PHz; 2.0×10^{-17} J; 1.240 keV.
(d) $1\,\mathrm{m^{-1}}$; 1 m; 300 MHz; $1.24\,\mu\mathrm{eV}$.
(e) $1.2407\,\mu\mathrm{m}$; 241.8 THz; 1.6022×10^{-19} J; 1 eV.

1.3

(a) $T = 300\,\mathrm{K} \equiv kT = 4.142 \times 10^{-21}\,\mathrm{J} \equiv kT/e = 25.85\,\mathrm{meV} \equiv \nu = kT/h = 6.625\,\mathrm{THz} \equiv kT/(hc) = 208\,\mathrm{cm^{-1}} \equiv hc/(kT) = 48\,\mu\mathrm{m}$.
(b) $1\,\mathrm{meV} \equiv 8.06\,\mathrm{cm^{-1}} \equiv 0.2418\,\mathrm{THz}$.
(c) $1\,\mathrm{cm^{-1}} \equiv 30\,\mathrm{GHz}$.
(c) $10\,\mathrm{cm^{-1}} \equiv 1.2408\,\mathrm{meV}$.

1.4 Power of the sun light and laser power

(a) 140 mW. (b) $1.8\,\mathrm{kW/cm^2}$. (c) $1.3\,\mathrm{kW/cm^2}$.

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Problems of Chap. 2

2.1 $5.6 \times 10^{22}\,\mathrm{m}^{-3}$; $5.6 \times 10^{25}\,\mathrm{m}^{-3}$; $5.6 \times 10^{28}\,\mathrm{m}^{-3}$.

2.2 Field amplitude

(a) $\varepsilon_0 A^2/2 = u$; $A = \sqrt{2u/\varepsilon_0}$; $\varepsilon_0 = 0.89 \times 10^{-11}\,\mathrm{A\,s\,V^{-1}\,m^{-1}}$; $A = 4.7 \times 10^5\,\mathrm{V\,m^{-1}}$.
(b) $Z = 10^6\,\mathrm{m}^{-3}$; $A = \sqrt{2h\nu Z/\varepsilon_0} = 6.3 \times 10^{-2}\,\mathrm{V\,m^{-1}}$; $u = 2 \times 10^{-14}\,\mathrm{J\,m^{-3}}$.
(c) $Z = 2 \times 10^{13}\,\mathrm{m}^{-3}$; A = $180\,\mathrm{V\,m^{-1}}$; $u = 1\,\mu\mathrm{J\,m^{-3}}$.

2.3 Occupation number

(b) $kT = 4.14 \times 10^{-21}\,\mathrm{J} = 25.8\,\mathrm{meV}$; $f_1^{\mathrm{Boltz}} - f_2^{\mathrm{Boltz}} \sim 1.8 \times 10^{-35}$.
(c) $f_1^{\mathrm{Boltz}} - f_2^{\mathrm{Boltz}} \sim 0.54 - 0.46 = 0.08$.

2.4 Oscillation condition

(a) In one case, the condition is $G_1 G_1 V u/2 = u/2$ and in an other case, $V G_1 G_1 u/2 = u/2$. Show that both cases lead to $GV = 1$.
(b) For both directions, we obtain the product GV and the same sum of the phases.

2.5 Brewster angle
(a) 54.4°. (b) 56.3°. (c) 61.2°. (d) 60.4°.

Problems of Chap. 3

3.1 $\delta\nu/(c/2L) = 2 \times 10^5$.
3.2 $V = R_1 R_2$; $s_{\mathrm{eff}} = 10$; $\tau_{\mathrm{p}} = 6.7\,\mathrm{ns}$; $l_{\mathrm{p}} = 2\,\mathrm{m}$; $Q = 63$.

3.3 Resonator with air

(a) $\nu_1 = c/(2nL)$.
(b) $\delta\nu = c/(2L) - (c/n)/(2L) = c/(2L)(1 - 1/n) = 160\,\mathrm{kHz}$; $\delta\nu/\nu \sim 3 \times 10^{-9}$.

3.4 Energy $= \varepsilon_0 a_1 T^{-1} \int_0^L \int_0^T E^2(z,t)\mathrm{d}z\mathrm{d}t = \varepsilon_0 a_1 a_2 A^2 T^{-1} \int_0^L \int_0^T \sin^2 kz \sin^2 \omega t \, \mathrm{d}z\mathrm{d}t = \varepsilon_0 a_1 a_2 L A^2/4$; $u = \varepsilon_0 A^2/4$.
3.5 $V = R_1 R_2$; $\tau_{\mathrm{p}} = 1/(1 - V)$.
3.6 Photon density

(a) $Z_{\mathrm{FP}}/\tau_{\mathrm{p}} = Z/T$; $Z_{\mathrm{FP}} = Q/(2\pi)Z$; $Q = 2\pi l/(1 - R)$.
(b) We obtain the same result, but $Q = \pi l/(1 - R)$.

3.7 $R_{\mathrm{FP}} = 1 - T_{\mathrm{FP}} = 4R(1 - R)^{-2} \sin^2 \delta/2[1 + 4R(1 - R)^{-2} \sin^2 \delta/2]^{-1}$.
3.8 $T_{\mathrm{FP}} = 1/[1 + 4R(1 - R)^{-2} \sin^2 \delta/2]$, where $R = \sqrt{R_1 R_2}$.

3.9 Fabry–Perot interferometer with absorbing mirrors

(a) $T_{\mathrm{FP}} = (1+(A_{\mathrm{m}}^2/T_{\mathrm{m}}^2)^{-1}(1+4R(1-R)^{-2}\sin^2\delta/2)^{-1}$; $T_{\mathrm{FP,max}} = (1+A_{\mathrm{m}}^2/T_{\mathrm{m}}^2)^{-1}$.
(b) $1/(1 + A_{\mathrm{m}}^2/T_{\mathrm{m}}^2) < 0.98$; $A_{\mathrm{m}}/T_{\mathrm{m}} < 0.1$.

3.10 Fabry–Perot interferometer for obliquely incident radiation

(a) $\delta = k \times 2L\cos\theta + 2\varphi$.
(b) $\varphi = 0$; $\delta = k \times 2L\cos\theta = zl \times 2\pi$; $2L\cos\theta = zl \times \lambda$.

Problems of Chap. 4

4.2 Absolute number of two-level systems
(a) $N_{\mathrm{tot}} = 10^{15}$. (b) $N_{\mathrm{tot}} = 10^{10}$. (c) $N_{\mathrm{tot}} = 10^4$.

Problems of Chap. 5

5.1 $L + (n-1)L' = 57.6\,\mathrm{cm}$.
5.2 Photon density

(a) The laser beam has only a slightly larger diameter at 10 m distance from the laser and the laser power is of the order of 1 W.
(b) Assuming that the luminescence radiation is emitted isotropically, the power reaching an area of 1 cm diameter is $P_{\mathrm{fluor}} = P_0 \times 2\pi\sin^2(\alpha/2)$, where α is the angle corresponding to the area. It follows that $\alpha \sim 5 \times 10^{-4}$; $P_{\mathrm{fluor}} \sim P_0 \times 2\pi \times \alpha^2/2 \sim 0.4\,\mu\mathrm{W}$.

5.3 $g(\lambda)\mathrm{d}\lambda = g(\nu)\mathrm{d}\nu$; $g(\nu) = g(\lambda)/|\mathrm{d}\nu/\mathrm{d}\lambda|$; $\nu = c/\lambda$; $\mathrm{d}\nu/\mathrm{d}\lambda = -c/\lambda^2$; $g(\nu) = \lambda^2 g(\lambda)/c$.
5.4 Population of the upper laser level

(a) $r = N_2/\tau_{\mathrm{rel}}^* = 3.3 \times 10^{29}\,\mathrm{m}^{-3}\,\mathrm{s}^{-1}$; volume $= \pi r^2 L' = 7.9 \times 10^{-10}\,\mathrm{m}^3$; $P_{\mathrm{pump}} = 1.5 \times 3.3 \times 10^{29} \times 7.9 \times 10^{-10} \times 2.4 \times 10^{-19}\,\mathrm{W} = 9.4\,\mathrm{W}$; the factor 1.5 takes account of the quantum efficiency.
(b) $N_{\mathrm{tot}} = 10^{24} \times 7.9 \times 10^{-10} = 7.9 \times 10^{14}$.
(c) Energy $= 10^{24} \times 7.9 \times 10^{-10} \times 1.5 \times 1.6 \times 10^{-19}\,\mathrm{J} = 190\,\mu\mathrm{J}$; energydensity $=$ energy/volume $= (N_{\mathrm{tot}}/\text{volume}) \times h\nu = 240\,\mathrm{kJ/m^3} = 240\,\mathrm{J}$ per liter.
(d) $P_{\mathrm{pump}} = 94\,\mathrm{W}$. [The reason is the stimulated emission (Sect. 8.8).]

Problems of Chap. 6

6.1 Photon density

(a) $Z = D(\nu)\mathrm{d}\nu\bar{n} = (8\pi\nu^2/c^3)kT/h\nu \sim 6 \times 10^7\,\mathrm{m}^{-3}$.

(b) $Z = 6 \times 10^{10}\,\text{m}^{-3}$.
(c) $Z = (8\pi\nu^2/c^3)\text{d}\nu\exp(-h\nu/kT) = 4 \times 10^{-34}\,\text{m}^{-3}$.

6.2 Number of thermal photons in a mode of a laser resonator

(a) $\bar{n} = \exp(-h\nu/kT) \sim 2 \times 10^{-29}$.
(b) $\bar{n} = 1/[\exp(h\nu/kT) - 1] \sim 0.25$.
(c) $\bar{n} = kT/h\nu \sim 6$.

Problems of Chap. 7

7.1 Amplification of radiation in titanium–sapphire

(a) $\alpha = 8\,\text{m}^{-1}$. (b) $G_1 - 1 = 0.5$.
(c) $\alpha(1\,\mu\text{m})/\alpha(\lambda_0) \sim 0.5$; $\alpha(1\,\mu\text{m}) \sim 4\,\text{m}^{-1}$; $G_1 - 1 = 0.25$.

7.2 $\sigma_{\text{nat}} = (\lambda/n)^2/2\pi = 3.2 \times 10^{-14}\,\text{m}^2$; $\tau_{\text{sp}} = 3.8\,\mu\text{s}$; $\Delta\nu_{\text{nat}} = 1/2\pi\tau_{\text{sp}} = 4.2 \times 10^4\,\text{Hz}$; $\Delta\nu_0 = 1.1 \times 10^{14}\,\text{Hz}$; $\sigma_{21}/\sigma_{\text{nat}} = 1.5\Delta\nu_{\text{nat}}/\Delta\nu_0 = 5.7 \times 10^{-10}$; $\sigma_{21} = 1.8 \times 10^{-23}\,\text{m}^2$.
7.3 Two-dimensional gain medium
(a) $\alpha = 2{,}000\,\text{m}^{-1} = 20\,\text{cm}^{-1}$. (b) $G_1 - 1 = 1.5 \times 10^{-3}$.

Problems of Chap. 8

8.1 $\tau_{\text{p}} = (2nL/c)(1 - R)^{-1} = 6 \times 10^{-8}\,\text{s}$; $l_{\text{p}} = (c/n)\tau_{\text{p}} = 10\,\text{m}$; $(N_2 - N_1)_{\text{th}} = 1/l_{\text{p}}\sigma_{21} = 3 \times 10^{21}\,\text{m}^{-3}$.
8.2 $r_{\text{th}} = (N_2 - N_1)_{\text{th}}/\tau^*_{\text{rel}} = 8 \times 10^{26}\,\text{m}^{-3}\,\text{s}^{-1}$;
$Z_\infty = (10r_{\text{th}} - r_{\text{th}})\tau_{\text{p}} = 9r_{\text{th}}\tau_{\text{p}} = 4.3 \times 10^{19}\,\text{m}^{-3}$; $P_{\text{out}} = Z_\infty a_1 a_2 L h\nu/\tau_{\text{p}} = 9\,\text{W}$; $r_{\text{out}}/r = Z_\infty/(\tau_{\text{p}} r) = 9r_{\text{th}}/10r_{\text{th}} = 0.9$.
8.3 $(N_2 - N_1)_0 = 10 \times (N_2 - N_1)_{\text{th}} = 3 \times 10^{22}\,\text{m}^{-3}$; $\gamma_0 = b_{21}(N_2 - N_1)_0 = 1.3 \times 10^8\,\text{s}^{-1}$; $\kappa = 1/\tau_{\text{p}} = 1.6 \times 10^7\,\text{s}^{-1}$; $Z_0 = 1/a_1 a_2 L = 2 \times 10^7\,\text{m}^{-3}$; $t_{\text{on}} = 18\,\text{ns}$.
8.4 If the active medium has a smaller length than the resonator, the threshold condition is $(N_2 - N_1)_\infty = -\ln V/(2nL'\sigma_{21})$, where L' is the length of the active medium. It follows for that case that the gain coefficient $\alpha = (N_2 - N_1)\sigma_{21}$ has to be larger than the reciprocal of the effective photon path length in the crystal, $l'_{\text{p}} = l_{\text{p}}L'/L, \alpha \geq 1/l'_{\text{p}} = -\ln V/(2nL')$ or $2\alpha L' \geq -\ln V$. We find, with $G = \exp(2\alpha L')$, that the condition of gain, $GV \geq 1$, is fulfilled.

Problems of Chap. 10

10.1 $\nu_{110} = \nu_{101} = \nu_{011} = c/(\sqrt{2}a) = 21.2\,\text{GHz}$; $\nu_{111} = \sqrt{3}c/(2a) = 26\,\text{GHz}$.

10.2 Degeneracy of modes of a rectangular cavity resonator
(a) 3. (b) 2. (c) No degeneracy. (d) 2.

10.3 Density of modes of a cavity resonator

(a) $D(\nu) = 8\pi n^3\nu^2/c^3 = 1.7 \times 10^5\,\mathrm{m^{-3}\,Hz^{-1}}$ for $\nu = 4.3 \times 10^{14}$ Hz; n = 1.
(b) $1.0 \times 10^6\,\mathrm{m^{-3}\,Hz^{-1}}$.
(c) $8.3 \times 10^6\,\mathrm{m^{-3}\,Hz^{-1}}$.

10.5 Mode density on different scales

(a) $D(\nu)\mathrm{d}\nu = D(h\nu)\mathrm{d}(h\nu)$; $D(h\nu) = D(\nu)\mathrm{d}\nu/\mathrm{d}(h\nu) = D(\nu)/h$.
(b) $D(\nu)\mathrm{d}\nu = D(\omega)\mathrm{d}\omega$; $D(\omega) = D(\nu)\mathrm{d}\nu/\mathrm{d}\omega = D(\nu)/(2\pi)$.
(c) $D(\nu)\mathrm{d}\nu = D(\lambda)\mathrm{d}\lambda$; $D(\lambda) = D(\nu) \times \mathrm{d}\nu/\mathrm{d}\lambda = cD(\nu)/\lambda^2$.

10.6 $\nu = (c/2)\sqrt{a_1^{-2} + L^{-2}} = c/(2a_1)\sqrt{1 + a_1^2/L^2} \sim c/(2a_1)[1 + a_1/(2L^2)]$; $\mathrm{d}\nu/\mathrm{d}L \sim -(ca_1/(2L^3)$; $\mathrm{d}\nu/\nu \sim (a_1^2/L^2)\mathrm{d}L/L$.

10.7 Density of modes in free space
We consider a propagating wave $E = A\exp[\mathrm{i}(\omega t - \boldsymbol{kr})]$. We apply periodic boundary conditions: $E(x+L, y+L, z+L) = E(x, y, z)$ for each value of t; L is the length of the periodicity interval assumed to be equal in all spatial directions. This leads to the conditions: $\exp(\mathrm{i}k_x L) = 1$; $\exp(\mathrm{i}k_y L) = 1$; $\exp(\mathrm{i}k_z L) = 1$. It follows that: $k_x = l \times 2\pi/L$; $k_y = m \times 2\pi/L$; $k_z = n \times 2\pi/L$; $k^2 = (2\pi/L)^2(l^2+m^2+n^2)$, with $l, m, n = 0, \pm 1, \pm 2, \ldots$. We find, with $\omega = ck$, that $\omega^2 = (2\pi c/L)^2(l^2+m^2+n^2)$. The mode density in k space is $D(k) = (L^3/\pi^2)k^2$ and in ω space $D^*(\omega) = \omega^2 L^3/(\pi^2 c^3)$. With $D^*(\omega)d\omega = D^*(\nu)\mathrm{d}\nu$, we obtain $D^*(\nu) = (8\pi\nu^2/c^3)L^3$.

Problems of Chap. 11

11.1 Gaussian beam

(b) The ratio of the intensity of the radiation within the beam radius r_0 to the total intensity is

$$\int_0^{r0} 2\pi r \exp(-r^2/r_0^2)\mathrm{d}r \Big/ \int_0^{\infty} 2\pi r \exp(-r^2/r_0^2)\mathrm{d}r = 1 - 1/\mathrm{e} = 0.63.$$

We used $\int 2xe^{-x^2}\mathrm{d}x = -\exp(-x^2)$.
(c) $r_p = I_p/I_{\mathrm{tot}} = 1 - \exp(-r_p^2/r_0^2)$; $r_p/r_0 = \sqrt{-\ln(1-p)}$.
(d) $r_p = 1.52\,r_0$.
(e) $r_p = 1.73\,r_0$.
(f) A Taylor expansion of $p(r_p)$ with respect to r_p yields $p \sim 1 - r_p^2/r_0^2$.

11.2 $\theta_{0,\mathrm{u}} = \sqrt{2}\lambda/(\pi r_0) = 1.3 \times 10^{-3}$(=4.5 arc minutes)
11.3 The angle of divergence is $\Theta = 0.1(\sqrt{2}/\pi)\lambda/r_0 = 2 \times 10^{-6}$.

11.4 Density of photons in a Gaussian beam. The number of photons emitted per second by the laser is $P_{out}/(h\nu) = 6 \times 10^{14}\,\mathrm{s}^{-1}$. A detector of diameter D monitors radiation within the angle $\vartheta = D/d$, where d is the distance from the laser. The portion of radiation within the angle ϑ is $\sin^2\vartheta/\sin\theta^2 \sim \vartheta^2/\theta^2$. It follows for the number of photons per second:

(a) $\vartheta = 10^{-7}$; $\vartheta^2/\theta^2 = 2 \times 10^{-3}$; $3 \times 10^{11}\,\mathrm{s}^{-1}$.
(b) $\vartheta = 7 \times 10^{-11}$; $\vartheta^2/\theta^2 \sim 10^{-9}$; $\sim 10^6\,\mathrm{s}^{-1}$.

Problems of Chap. 13

13.1 Ultrashort pulses
(a) $t_p \sim 1/\Delta\nu_0 \sim 0.3\,\mathrm{ps}$. (b) $t_p \sim 10\,\mathrm{ps}$. (c) $t_p = 30\,\mathrm{ps}$.
13.2 Excited Ti^{3+} ions are collected during the round trip time $T = 10^{-8}\,\mathrm{s}$. The density of excited Ti^{3+} is $rT = 3 \times 10^{20}\,\mathrm{m}^{-3}$. Accordingly, the energy in a pulse is $rTa_1a_2L' \times h\nu = 19\,\mathrm{nJ}$ and the pulse power = 1.9 MW. The average power is 1.9 W.
13.3 $2 \times 10^7\,\mathrm{W}$.

Problems of Chap. 14

14.1 Helium–neon laser: line broadening and gain cross section

(a) $\Delta\nu_D = 1.5 \times 10^9\,\mathrm{Hz}$. (b) $\Delta\nu_c \sim 10^6\,\mathrm{Hz}$. (c) $\Delta\nu_{nat} = 1.6 \times 10^6\,\mathrm{Hz}$.
(d) $\Delta\nu_0 = 1.6 \times 10^7\,\mathrm{Hz}$. (e) $\sigma_{21}(\nu_0) = (1/c)h\nu B_{21}g_G(\nu_0) = 1.0 \times 10^{-16}\,\mathrm{m}^2$.

14.2 Helium–neon laser: threshold condition, output power and oscillation onset time

(a) $\tau_p = T/(1-R_1R_2) = 1.5\times10^{-7}\,\mathrm{s}$; $l_p = c\tau_p = 45\,\mathrm{m}$. $(N_2-N_1)_{th} = 1/(\sigma_{21}l_p) = 2 \times 10^{14}\,\mathrm{m}^{-3}$.
(b) $(N_2 - N_1)_{th} \times a_1a_2L = 2 \times 10^8$; number of excited neon atoms in the laser.
(c) $r_{th} = (N_2 - N_1)_{th} \times a_1a_2L/\tau^*_{rel} = 2 \times 10^{15}\,\mathrm{s}^{-1}$; $r_{out} \times a_1a_2L = 9r_{th}a_1a_2L = 2 \times 10^{16}\,\mathrm{s}^{-1}$; $P_{out} = r_{out} \times a_1a_2Lh\nu = 13\,\mathrm{mW}$.
(d) $Z_0 = (a_1a_2L)^{-1} = 5 \times 10^5\,\mathrm{m}^{-3}$; $Z_\infty = r_{out}\tau_p = 2.7 \times 10^9\,\mathrm{m}^{-3}$; $\alpha_{th} = (N_2 - N_1)_{th} \times \sigma_{21} = 0.02\,\mathrm{m}^{-1}$; $\gamma_{th} = c\alpha_{th} = 6 \times 10^6\,\mathrm{s}^{-1}$; $\kappa = 6 \times 10^6\,\mathrm{s}^{-1}$ (because $\kappa = \gamma_{th}$ at threshold); $\gamma_0 = 10\,\gamma_{th} = 6 \times 10^7\,\mathrm{s}^{-1}$; $t_{on} = \ln(Z_\infty/Z_0)/(\gamma_0 - \kappa) = 160\,\mathrm{ns}$.

14.3 Doppler effect

(a) $\nu = \nu_0(1 \pm \mathrm{v}/c)$; $\delta\nu = (2\mathrm{v}/c)\nu_0 = 3 \times 10^9\,\mathrm{Hz}$.
(b) The homogeneous width of the line due to $2 \rightarrow 1$ spontaneous transitions is $\Delta\nu = 1/(2\pi\tau_{rel}) = 1.6 \times 10^8\,\mathrm{Hz}$, where τ_{rel} is the lifetime of the lower laser level. This corresponds to a velocity range $-\mathrm{v}, \mathrm{v}$ or to $|\mathrm{v}| = c\delta\nu/\nu_0 = 100\,\mathrm{m\,s^{-1}}$.

(c) The gain curve has a minimum of a halfwidth of 160 kHz. In the line center, the gain is smaller than outside because outside (80 kHz away from the center) ions of the velocity +v contribute to gain in half a round trip and ions of the velocity −v contribute during the other half round trip. The Lamb dip can be used for frequency stabilization of a helium–neon laser.

14.4 CO_2 laser

(a) $\Delta\nu_D = 2\nu_0\sqrt{(2kT/mc^2)\ln 2} = 5.6\times 10^7$ Hz; $m = m_C + 2m_O = 44\,m_p = 7.3\times 10^{-26}$ kg; m_p = proton mass. $\sigma_{21}(\nu_0) = 0.94c^2A_{21}/(8\pi\nu^2\Delta\nu_D) = 1\times 10^{-21}\,\mathrm{m}^2$. $G_{th}V = 1$; $V = 0.7$; $G_{th} = 1.43$; $G_{th} = \exp[\alpha_{th}\times 2L]$; $\alpha_{th} = \ln(G_{th})/2L = 0.18\,\mathrm{m}^{-1}$. $(N_2 - N_1)_{th} = \alpha_{th}/\sigma_{21} = 1.8\times 10^{20}\,\mathrm{m}^{-3}$. $r_{th} = (N_2 - N_1)_{th}/\tau^*_{rel} = 4.5\times 10^{19}\,\mathrm{m}^{-3}\,\mathrm{s}^{-1}$; $P = ra_1a_2Lh\nu$; $r = P/a_1a_2Lh\nu) = 3\times 10^{25}\,\mathrm{m}^{-3}\,\mathrm{s}^{-1}$; $r \sim 10^6\,r_{th}$; the pump rate is about 10^6 times larger than the threshold pump rate.

(b) Because of the extremely long lifetime of the upper laser level with respect to spontaneous emission, the oscillation builds up as soon as the population difference exceeds $(N_2 - N_1)_{th}$. Stronger pumping then leads to generation of laser radiation. By collisions of the CO_2 molecules with each other, a quasithermal distribution of the populations of the different rotational levels of the excited state is maintained and the pump energy is converted to laser radiation (and energy of relaxation).

(c) We treat, for simplicity, the CO_2 gas as an ideal gas. At 273 K and normal pressure, an ideal gas (mole volume 22.4 l) contains 6×10^{23} molecules. This corresponds to about $3\times 10^{25}\,\mathrm{m}^{-3}$. We use this number for CO_2 at room temperature and normal pressure (1 bar). At a pressure of 10 mbar, the density of available CO_2 molecules is $3\times 10^{22}\,\mathrm{m}^{-3}$. At room temperature, excited CO_2 molecules are in different rotational states. About 1% of the molecules are in a particular rotational state. Thus, about 3×10^{21} molecules per m^3 are available for laser transitions. Assuming that half of the molecules are in an excited state we find that the density of molecules in a vibrational-rotational state is $1.5\times 10^{21}\,\mathrm{m}^{-3}$. This leads to $\alpha \sim 8\times\alpha_{th} \sim 1.4\,\mathrm{m}^{-1}$ and to a single path gain of $G_1 = \exp(\alpha L) = 4$.

(d) For a collision-broadened line, the gain cross section is $\sigma_{21} = c^2A_{21}/(8\pi\nu^2)\,g(\nu)$. With increasing pressure $g(\nu)$ broadens and the cross section in the line center, $\sigma_{21}(\nu_0) = c^2A_{21}/(8\pi\nu^2)\times 2/(\pi\Delta\nu_c)$, is inversely proportional to the gas pressure p. It follows that $\alpha(\nu_0)$ is independent of pressure above a pressure of about 10 mbar. At this pressure, $2\Delta\nu_c \sim \Delta\nu_D$, the gain coefficient we calculated is the maximum gain coefficient for the TEA and the high-pressure CO_2 laser.

In a TEA laser, the pulse duration of the radiation is about 200 ns. It is much larger than the duration (20 ns) of the electrical excitation pulse. During about 20 round trip transits of the radiation through the active medium, a fast redistribution occurs for the population of the levels involved in a laser oscillation. If we assume that about 1% of the excited molecules contribute to the laser oscillation, we find the pulse energy $E_{pulse} = 0.3$ J and the pulse power $E_{pulse}/t_{pulse} \sim 1$ MW.

(e) $Z_0 = (a_1 a_2 L)^{-1} = 10^4\,\mathrm{m}^{-3}$; $\tau_p = (2L/c)/(1-R) = 2.2 \times 10^{-8}\,\mathrm{s}$; $Z_\infty = P_{out}\tau_p/(a_1 a_2 L h\nu) = 1.1 \times 10^{21}\,\mathrm{m}^{-3}$; $t_{on} = T \ln(Z_\infty/Z_0)/(GV) = 24\,\mathrm{ns}$.

Problems of Chap. 15

15.2 σ_{21}(YAG)/σ_{21}(TiS) = $\Delta\nu_0$ (TiS)/$\Delta\nu_0$ (YAG) $\times\tau_{sp}$ (TiS)/τ_{sp} (YAG) $\times\lambda^2$(YAG)/λ^2 (TiS) ~ 10.

15.3 An N_2 molecule consists of two atoms, a molecule has a single vibrational frequency (and all molecules in an N_2 gas have the same frequency) while the Ti^{3+} ions in Al_2O_3 belong to a system with a large number of atoms (ions), namely of $N \sim 10^{25}\,\mathrm{m}^{-3}$, with $3N$ vibrational frequencies.

15.4 Laser tandem pumping

(a) $\eta = \eta_1 \times \eta_2 \times \eta_3 \times \eta_4 \sim 25\%$; $\eta_1 \sim 0.8$ (efficiency of a semiconductor laser); $\eta_2 \sim 0.8$ (Nd^{3+}:YVO_4 laser); $\eta_3 = 0.5$ (frequency doubling); $\eta_4 = 0.53\,\mu\mathrm{m}/0.68\,\mu\mathrm{m} = 0.78$.

(b) The Nd^{3+}:YVO_4 laser produces a laser beam with a small angle of aperture. Therefore, a column of a small diameter can be excited in titanium–sapphire allowing generation of a narrow laser beam. Direct pumping with a semiconductor laser beam, which has a large divergence, leads to excitation of the whole titanium–sapphire crystal. This results in strong heating.

Problems of Chap. 16

16.1 Dye laser

(a) GV; $V = 0.7$; $G_{th} = 1.43$; $G_{th} = \exp(\alpha_{th} L')$; $\alpha_{th} = (1/L') \ln G_{th} = 350\,\mathrm{m}^{-1}$; $(N_2 - N_1)_{th} = \alpha_{th}/\alpha_{21} = 3.5 \times 10^{21}\,\mathrm{m}^{-3}$.

(b) $r_{th} = (N_2 - N_1)_{th}/\tau^*_{rel} = 7 \times 10^{29}\,\mathrm{m}^{-3}\,\mathrm{s}^{-1}$; $r_{th} a_1 a_2 L' = 3 \times 10^{19}\,\mathrm{s}^{-1}$; $P_{out} = 9 r_{th} a_1 a_2 L' h\nu = 0.8\,\mathrm{W}$.

16.2 $P_{pulse} = 190\,\mathrm{MW}$; $P = 1.9\,\mathrm{mW}$.

Problems of Chap. 19

19.1 Acceleration energies

(a) $E = \sqrt{\lambda_w/\lambda}\, m_0 c^2 = 2.9\,\mathrm{MeV}$. (b) $E = 500\,\mathrm{MeV}$.

19.2 $E = \sqrt{\lambda_w(1+k^2)8 m_0 c^2)/2}/\sqrt{\lambda}$; $\mathrm{d}E = \mathrm{d}\lambda = \sqrt{\cdots}(-1)/(2\lambda^{3/2})$; $\mathrm{d}\nu/\nu = -\mathrm{d}\lambda/\lambda = 2\mathrm{d}E/E$.

Problems of Chap. 20

20.1 $\lambda_{\mathrm{deBroglie}} = h/(m\mathrm{v})$. The ratio is $m_0 = m_{\mathrm{e}} = 1/0.07 = 14$.
20.2 Number of states

(a) $\epsilon = 26\,\mathrm{meV} = 4.2 \times 10^{-21}\,\mathrm{J}$; $\mathrm{d}\epsilon = 1.6 \times 10^{-22}\,\mathrm{J}$; $D(\epsilon)\mathrm{d}\epsilon = 1.5 \times 10^{23}\,\mathrm{m}^{-3}$.
(b) $D^{2\mathrm{D}}(\epsilon)d\epsilon = 1.0 \times 10^{16}\,\mathrm{m}^{-2}$.
(c) $D^{1\mathrm{D}}(\epsilon)d\epsilon = 7.6 \times 10^{7}\,\mathrm{m}^{-1}$.

20.3 $\delta\nu = c/(2nL) = 41.6\,\mathrm{GHz}$.

Problems of Chap. 21

21.1 Wave vector of nonequilibrium electrons in GaAs
$k = (1/\hbar)\sqrt{2m_{\mathrm{e}}\epsilon} = 4.3 \times 10^{8}\,\mathrm{m}^{-1}$; $1.4 \times 10^{8}\,\mathrm{m}^{-1}$; $4.3 \times 10^{7}\,\mathrm{m}^{-1}$; $\lambda_{\mathrm{g}} = hc/E_{\mathrm{g}} = 870\,\mathrm{nm}$; gap wavelength (vacuum wavelength); $q_{\mathrm{p}} = 2n\pi/\lambda_{\mathrm{g}} = 2.6\times 10^{7}\,\mathrm{m}^{-1}$. It follows that q_{p} is small compared to k for the first two values of k.
(b) $q_{\mathrm{p}} = k$ for $\epsilon = \hbar^2 q_{\mathrm{p}}^2/(2m_{\mathrm{e}}) = 0.4$ meV.
21.2 $q_{\mathrm{p}} \ll k_1 = (1/\hbar)\sqrt{2m_{\mathrm{e}}\epsilon_{\mathrm{c}}}$; $\epsilon_{\mathrm{c}} \gg \hbar^2 q_{\mathrm{p}}^2/(2m_{\mathrm{e}}) = 0.4\,\mathrm{meV}$; $q_{\mathrm{p}} \ll k_2 = (1/\hbar)\sqrt{2m_{\mathrm{h}}\epsilon_{\mathrm{v}}}$; $\epsilon_{\mathrm{v}} \gg \hbar^2 q_{\mathrm{p}}^2/(2m_{\mathrm{h}}) = 0.4/6\,\mathrm{meV}$.
21.3 Electron and holes in an undoped GaAs quantum film in thermal equilibrium

(a) $E_{\mathrm{Fe}} = E_{\mathrm{Fh}} = E_{\mathrm{F}}$.
(b) Since $E_{\mathrm{Fe}} = E_{\mathrm{c}} + \epsilon_{\mathrm{Fe}} = E_{\mathrm{F}}$ and $E_{\mathrm{Fv}} = E_{\mathrm{V}} - \epsilon_{\mathrm{Fh}} = E_{\mathrm{F}}$, it follows that $E_{\mathrm{c}} + \epsilon_{\mathrm{Fe}} = E_{\mathrm{v}} - \epsilon_{\mathrm{Fh}}$ and $-\epsilon_{\mathrm{Fe}} - \epsilon_{\mathrm{Fh}} = E_{\mathrm{g}}$. The gap energy is positive because ϵ_{Fe} and ϵ_{Fh} have negative signs for small electron and hole densities.
(c) $N^{2\mathrm{D}}_{\mathrm{thermal}} = \sqrt{D^{2\mathrm{D}}_{\mathrm{e}} D^{2\mathrm{D}}_{\mathrm{h}} kT \exp[-E^{2\mathrm{D}}_{\mathrm{g}}/(2kT)]} = 2 \times 10^{4}\,\mathrm{m}^{-2}$; with $kT = 26\,\mathrm{meV}$; $E^{2\mathrm{D}}_{\mathrm{g}} = 1.4\,\mathrm{eV}$; $N^{2\mathrm{D}}_{\mathrm{thermal}}$ is by many orders of magnitude smaller than $N^{2\mathrm{D}}_{\mathrm{tr}} = 1.4 \times 10^{16}\,\mathrm{m}^{-2}$.

Problems of Chap. 22

22.1 $N^{2\mathrm{D}} \sim 3.3 \times 10^{16}\,\mathrm{m}^{-2}$; $\epsilon_{\mathrm{Fv}} \sim 6\,\mathrm{meV}$; $\epsilon_{\mathrm{Fc}} \sim 84\,\mathrm{meV}$; $E_{\mathrm{Fc}} - E_{\mathrm{Fv}} \sim 78\,\mathrm{meV}$.
22.2 Quantum well laser
Average photon path length $l_{\mathrm{p}} = 0.9L = 0.45\,\mathrm{mm}$; $N^{2\mathrm{D}}_{\mathrm{th}} - N^{2\mathrm{D}}_{\mathrm{tr}} = (1/3) \times 1.3(\sigma_{\mathrm{eff}} l_{\mathrm{p}}/a_1)^{-1} = 1.3 \times 10^{15}\,\mathrm{m}^{-2}$: $j = 3N^{2\mathrm{D}}_{\mathrm{th}} e/\tau_{\mathrm{sp}} = 3.1 \times 10^{6}\,\mathrm{A\,m}^{-2}$; $I = 0.3\,\mathrm{A}$.
22.3 Photons in a quantum well laser

(a) $h\nu = 1.42\,\mathrm{eV} = 2.3 \times 10^{-19}\,\mathrm{J}$; $\tau p = 5 \times 10^{-12}\,\mathrm{s}$; $Z a_1 a_2 L h\nu/\tau_{\mathrm{p}} = 2P_{\mathrm{out}}$; $Z = 2.0 \times 10^{18}\,\mathrm{m}^{-3}$.
(b) $Z_{\mathrm{tot}} = Z a_1 a_2 L = 10^{5}$.

(c) $N_{\text{tot}} \sim N_{\text{tr}}^{2\text{D}} a_2 L = 1.2 \times 10^9$; Z_{tot} is much smaller than N_{tot}.

Problems of Chap. 23

23.1 $q_{\text{p}} = 2\pi n/\lambda_{\text{g}} = 2.6 \times 10^7\,\text{m}^{-1}$; $\pi/(a/2) = 1.1 \times 10^{10}\,\text{m}^{-1} \gg q_{\text{p}}$.
23.2 Indirect gap semiconductor

(a) $h\nu = E_{\text{g}}^{\text{ind}} + \hbar\omega_{\text{phonon}}$; $0 = 2\pi/a + q_{\text{phonon}}$.
(b) $E_{\text{g}}^{\text{ind}} = h\nu + \hbar\omega_{\text{phonon}}$; $2\pi/a = q_{\text{phonon}}$.

Problems of Chap. 24

24.1 GaN quantum well

(a) $D^{2\text{D}}(\text{GaN}) = 3 \times D^{2\text{D}}$ (GaAs).
(b) To obtain the same occupation number difference, the nonequilibrium electron density has to be larger by a factor of three. If the Einstein coefficient B_{21} has the same value, the gain is by a factor $\nu_2 = \nu_1$ larger for GaN (ν_2 = frequency of a laser with a GaN-based quantum well and ν_1 = frequency of a laser with a GaAs-based quantum well).

Problems of Chap. 26

26.1 We consider a two-dimensional plane wave,

$$\Psi = \Psi_0 \text{e}^{\text{i}[\boldsymbol{kr} - (E/\hbar)t]},$$

where $\boldsymbol{k}$ and $\boldsymbol{r}$ are two-dimensional vectors within the plane. We apply periodic boundary conditions for the x and y direction, $k_x L = m \times 2\pi$ and $k_y L = \text{n} \times 2\pi$, where m and n are integers and L is the periodicity length (for the directions along x and y). In k space, the area of a ring of radius k and width dk is $2\pi k\,\text{d}k$. The area containing one k point is $(2\pi/L)^2$. The density of k states is $\bar{D}^{2\text{D}}(k) = k\text{d}k/(2\pi)L^2$. The density of states in the energy space follows from the relation $\bar{D}^{2\text{D}}(\epsilon)\text{d}\epsilon = 2\bar{D}^{2\text{D}}(k)\text{d}k$, where the factor 2 takes into account that there are two spin directions for an electron. Making use of the dispersion relation $\epsilon = \hbar^2 k^2/(2m)$ and of $\text{d}\epsilon/\text{d}k = \hbar^2 k/m$ we find $\bar{D}^{2\text{D}} = 2\bar{D}^{2\text{D}}(k)\ \text{d}k/\text{d}\epsilon = m/(\pi\hbar^2)L^2$ and $D^{2\text{D}}(\epsilon) = m/(\pi\hbar^2)$ (=density of states per unit of energy and unit of area).

26.2 Subpicosecond quantum well laser

(a) Yes. It is in principle possible to have a gain profile of a halfwidth of about 50 meV. The necessary frequency width is $\Delta\nu_0 = \Delta E_0/h = 12\,\text{THz}$; $t_{\text{pulse}} \sim 1/\Delta\nu_0 \sim 10^{-13}\,\text{s} = 100\,\text{fs}$.
(b) The pulse separation is $T = 2nL/c = 2 \times 10^{-11}$ s. Most likely, the number of photons available in a pulse is not sufficient for the saturation of a semiconductor reflector.
(c) If an external reflector is used, an active Q-switching technique should be applicable and the generation of subpicosecond pulses should be possible.

Problems of Chap. 27

27.1 The periodic boundary condition for a one-dimensional system yields the k values $k = s \times 2\pi/L$. The density of states in k space is $\bar{D}^{1\text{D}}(k) = 2 \times L/(2\pi)$ because there are two states ($\pm k$) in an interval $2\pi/L$.
$\epsilon = \hbar^2 k^2/2m$; $\text{d}\epsilon/\text{d}k = (\hbar/m)k = \hbar\sqrt{2\epsilon/m}$; $\bar{D}^{1\text{D}}(\epsilon)\,\text{d}\epsilon = 2\bar{D}^{1\text{D}}(k)\,\text{d}k$;
$\bar{D}^{1\text{D}}(\epsilon) = (2L/\pi)\,\text{d}k/\text{d}\epsilon = L/(\pi\hbar)\sqrt{2m/\epsilon}$ and $D^{1\text{D}} = (\pi/\hbar)\sqrt{2m/\epsilon}$.
27.2 $f_2 = 0$; $f_1 = 1$; $G - 1 = (n/a_2 c)h^2 \text{v} B_{21} n^{1\text{D}}_{0,\text{nat}}/\Delta\epsilon_{\text{nat}} = 0.26$.
27.3 $I_{\text{th}} = 0.8 \times Ne/\tau_{\text{sp}} = 0.75$ nA.
27.4 Bipolar laser as two-level laser

(a) $D_{\text{r}}(E)$; $E = E_{\text{g}} + \epsilon =$ pair level energy; $\epsilon = \epsilon_2 + \epsilon_1$; ϵ_2 and ϵ_1 are the energies of the electron and the hole that constitute a radiative pair. $D_{\text{r}}(\epsilon)$ is the 3D, 2D, 1D or 0D density of states, depending on the dimensionality of the semiconductor.
(b) The gain characteristic H_{21} is proportional to $f_{\text{p}} - \bar{f}_{\text{p}}$, where f_{p} is the probability that the pair level is occupied and $\bar{f}_{\text{p}}$ is the probability that the pair level is empty; $f_{\text{p}} - \bar{f}_{\text{p}} = 2f_{\text{p}}$.
(c) $f_{\text{p}} - \bar{f}_{\text{p}} > 1/2$, since the absorption coefficient is proportional to $\bar{f}$ and the stimulated emission to f.
The condition must correspond to the condition $f_2 - f_1 = 0$ or $f_2 - f_1 = f_{\text{e}} - (1 - f_{\text{h}} = 0$. It follows that $f_{\text{p}} - \bar{f}_{\text{p}} = f_2 - f_1 + 1/2 = f_{\text{e}} + f_{\text{h}} - 1/2$. f_2 and f_1 are the occupation numbers for the electrons in the conduction band and the valence band and f_{e} and f_{h} are, in the electron-hole picture, the occupation numbers for the electrons and holes, respectively (see Sect. 21.10).

27.5 Laser operated with a gain medium with a naturally broadened line

(a) To the knowledge of the author: no.

An electron-hole pair can be considered as an occupied single electron pair level (=occupied upper laser level). The lower laser level is the vacuum level. The lifetime of the vacuum level is infinitely large (or large compared to the spontaneous lifetime of the pair). Thus, we have no lifetime broadening of the lower laser level, supposed that the population of the levels of the electron and the hole, which constitute a radiative electron-hole pair, occurs sufficiently fast.

(b) $\sigma_{21} = (\lambda/n)^2/2\pi = 9 \times 10^{-15}\,\text{m}^2$ for $n = 3.6$.

Problems of Chap. 31

31.3

(a) $A_\infty = 0.18\,\text{V}$.

(b) $\gamma_0 = a/C = 10 \times 10^9\,\text{s}^{-1}$; $\kappa = G/C = 0.77 \times 10^9\,\text{s}^{-1}$; $\gamma_0 - \kappa = 2.3 \times 10^8\,\text{s}^{-1} \ll \omega_0 = 6 \times 10^{10}\,\text{s}^{-1}$.

31.4 Van der Pol equation

(a) The equation follows from (31.47) by introducing $\epsilon = (\gamma_0 - k)/\omega_0$ and $y = U\sqrt{(3\omega_0 b/C)(\gamma_0 - k)^{-1}}$.

(b) The ansatz $y = A \cos \tau$ leads to $A = 2$.

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Index

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