

# Bibliography

1. E. Cartan, *Leçons sur les Invariants Intégraux* (Hermann, Paris, 1922)
2. V.I. Arnold, *Mathematical Methods of Classical Mechanics*, 2nd edn. (Springer, New York, 1989)
3. A.A. Kirillov, *Elements of the Theory of Representations* (Springer, Berlin, 1976)
4. V.P. Maslov, M.V. Fedoruk, *Semiclassical Approximation in Quantum Mechanics* (D. Reidel Publishing Company, Dordrecht, 1981)
5. A.T. Fomenko, *Symplectic Geometry* (Gordon and Breach, New York, 1988)
6. J.M. Souriau, *Structure des systèmes dynamiques* (Dund, Paris, 1970)
7. J.E. Marsden, R.H. Abraham, *Foundations of Mechanics*, 2nd edn. (Benjamin-Cummings Publishing Company, Inc., Reading, 1978)
8. P.A.M. Dirac, *Can. J. Math.* **2**, 129 (1950); *Lectures on Quantum Mechanics* (Yeshiva University, New York, 1964)
9. A.A. Slavnov, L.D. Faddeev, *Introduction in Quantum Theory of Gauge Fields* (Nauka, Moscow, 1978)
10. D.M. Gitman, I.V. Tyutin, *Quantization of Fields with Constraints* (Springer, Berlin, 1990)
11. M. Henneaux, C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, 1992)
12. H. Goldstein, *Classical Mechanics*, 2nd edn. (Addison-Wesley, Reading, 1980)
13. L.D. Landau, E.M. Lifshits, *Mechanics* (Pergamon Press, Oxford, 1976)
14. F.R. Gantmacher, *Lectures on Analytical Mechanics* (MIR, Moscow, 1970)
15. S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972)
16. L.D. Landau, E.M. Lifshits, *The Classical Theory of Fields* (Pergamon Press, Oxford, 1980)
17. W. Pauli, *Theory of Relativity* (Pergamon Press, Oxford, 1958)
18. P.G. Bergmann, *Introduction to the Theory of Relativity* (Academic Press, New York, 1967)
19. V.A. Ugarov, *Special Theory of Relativity* (Mir Publishers, Moscow, 1979)
20. R. Feynman, P. Leighton, M. Sands, *The Feynman Lectures on Physics: Commemorative Issue*, vol. 2 (Addison-Wesley, Reading, 1989)
21. H. Hertz, *The Principles of Mechanics Presented in a New Form* (Dover Publications, New York, 1956)
22. P.S. Wesson, *Five-Dimensional Physics: Classical and Quantum Consequences of Kaluza-Klein Cosmology* (World Scientific, Singapore, 2006)
23. V.S. Vladimirov, *Equations of Mathematical Physics*, 3rd edn. (Izdatel'stvo Nauka, Moscow, 1976), 528p. In Russian. (English translation: *Equations of Mathematical Physics*, ed. by V.S. Vladimirov (M. Dekker, New York, 1971))
24. A.A. Deriglazov, *Phys. Lett. B* **626** 243–248 (2005)
25. W. Ehrenberg, R.E. Siday, *Proc. R. Soc. Lond. B* **62**, 8 (1949)

26. Y. Aharonov, D. Bohm, Phys. Rev. **115**, 485 (1959)
27. E. Schrödinger, Ann. Phys. **81**, 109 (1926); See also letters by Schrödinger to Lorentz in: K. Przibram, Briefe zur Wellenmechanik (Wien, 1963)
28. H. von Helmholtz, J. Math. C, 151 (1886)
29. K.S. Stelle, Phys. Rev. **D16**, 953–969 (1977)
30. R.P. Woodard, *How Far Are We from the Quantum Theory of Gravity?* arXiv:0907.4238 [gr-qc]
31. M.V. Ostrogradsky, Mem. Ac. St. Petersburg **VI 4**, 385 (1850)
32. D. Bohm, Phys. Rev. **85**, 166, 180 (1952)
33. F. Mandl, *Introduction to Quantum Field Theory* (Interscience Publishers Inc., New York, 1959)
34. W. Yourgrau, S. Mandelstam, *Variational Principles in Dynamics and Quantum Theory* (Pitman/W. B. Sanders, London/Philadelphia, 1968)
35. R.M. Wald, *General Relativity* (The University of Chicago Press, Chicago/London, 1984)
36. P.A.M. Dirac, *Quantum Mechanics*, 4th edn. (Oxford University Press, London, 1958)
37. J.D. Bjorken, S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, New York, 1964)
38. P.J. Olver, *Applications of Lie Groups to Differential Equations* (Springer, New York, 1986)
39. J.L. Anderson, P.G. Bergmann, Phys. Rev. **83**, 1018 (1951); P.G. Bergmann, I. Goldberg, Phys. Rev. **98**, 531 (1955)
40. A.A. Deriglazov, Phys. Lett. A **373** 3920–3923, (2009)
41. D.J. Griffiths, *Introduction to Quantum Mechanics*, 2nd edn. (Pearson Prentice Hall, Upper Saddle River, 2005)
42. F.A. Berezin, M.S. Marinov, JETP Lett. **21**, 320 (1975); Ann. Phys. **104**, 336 (1977)
43. V.A. Borokhov, I.V. Tyutin, Phys. At. Nucl. **61**, 1603 (1998); Phys. At. Nucl. **62**, 10 (1999)
44. D.M. Gitman, I.V. Tyutin, Int. J. Mod. Phys. A **21**, 327 (2006)
45. A.A. Deriglazov, K.E. Evdokimov, Int. J. Mod. Phys. A **15**, 4045 (2000)
46. A.A. Deriglazov, J. Math. Phys. **50**, 012907 (2009)
47. M. Henneaux, C. Teitelboim, J. Zanelli, Nucl. Phys. B **332**, 169 (1990)
48. A.A. Deriglazov, Z. Kuznetsova, Phys. Lett. B **646**, 47 (2007)
49. S. Weinberg, *The Quantum Theory of Fields*, vol. 1 (Cambridge University Press, Cambridge, 1995)
50. S. Weinberg, *Lectures on Quantum Mechanics*, vol. 1 (Cambridge University Press, Cambridge, 2013)
51. J. Frenkel, Die elektrodynamik des rotierenden elektrons. Z. Phys **37**, 243 (1926)
52. L.H. Thomas, The kinematics of an electron with an axis. Philos. Mag. J. Sci. **3** S.7, No.13, 1 (1927)
53. M. Mathisson, Neue mechanik materieller systeme. Acta Phys. Polon. **6**, 163 (1937); Republication: Gen. Rel. Grav. **42**, 1011 (2010)
54. A. Papapetrou, Spinning test-particles in general relativity. I. Proc. R. Soc. Lond. **A 209**, 248 (1951)
55. W.M. Tulczyjew, Motion of multipole particles in general relativity theory binaries. Acta Phys. Polon. **18**, 393 (1959)
56. W.G. Dixon, A covariant multipole formalism for extended test bodies in general relativity. Nuovo Cimento **34**, 317 (1964)
57. F.A.E. Pirani, Acta. Phys. Polon. **15**, 389 (1956)
58. H.C. Corben, *Classical and Quantum Theories of Spinning Particles* (Holden-Day, San Francisco, 1968)
59. A.O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (MacMillan, New York, 1964)
60. I.B. Khriplovich, A.A. Pomeransky, Equations of motion of spinning relativistic particle in external fields. J. Exp. Theor. Phys. **86**, 839 (1998)
61. I.L. Buchbinder, S.M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace* (Institute of Physics Publishing, Bristol and Philadelphia, 1995/1998)

62. R.D. Pisarski, Theory of curved paths. *Phys. Rev.* **D 34**, 670 (1986)
63. A.A. Deriglazov, A. Nersessian, Rigid particle revisited: extrinsic curvature yields the Dirac equation. *Phys. Lett.* **A 378**, 1224–1227 (2014)
64. E. Schrödinger, *Sitzunger. Preuss. Akad. Wiss. Phys.-Math. Kl.* **24**, 418 (1930)
65. R.P. Feynman, *Quantum Electrodynamics* (W.A. Benjamin, New York, 1961)
66. M.H.L. Pryce, The mass-centre in the restricted theory of relativity and its connexion with the quantum theory of elementary particles. *Proc. R. Soc. Lond. Ser. A Math. Phys. Sci.* **195**, 62 (1948)
67. L.L. Foldy, S.A. Wouthuysen, On the Dirac theory of spin 1/2 particles and its non-relativistic limit. *Phys. Rev.* **78**, 29 (1950)
68. A.A. Deriglazov, A.M. Pupasov-Maksimov, Geometric constructions underlying relativistic description of spin on the base of non-grassmann vector-like variable. *SIGMA* **10**, 012 (2014)
69. E. Wigner, On unitary representations of the inhomogeneous Lorentz group. *Ann. Math.* **40**(1), 149 (1939)
70. V. Bargmann, E.P. Wigner, Group theoretical discussion of relativistic wave equations. *Proc. Natl. Acad. Sci. USA* **34**(5), 211 (1948)
71. A.J. Hanson, T. Regge, The relativistic spherical top. *Ann. Phys.* **87**(2), 498 (1974)
72. S.S. Stepanov, Thomas precession for spin and for a rod. *Phys. Part. Nucl.* **43**, 128 (2012)
73. J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975)
74. A. Staruszkiewicz, Fundamental relativistic rotator. *Acta Phys. Polon. B Proc. Suppl.* **1**, 109 (2008)
75. A.A. Deriglazov, A.M. Pupasov-Maksimov, Frenkel electron on an arbitrary electromagnetic background and magnetic Zitterbewegung. *Nucl. Phys.* **B 885**, 1 (2014)
76. A. Trautman, Lectures on general relativity. *Gen. Rel. Grav.* **34**, 721 (2002)
77. A.A. Deriglazov, A. Pupasov-Maksimov, Relativistic corrections to the algebra of position variables and spin-orbital interaction. *Phys. Lett.* **B 761**, 207 (2016)
78. A.A. Deriglazov, A.M. Pupasov-Maksimov, Lagrangian for Frenkel electron and position's non-commutativity due to spin. *Eur. Phys. J. C* **74**, 3101 (2014)
79. R.P. Feynman, M. Gell-Mann, Theory of the Fermi interaction. *Phys. Rev.* **109**, 193 (1958)
80. W. Guzmán Ramírez, A.A. Deriglazov, A.M. Pupasov-Maksimov, Frenkel electron and a spinning body in a curved background. *J. High Energy Phys.* **1403**, 109 (2014)
81. W.G. Ramirez, A.A. Deriglazov, Lagrangian formulation for Mathisson-Papapetrou-Tulczyjew-Dixon (MPTD) equations. *Phys. Rev.* **D 92**, 124017 (2015)
82. A.A. Deriglazov, Lagrangian for the Frenkel electron. *Phys. Lett. B* **736**, 278 (2014)
83. J. Magueijo, L. Smolin, Gravity's rainbow. *Class. Quantum Gravity* **21**, 1725 (2004)
84. J.B. Conway, *A Course in Functional Analysis* (Springer, Berlin, 1990)

# Index

## A

### Action

- of first order, 129
- Hamiltonian, 121
- Lagrangian, 235, 240

## B

### Boost

- of Galileo, 14
- of Lorentz, 25

### Bracket

- of Dirac, 296
- fundamental, 102
- Poisson, 102
- Poisson non canonical, 118

## C

Canonical quantization, 297

Cauchy problem, 2

Center of mass, 259

Change of variables, 92

Charge, 110

- functionally independent, 111

Christoffel symbols, 198

Closed two-form, 121

Configuration space, 37

Conjugate momenta, 98

### Connection

- affine, 206
- Riemann, 198, 208

Conservation law, 110

- of angular momentum, 5
- of energy, 4
- of momentum, 7

Conservative force, 3

Conserved quantity, 103, 110

### Constraints

- equivalent, 289
- of first class, 289
- holonomic, 54
- kinematic, 54
- primary, 273, 278
- secondary, 274
- of second class, 289
- of second stage, 274, 280

Continuity equation, 128

Contraction of a moving body, 27

Conversion of second-class constraints,  
343

Coordinate basis of tangent space, 217

### Coordinates

- of center of mass, 44
- generalized, 37
- special, 292
- transformations, 240

Coulomb law, 67

Covariance, 13

Covariant derivative, 206

- along the curve, 208
- along the field, 208

Covariant equation, 210, 211

Curve, 204

## D

Decrease of order of a system, 91–92

Dipole electric moment, 374, 390

Dirac constraints, 165

Distance, 220

Dynamical invariant, 110

**E**

- Energy
  - as a conjugated momentum, 307
  - kinetic, 3
  - potential, 3
  - total, 4
- Equations
  - of continuity, 69
  - of Euler-Lagrange, 33
  - Hamiltonian, 98
  - invariant, 19
  - of Klein-Gordon, 422
  - Lagrangian, 33, 95
  - of Maxwell, 66
  - of normal form, 36
  - of Pauli, 368
  - of second stage, 280
  - of third-stage, 280
- Euler-Lagrange equations, 95
- Event, 19
  - space-like, 21

**F**

- Field, 46
  - central, 5
  - potential, 3
- First integral, 110
- First order form of a system, 92
- Fixation of gauge, 294

**G**

- Galileo boosts, 244
- Gauge
  - condition, 77, 295
  - of Lorentz, 77
  - symmetry, 321
  - unitary, 77
- Generalized velocities, 95
- General solution, 108
- Generating function, 158, 159, 168
  - of canonical transformation, 140
- Generator
  - of infinitesimal canonical transformation, 154
  - of local symmetry, 321
  - of Lorentz boost, 24
  - of rotation, 16
- Geodesic equation
  - in canonical parametrization, 197
  - covariant under reparametrizations, 214
  - in dynamical parametrization, 224

- Geodesic line, 197, 214
  - in canonical parametrization, 216
  - in natural parametrization, 216
  - reparametrization covariant equation, 214
- Geometry
  - rainbow, 401
- Group
  - of Galileo, 246
  - of Lie, 239
- Gyromagnetic ratio, 355

**H**

- Hamiltonian, 98, 278
  - complete, 273, 278
  - equations generalized, 118
  - extended, 332
- Hamiltonization procedure, 99
- Hamilton-Jacobi equation, 170
- Hessian matrix, 35
- Homogeneity
  - in space, 259
  - in time, 244

**I**

- Inertial frame, 12
- Integral invariant
  - of first order, 178
  - of the Poincaré, 179, 184
  - of the Poincaré-Cartan, 179
  - universal, 178
- Integral of motion, 110
- Interaction
  - minimal, 82
  - non minimal, 84
- Interval
  - of Minkowski, 20
  - time-like, 21
- Invariance
  - of action, 236
  - of the Poisson bracket, 139

**J**

- Jacobi matrix, 93

**K**

- Kaluza-Klein theory, 65
- Kepler's problem, 5, 245

**L**

Lagrange bracket, 150  
 Lagrangian  
   action functional, 31  
   extended, 330  
   function, 29  
   multiplier, 273  
   nonsingular, 95  
   singular, 221  
 Law  
   of Biot-Savart, 68  
 Legendre transformation, 98  
 Leibnitz rule, 207  
 Lie group, 239  
 Line, 204  
   geodesic, 65  
   integral, 177  
   length, 219  
 Local coordinate system, 200  
 Lorentz gauge, 77

**M**

Magnetic moment, 355  
 Manifold  
   of Lorentz, 225  
 Matrices  
   of Dirac, 371  
 Maupertuis principle, 189  
 Metric, 197, 205  
   of configuration space, 39  
   effective, 388  
   Euclidean, 37  
   induced, 217  
   of Minkowski, 19  
 Michelson-Morley experiment, 18  
 Moment  
   gravimagnetic, 402

**N**

Newton's law  
   first, 1, 12  
   third, 7  
 Noether charge, 251  
 Noether identities, 252  
 Normal form of a system, 92

**O**

Observable, 277, 292  
 Off-shell, 110

On-shell, 110  
 Operator of spin  
   non relativistic, 355  
 Operators  
   of Pryce, 423, 432

**P**

Parallel transport, 209  
 Particle  
   relativistic, 80, 222, 237  
 Pauli equation, 368  
 Pauli matrices, 355  
 Pendulum  
   of Thomson-Tait, 56  
 Phase space, 98  
   extended, 129, 278  
 Precession  
   frequency, 368  
   of Thomas, 381  
 Principle  
   of Galilean relativity, 13  
   of Hamilton, 30  
   of least action, 30, 189  
   of Maupertuis, 192, 194, 196  
   of Newton's determinism, 3  
   of special relativity, 18  
 Probability  
   relativistic invariant, 371  
 Projector, 222  
 Proper time, 29

**R**

Reference frame, 12  
 Reparametrization independence, 210  
 Reparametrization invariance, 41, 220  
 Riemann connection, 208  
 Riemann space, 197, 205  
 Rigid particle, 382  
 Routhian, 130

**S**

Scalar function, 21, 202  
 Scalar product, 205  
 Schrödinger equation, 122  
 Second Newton law, 212  
 Second quadratic form of a surface, 218  
 Separation of variables, 191  
 Singular theory  
   degenerate, 277  
   non-degenerate, 277

- Solution
  - complete, 170
  - general, 2
- Space
  - configuration, 95
  - configuration-velocity, 95, 282
  - of Minkowski, 19
- Spin
  - of Bargmann-Michel-Telegdi, 377
  - tensor of Frenkel, 374
- Spinor
  - adjoint, 370
  - of Dirac, 369
  - of Weyl, 426
- Standing wave, 52
- String tension, 46
- Stuckelberg field, 350
- Symbol
  - of Levi-Civita, 21, 376
- Symmetry
  - of action, 39, 242
  - global, 39, 78, 238
  - infinitesimal, 254, 265
  - infinitesimal trivial, 255
  - local, 41, 78, 238, 262
  - local or gauge, 78
  - manifest, 62
  - of spin-plane, 357
  - variational, 242
- Symmetry transformation of action, 236
- Symplectic matrix, 103
- System of equations
  - equivalent, 91
  - Hamiltonian, 94
  - normal, 2
  
- T**
- Tangent space, 202
- Tangent vector, 205
- Tensor, 201
  - of Riemann curvature, 402
- Theory
  - degenerate, 277
  - non singular, 35
  - singular, 35
  - of special relativity, 18
- Total
  - angular momentum, 260
  - energy, 259
  - momentum, 259
- Trajectory, 190
  
- Transformation
  - of the action, 243
  - canonical, 96, 116
  - canonical free, 139, 156
  - canonical infinitesimal, 153
  - canonoid, 116
  - contact, 137
  - of coordinates, 201
  - of form of function, 254
  - of Galileo, 12
  - infinitesimal, 251, 253
  - of Lorentz, 20
  - phase-space, 150
  - of phase-space, time-dependent, 113
  - of Poincaré, 20
  - univalent, 117
- Transition function, 178
- Tube, 175
  
- U**
- Unitary gauge, 77
  
- V**
- Variables
  - auxilliary, 63
  - non physical, 277
  - physical, 277
  - unphysical, 63
  - weak, 299
- Variational problem, 31
  - dynamical, 306
  - non dynamical, 306
- Variation in form, 155
- Variation of functional, 32
- Vector
  - of center of mass, 43
  - contravariant, 21
  - covariant, 21
  - Hamiltonian, 7
  - of Pauli-Lubanski, 376
  - of Runge-Lenz, 7
  - of spin, four-dimensional, 376
- Vector field
  - conservative, 110
  - contravariant, 201
  - covariant, 202
  - covariantly constant, 207
  - divergenceless, 110
  - Hamiltonian, 109
  - parallel, 209

Vector potential, 71, 74  
Vector product, 2

**W**

Wave equation, 50  
Wave function, 122

Whittaker's equations, 193  
Worldline, 19, 27  
World time, 226

**Z**

Zitterbewegung, 371