Appendix A
Practical Questions Related to AHP Modeling

1. **What is the best kind of decision problems for AHP?**

While AHP can be used in a wide number of decision-making problems, AHP is traditionally used in selection, prioritization, and forecasting. AHP assumes that the decision-makers know or will come up with, individually or collectively, implicitly or explicitly, the criteria or objectives and alternatives associated with the decision. AHP is also particularly useful for situations in which we have both tangible and intangible criteria to consider in the decision.

2. **How many hierarchies are needed to perform AHP Analysis?**

When working with a single type of stakeholder, one hierarchy may be enough (or 4 if you perform a BOCR analysis); however, when working with different types of stakeholders, a hierarchy for each perspective may be needed. In any case, there are no rules about the number of hierarchies to analyze a problem.

3. **How many criteria are needed for the AHP hierarchy?**

Saaty's scale intensity, as well as AHP as a whole, is based on the findings from cognitive science that suggest that a person's working memory capacity is in the order of 7 ± 2; that is between 5 and 9 elements. This suggests that 5–9 criteria should be the ideal. If you have more than that you may consider grouping some of them into an overall criterion and creating sub-criteria for it (e.g., cost can group sub-criteria such as acquisition cost and maintenance cost). An important step in the process, which is not usually properly addressed, is the importance of modeling the problem with a correct hierarchy. If the criteria are incomplete or they are not clearly defined and different from each other, the model will not be a good fit for the decision at hand and any decision obtained this way will be sub par.

4. **How many levels should an AHP hierarchy have?**

The same rationale from the previous question can be applied here. While there is not a limit to the number of levels in a hierarchy, you may want to keep it within the 7 ± 2 limit, if possible. One way to do this is by decomposing the problem into a set of hierarchies rather than using a big gigantic hierarchy.

E. Mu and M. Pereyra-Rojas, *Practical Decision Making*,
SpringerBriefs in Operations Research, DOI 10.1007/978-3-319-33861-3

5. **Does AHP eliminate Cognitive Bias problems?**

While cognitive biases may certainly affect the judgments we make when comparing elements in the model, the visibility and transparency of the decision-making process allows us to detect potential biases much more easily, in particular during the sensitivity analysis.

6. **In a nutshell, what are the advantages of using AHP?**

In terms of advantages, the most important ones are: (a) the ability of structuring a problem in a way that is easily manageable, (b) making the decision criteria explicit and the decision-making process transparent as a whole, (c) deriving priorities through a rigorous mathematical process using ratio scales, (d) allowing measuring and comparison of tangible and intangible elements and (e) allowing easy sharing of the decision-making process for feedback and buy-in.

7. **What are the potential limitations of using AHP?**

Based on our experience in the use of AHP, the following limitations have been found: (a) the comparison process may be long if the decision is complex (b) the comparison judgment may be unreliable if the participants are not fully engaged in the process (c) the decision-making transparency may be counter-productive for managers who are interested in manipulating the results (d) group decision-making may make difficult to handle consistency problems.

Appendix B
AHP Basic Theory

We present here, for the purpose of completeness, the basics of the AHP theory.[1] While the theoretical fundamentals were presented by Saaty (2012). Brunnelli (2015) and Ishizaka and Nemery (2013) also do a good job of presenting the AHP theoretical fundamentals in a very accessible way. AHP methodology requires the following steps: first, development of the hierarchy (goal, criteria, and alternatives); second, assessing relative weights of the criteria; third, assessing the alternatives relative priority with respect to criteria and finally, calculating the overall priorities. These steps will be explained with a simple model (Fig. B.1).

Development of the Hierarchy

In a basic AHP hierarchy, we may consider three levels (as shown in Fig. B.1): the goal, the criteria[2] and the alternatives.

Assessing Criteria Relative Importance

In the AHP example shown in Fig. B.1, the C_1–C_3 criteria are used to evaluate the alternatives. However, not all the criteria have the same importance for the decision-makers. It could be that for one institution C_3 has greater importance than C_2. In AHP, the criteria need to be compared pairwise with respect to the goal to establish their relative importance using an intensity scale developed for this purpose as shown in Fig. B.2.

---

[1]This appendix is optional and some basic knowledge of linear algebra and vector notation is required.

[2]In more complex hierarchies, the criteria may have sub-criteria and it is also possible that alternatives may have sub-alternatives.

E. Mu and M. Pereyra-Rojas, *Practical Decision Making*,
SpringerBriefs in Operations Research, DOI 10.1007/978-3-319-33861-3

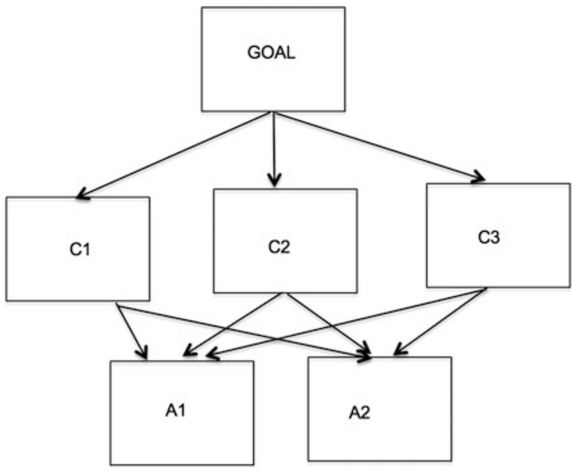

Fig. B.1 Basic AHP model example

Relative Intensity	Importance	Explanation
1	Equal	Both criteria are equally important
3	Moderately	One criterion is moderately more important than the other
5	Strong	One criterion is strongly more important than the other
7	Very Strong	One criterion is very strongly more important than the other
9	Extreme	One criterion is extremely more important than the other
2,4,6,8	Intermediate Values	Compromise is needed

Fig. B.2 Intensity scale for criteria pairwise comparison

Using the scale from Fig. B.2 we will ask questions such as: With respect to the purpose of this decision, which is more important criterion "C_3" or "C_2"? If we consider that C_3 is moderately more important than C_2 we are mathematically stating $C_3/C_2 = 3$ (using the scale from Fig. B.3). Notice that this judgment automatically implies that the comparison of C_2 with C_3 will yield the ratio $C_2/C_3 = 1/3$. This constitutes the reciprocity rule that can be expressed mathematically as $C_{ij} = 1/C_{ji}$ where i and j are any element (i corresponds to the row and j refers to the column) in the comparison matrix.

These judgments are recorded in a comparison matrix as shown in Fig. B.3. Notice that the judgment diagonal, given that the importance of a criterion compared with itself (C_{ij}/C_{ij}), will always be equal and is 1 in the comparison matrix. Also, only the comparisons that fill in the upper part of the matrix (shaded area) are needed. The judgments in the lower part of the comparison matrix are the reciprocals of the values in the upper part, as shown in Fig. B.3.

Another important consideration when completing the comparison matrix is the extent to which it respects the transitivity rule. If the importance of $C_1/C_2 = 1/5$, and the importance of $C_2/C_3 = 1/3$, then it is expected that $C_1/C_3 = (1/5) \times (1/3) = 1/15$. In other words, $C_{ij} = C_{ik} \times C_{kj}$ where C_{ij} is the comparison of criteria i and j. However, this is not the case in Fig. B.3 where $C_1/C_3 = 1$ as indicated by the decision-maker. This means there is some inconsistency in this matrix of judgment as will be explained next.

Checking Consistency of Judgments

Any comparison matrix that fulfills the reciprocity and transitivity rules is said to be consistent. The reciprocity rule is relatively easy to respect, whenever you elicit the judgment C_{ij} you make a point of recording the judgment C_{ji} as the reciprocal value in the comparison. However, it is much harder to comply with the transitivity rule because of the use of English language verbal comparisons from Fig. B.2 such as "strongly more important than," "very strongly more important than," "extremely more important than," and so forth.

Deriving criteria weights in AHP only makes sense if the comparison matrix is consistent or near consistent, and to assess this Saaty (2012) has proposed a consistency index (CI) as follows:

$$\mathrm{CI} = (\lambda_{\max} - \mathrm{N})/(\mathrm{N} - 1)$$

where $\lambda_{\max}$ is the matrix maximal eigenvalue. This is used to calculate the consistency ratio defined as:

$$\mathrm{CR} = \mathrm{CI}/\mathrm{RI}$$

where RI is the random index (the average CI of 500 randomly filled matrices which is available in published tables). CR less than 10 % means that the inconsistency is less than 10 % of 500 random matrices. CR values of 0.1 or below constitute acceptable consistency.

Fig. B.3 Pairwise comparison matrix

	C1	C2	C3	Weights
C1	**1**	1/5	1	0.481
C2	5	**1**	1/3	0.114
C3	1	3	**1**	0.405

C. R. = 0.028

For the comparison matrix used in our example analysis, CR can be calculated as being 0.028, which constitutes an acceptable consistency and means that we can proceed to calculate the priorities (weights) for our criteria comparison matrix shown in Fig. B.3.[3]

Deriving Criteria Weights

The vector of priorities (or weights) **p** for the criteria matrix, given that it is consistent, is calculated by solving the equation (Ishizaka and Nemery 2013):

$$\mathbf{Cp} = n\mathbf{p}$$

where n is the matrix dimension of **C**, the criteria matrix, and $\mathbf{p} = (p_1, p_2, \ldots p_n)$.

Saaty (2012) demonstrated that for a consistent matrix, the priority vector is obtained by solving the equation above. However, for an inconsistent matrix, this equation is no longer valid. Therefore, the dimension n is replaced by the unknown λ. The calculation of λ and p is constituted by solving the eigenvalue problem $\mathbf{Cp} = \lambda\mathbf{p}$. Any value λ satisfying this equation is called an eigenvalue and **p** is its associated eigenvector. Based on Perron theory, a positive matrix has a unique positive eigenvalue called the maximum eigenvalue λ_{max}. For perfectly consistent matrices, $\lambda_{max} = n$; otherwise the difference $\lambda_{max} - n$ is a measure of the inconsistency. Software packages[4] calculate the eigenvector[5] associated to the maximum eigenvalue by elevating the comparison matrix to successive powers until the limit matrix, where all the columns are equal, is reached. Any column constitutes the desired eigenvector. The calculated priorities, using this eigenvalue method, for our tentative criteria comparison matrix is shown in the rightmost column (under the heading *Weights*) in Fig. B.3.

[3]Given the extensive availability of commercial (e.g., Decision Lens, Expert Choice) and freely available software (e.g., SuperDecisions, MakeItRational), we do not show the calculations here but simply report the consistency reported by the software package.

[4]In our applications, the open software SuperDecisions was used to perform the comparison matrix calculations to obtain the eigenvector (criteria and sub-criteria weights) as well as ensuring that C.R. was less or equal 0.1 (SuperDecisions 2014).

[5]Naturally, there is the question if the eigenvalue is still valid for inconsistent matrices. Saaty (2012) justified this using perturbation theory which says that slight variations in a consistent matrix imply only slight variations of the eigenvector and eigenvalue (Ishizaka and Nemery 2013).

References

Brunnelli, M. (2015). Introduction to the analytic hierarchy process. Springer.

Ishizaka, A., & Nemery, P. (2013). *Multi-criteria decision analysis: Methods and software*. West Sussex, UK: John Wiley and Sons.

Saaty, T. L. (2012). *Decision making for leaders: The analytic hierarchy process for decisions in a complex world* (Third Revised Edition ed.). Pittsburgh: RWS Publications.