Appendix A
Extended Standard Model in Euclidean Spacetime

In Eq. (5.39), the gauge-fixed imaginary-time Lagrangian of QCD was given. Here we display the corresponding structure for the Standard Model. For simplicity, terms related to gauge fixing are omitted. On the other hand, we include right-handed neutrinos as degrees of freedom, even though they are not considered to be part of the "classic" Standard Model, in which neutrinos are postulated to be massless. We include them because this does not change any of the construction principles and yet offers for a simple way to solve a number of short-comings of the classic Standard Model (for a review, see [1]). A reader preferring not to include right-handed neutrinos may decouple them by setting the corresponding Yukawa coupling matrix h_ν to zero.

In QCD literature, it is common to keep the number of colours, N_c, as a free parameter. When considering the Standard Model, however, its proper inclusion requires care [2, 3]. For simplicity we restrict to $N_c = 3$ in the following. The gauge group is then $U_Y(1)\times SU_L(2)\times SU_c(3)$, where Y refers to the hypercharge degree of freedom and L to left-handed fermions.

The fermionic matter fields of the Standard Model carry a specific chirality. Denoting chiral projectors by $a_L \equiv (1-\gamma_5)/2$ and $a_R \equiv (1+\gamma_5)/2$, left-handed doublets are defined as

$$Q_a \equiv a_L Q_a \equiv \begin{pmatrix} a_L u_a \\ a_L d_a \end{pmatrix} , \quad L_a \equiv a_L L_a \equiv \begin{pmatrix} a_L \nu_a \\ a_L e_a \end{pmatrix} , \tag{A.1}$$

where $a \in \{1,2,3\}$ is a "family" or "generation" index. Here the fermion fields are 4-component Dirac spinors. The quark fields carry an additional colour index that has been suppressed in the notation, i.e. they are really 12-component spinors. Subsequently we redefine the notation in order to denote the right-handed components by

$$u_a \equiv a_R u_a , \quad d_a \equiv a_R d_a , \quad \nu_a \equiv a_R \nu_a , \quad e_a \equiv a_R e_a . \tag{A.2}$$

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The scalar (Higgs) doublet is denoted by ϕ, and $\tilde{\phi} \equiv i\sigma_2\phi^*$, where σ_2 is a Pauli matrix, is a conjugated version thereof, which transforms in the same way under $SU_L(2)$ but has an opposite "charge" under $U_Y(1)$. Note that the right-handed neutrino field ν_a was denoted by N in Eq. (8.69).

With this field content, the Euclidean Lagrangian can be written as ($Q = (Q_1\, Q_2\, Q_3)^T$, etc.)

$$\begin{aligned} L_E \equiv\; & \frac{1}{4}F^{a_i}_{\mu\nu}F^{a_i}_{\mu\nu} \;+\; (D_\mu\phi)^\dagger D_\mu\phi \;-\; m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \\ & +\bar{Q}\not{D}Q + \bar{u}\not{D}u + \bar{d}\not{D}d + \bar{L}\not{D}L + \bar{\nu}\not{D}\nu + \bar{e}\not{D}e + \frac{1}{2}\big(\bar{\nu}^c M\nu + \bar{\nu}\, M^\dagger\nu^c\big) \\ & +\bar{Q}\, h_u\, u\, \tilde{\phi} + \bar{Q}\, h_d\, d\, \phi + \bar{L}\, h_\nu\, \nu\, \tilde{\phi} + \bar{L}\, h_e\, e\, \phi \\ & +\tilde{\phi}^\dagger\bar{u}\, h_u^\dagger Q + \phi^\dagger\bar{d}\, h_d^\dagger Q + \tilde{\phi}^\dagger\bar{\nu}\, h_\nu^\dagger L + \phi^\dagger\bar{e}\, h_e^\dagger L \;. \end{aligned} \tag{A.3}$$

A number of undefined symbols will be explained below. Starting from the end of the second row, $\nu^c \equiv C\bar{\nu}^T$ denotes a charge-conjugated spinor, with the charge conjugation matrix defined for instance as $C \equiv i\gamma^2\gamma^0$, where γ^μ are Dirac matrices. It is possible to verify that the "Majorana" mass matrix M is symmetric, $M^T = M$; as a symmetric matrix it could also be diagonalized.

The theory defined by Eq. (A.3) contains a number of parameters: the real gauge couplings g_1, g_2, g_3; the Higgs mass parameter m^2 and self-coupling λ; the complex 3×3 Yukawa matrices h_u, h_d, h_ν and h_e; and the complex 3×3 Majorana mass matrix M. In the quantized theory, all of these are to be understood as bare parameters. Note that there is a fairly large redundancy in this parameter set, which could be reduced by various field redefinitions, but for simplicity we display the general expression.

Let us now define the gauge interactions. In the fermionic case, gauge interactions reside in $\not{D} = \gamma_\mu D_\mu$, where D_μ is a covariant derivative. When acting on the Higgs doublet the covariant derivative takes the form

$$D_\mu\phi \;\equiv\; \Big(\partial_\mu + \frac{ig_1}{2}A_\mu - ig_2T^{a_2}B^{a_2}_\mu\Big)\,\phi \;, \tag{A.4}$$

where g_1 and g_2 are gauge couplings related to the $U_Y(1)$ and $SU_L(2)$ gauge fields A_μ and $B^{a_2}_\mu$, respectively, and T^{a_2} are Hermitean generators of $SU_L(2)$, normalized as $\mathrm{Tr}\,[T^{a_2}T^{b_2}] = \frac{1}{2}\delta^{a_2b_2}$. We employ a notation whereby the index $a_2 \in \{1,\ldots,d_2 \equiv 3\}$ implies the use of SU(2) generators, and repeated indices are summed over (in addition, $d_1 \equiv 1$, and sums over a_1 are omitted whenever possible). When acting on leptons, the covariant derivative reads

$$D_\mu L_a \equiv \Big(\partial_\mu - \frac{ig_1}{2}A_\mu - ig_2T^{a_2}B^{a_2}_\mu\Big)\,L_a \;, \tag{A.5}$$

$$D_\mu\nu_a \equiv \Big(\,\partial_\mu\,\Big)\,\nu_a \;, \tag{A.6}$$

$$D_\mu e_a \equiv \left(\partial_\mu - ig_1 A_\mu\right) e_a \ . \tag{A.7}$$

In the case of quarks, the $\mathrm{SU_c}(3)$ gauge coupling g_3 and the generators T^{a_3} and the gauge fields $C_\mu^{a_3}$ appear as well:

$$D_\mu Q_a \equiv \left(\partial_\mu + \frac{ig_1}{6} A_\mu - ig_2 T^{a_2} B_\mu^{a_2} - ig_3 T^{a_3} C_\mu^{a_3}\right) Q_a \ , \tag{A.8}$$

$$D_\mu u_a \equiv \left(\partial_\mu + \frac{2ig_1}{3} A_\mu - ig_3 T^{a_3} C_\mu^{a_3}\right) u_a \ , \tag{A.9}$$

$$D_\mu d_a \equiv \left(\partial_\mu - \frac{ig_1}{3} A_\mu - ig_3 T^{a_3} C_\mu^{a_3}\right) d_a \ . \tag{A.10}$$

The colour index a_3 is summed over the set $a_3 \in \{1, \ldots, d_3 \equiv 8\}$. Finally, field strength tensors are defined in accordance with Eq. (5.1),

$$F_{\mu\nu}^{a_1} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \ , \tag{A.11}$$

$$F_{\mu\nu}^{a_2} \equiv \partial_\mu B_\nu^{a_2} - \partial_\nu B_\mu^{a_2} + g_2\, \epsilon^{a_2 b_2 c_2} B_\mu^{b_2} B_\nu^{c_2} \ , \tag{A.12}$$

$$F_{\mu\nu}^{a_3} \equiv \partial_\mu C_\nu^{a_3} - \partial_\nu C_\mu^{a_3} + g_3 f^{a_3 b_3 c_3} C_\mu^{b_3} C_\nu^{c_3} \ , \tag{A.13}$$

where $\epsilon^{a_2 b_2 c_2}$ is the Levi-Civita symbol and $f^{a_3 b_3 c_3}$ are the structure constants of SU(3).

We remark that we have not included so-called θ-terms in Eq. (A.3), which would have the form $\delta L_E = i \sum_{n=1}^{3} \theta_n \frac{\epsilon_{\mu\nu\rho\sigma} g_n^2 F_{\mu\nu}^{a_n} F_{\rho\sigma}^{a_n}}{64\pi^2}$. This is because our Yukawa couplings are complex. The complex Yukawa couplings corresponding to quark masses can be tuned to be real by a chiral rotation, but this induces a QCD θ-term, leading to the so-called strong CP problem, i.e. the unnatural-looking fact that phenomenologically $\theta_3 \approx 0$. (It is an interesting exercise to contemplate why the $\mathrm{U_Y}(1)$ and the $\mathrm{SU_L}(2)$ θ-angles, θ_1 and θ_2, do not pose similar problems.)

Finally we recall that the quantization of chiral gauge theories is highly non-trivial. Even staying within perturbation theory, γ_5 has to be carefully defined in the context of dimensional regularization [4, 5], but this tends to break spacetime and/or gauge symmetries, leading e.g. to a complicated pattern of operator mixings [6, 7].

References

1. L. Canetti, M. Drewes, T. Frossard, M. Shaposhnikov, Dark matter, baryogenesis and neutrino oscillations from right handed neutrinos. Phys. Rev. D **87**, 093006 (2013) [1208.4607]
2. A. Abbas, Anomalies and charge quantization in the Standard Model with arbitrary number of colors. Phys. Lett. B **238**, 344 (1990)
3. O. Bär, U.-J. Wiese, Can one see the number of colors? Nucl. Phys. B **609**, 225 (2001) [hep-ph/0105258]

4. G. 't Hooft, M.J.G. Veltman, Regularization and renormalization of gauge fields. Nucl. Phys. B **44**, 189 (1972)
5. P. Breitenlohner, D. Maison, Dimensional renormalization and the action principle. Commun. Math. Phys. **52**, 11 (1977)
6. J.G. Körner, N. Nasrallah, K. Schilcher, Evaluation of the flavor-changing vertex $b \to sH$ using the Breitenlohner – Maison – 't Hooft – Veltman γ_5 scheme. Phys. Rev. D **41**, 888 (1990)
7. A.J. Buras, P.H. Weisz, QCD nonleading corrections to weak decays in dimensional regularization and 't Hooft-Veltman schemes. Nucl. Phys. B **333**, 66 (1990)

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