

Appendix

Numerical Data for Illustrative Examples

A.1 Hyperplane Arrangements in \mathbb{R}^2 with 4 Hyperplanes

The hyperplane arrangement used in the illustrative example of Sect. 2.1 and the numerical observations from Sect. 2.3.2 is given by¹:

$$H = \begin{bmatrix} 0.673 & 0.740 \\ 0.857 & -0.514 \\ -0.476 & 0.879 \\ 0.000 & 1.000 \end{bmatrix}, \quad k = \begin{bmatrix} 0.336 \\ 0.000 \\ 0.183 \\ 2.000 \end{bmatrix}. \quad (\text{A.1})$$

The perturbed hyperplane arrangement described in the illustrative example of Sect. 2.1 and also analyzed in the illustrative example for region counting in Sect. 2.1.1 is given by:

$$H = \begin{bmatrix} 0.673 & 0.740 \\ 0.857 & -0.514 \\ 0.600 & -0.800 \\ 0.000 & 1.000 \end{bmatrix}, \quad k = \begin{bmatrix} 0.336 \\ 0.000 \\ 1.000 \\ 2.000 \end{bmatrix}. \quad (\text{A.2})$$

A.2 Hyperplane Arrangements in \mathbb{R}^3 with 4 Hyperplanes

The parametrized hyperplane arrangement lifted in \mathbb{R}^3 as in (2.10) and presented in the illustrative example for hyperplane parametrization of Sect. 2.1.2 is given by the following numerical data:

¹Matrices H and k store row-wise the elements defining the current hyperplane arrangement as in (2.1).

$$H = \begin{bmatrix} 0.2184 & -0.3944 & -0.4092 \\ 0.4686 & 0.1110 & -0.2335 \\ 0.0313 & 0.2788 & -0.3463 \\ -0.1749 & -0.0765 & -0.2190 \end{bmatrix}, \quad k = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \quad (\text{A.3})$$

A.3 Hyperplane Arrangements in \mathbb{R}^2 with 3 Hyperplanes

The following hyperplane arrangement is used for the numerical observations provided in Sects. 2.3.2 and 3.6:

$$H = \begin{bmatrix} -0.0250 & -0.9997 \\ -0.6190 & 0.7854 \\ 0.9983 & -0.0583 \end{bmatrix}, \quad k = \begin{bmatrix} 4.7798 \\ 2.2037 \\ 5.9584 \end{bmatrix}. \quad (\text{A.4})$$

The collection of forbidden tuples (see the description provided in (2.19)) of the above hyperplane arrangement used in the numerical consideration of Sects. 2.3.2 and 3.6 is: $\Sigma^{\bullet,1} = \{(+++)\}$, $\Sigma^{\bullet,2} = \{(+ - +), (+ - -)\}$.

A.4 Hyperplane Arrangement in \mathbb{R}^2 with 8 Hyperplanes

The following hyperplane arrangement is used first in the illustrative example of non-convex regions in Sect. 2.3 and continued throughout the book in the illustrative example of merging procedures Sect. 2.3.1, numerical considerations Sect. 2.3.2, illustrative example for tuple allocation Sect. 3.2, illustrative example for mixed-integer formulations for the complement of a union of convex sets Sect. 3.3, illustrative example for mixed-integer formulations for the union of feasible cells Sect. 3.4, illustrative example for mixed-integer formulations for the feasible region characterized directly through the arrangement Sect. 3.5 and the numerical considerations in Sect. 3.6:

$$H = \begin{bmatrix} 0.7061 & -0.7081 \\ 0.0532 & -0.9986 \\ -0.9584 & -0.2855 \\ 0.1784 & 0.9840 \\ 1.0000 & 0 \\ 0.7577 & 0.6526 \\ -0.3427 & 0.9394 \\ 0.9172 & 0.3984 \end{bmatrix}, \quad k = \begin{bmatrix} 1.1403 \\ 2.9327 \\ 8.6224 \\ 1.5864 \\ 7.7650 \\ 8.6885 \\ 4.4814 \\ 3.9327 \end{bmatrix}. \quad (\text{A.5})$$

Furthermore, the associated collection of forbidden tuples (see the description provided in (2.19)) of the above hyperplane arrangement used in the numerical consideration of Sects. 2.3.2 and 3.6 is:

$$\begin{aligned} \Sigma^{\bullet,1} &= \{(+ - + + + + +), (+ + + - + + +), (- + + - + + +)\}. \\ \Sigma^{\bullet,2} &= \{(- + + + + + +), (+ - + + + + +), (+ + + - + + +), (- + + - + + +), (- + + - + + +)\}. \end{aligned}$$

A.5 Hyperplane Arrangement in \mathbb{R}^2 with 10 Hyperplanes

The following hyperplane arrangement is used for the numerical observations provided in Sects. 2.3.2 and 3.6:

$$H = \begin{bmatrix} -0.0134 & -0.9999 \\ -0.7822 & -0.6231 \\ -0.9999 & 0.0173 \\ -0.6312 & -0.7756 \\ 0.9844 & 0.1760 \\ 0.8663 & -0.4995 \\ -0.1076 & 0.9942 \\ -0.8091 & 0.5876 \\ 0.1496 & 0.9887 \\ -0.8177 & 0.5756 \end{bmatrix}, \quad k = \begin{bmatrix} 9.8619 \\ 10.0495 \\ 9.2534 \\ 0.6254 \\ 7.9396 \\ 7.2362 \\ 5.3507 \\ 2.9736 \\ 8.6598 \\ 7.3975 \end{bmatrix}. \quad (\text{A.6})$$

The associated collection of forbidden tuples of the hyperplane arrangement in (A.6) is given by:

$$\begin{aligned} \Sigma^{\bullet,1} &= \{(+ - + - + + + + +), (+ - + - + + + - + +), (+ + + + + + - - + +), (+ + + + + - + + + +)\}. \\ \Sigma^{\bullet,2} &= \{(+ + - - + + + - + +), (+ - - - + + + - + +), (+ + - - + + + - + -), (+ + + + + - + + + +), (+ + + + + + + + +), (+ + + + + + - + + +), (+ + + + - - + + - +)\}. \end{aligned}$$

A.6 Hyperplane Arrangement in \mathbb{R}^2 with 15 Hyperplanes

The hyperplane arrangement with 15 hyperplanes used for the numerical observations in Sects. 2.3.2 and 3.6 is given by:

$$H = \begin{bmatrix} 0.2810 & -0.9597 \\ -0.9994 & 0.0332 \\ -0.2261 & -0.9741 \\ -0.8722 & 0.4892 \\ -0.9016 & -0.4325 \\ 0.5457 & -0.8380 \\ 0.9818 & -0.1900 \\ 0.4778 & 0.8785 \\ -0.4713 & 0.8820 \\ 0.8572 & 0.5150 \\ -0.4854 & -0.8743 \\ 0.0298 & -0.9996 \\ 0.9940 & -0.1091 \\ -0.1431 & -0.9897 \\ 0.6968 & -0.7173 \end{bmatrix}, \quad k = \begin{bmatrix} 3.6436 \\ 3.5455 \\ 0.1421 \\ 7.1289 \\ 9.1531 \\ 1.0230 \\ 8.1203 \\ 11.8156 \\ 4.9609 \\ 6.1583 \\ 5.4883 \\ 9.1173 \\ 9.6930 \\ 2.8675 \\ 6.5968 \end{bmatrix}. \tag{A.7}$$

The associated collection of forbidden tuples of the above hyperplane arrangement is:

$$\begin{aligned}
 \Sigma^{\bullet,1} &= \{(+--+++++--++-+), (+--+++++--++++), (+- \\
 &+ + + + + + + + + + +), (+--+++++ + + + + + - +), (+- - + + - \\
 &+ + + + - + + - +), (+- - + + - + + + + + + - +), (- + - + + - + + + + + \\
 &+ - -), (- + - + + - - + + + + + - -), (+ + + + + + + + - + + + + +)\}. \\
 \Sigma^{\bullet,2} &= \{(+--+++++--++-+), (+--+++++ + + + + + - +), (+- \\
 &- + + + + + + + + + + + + +), (+- - + + + + + + + - + + + +), (+- - + + - + + \\
 &+ + + + + - +), (+- - + + - + + + + - + + - +), (- + - + + - + + + + + + - \\
 &-), (- + - + + - - + + + + + - -), (- + - + + - + + + + - + + - -), (- + \\
 &- + + - + + + + + + + -), (- + - + + - - + + + + + + -), (- + + + + - - + \\
 &+ + + + + -), (- + - + + - - + + - + + + + -), (- + + + + - - + + + + + + \\
 &-), (- + + + + - + + + + + + + -), (+ + + + + + + + - + + + + +), (+ + \\
 &+ - + + + + - + + + + + +), (+ + + - + + + - - + + + + +), (+ + + + + - \\
 &- + + - + + - + +), (+ + + + + - - - + - + + - + +), (+ + + + + + - + + - + + \\
 &- + +), (+ + + + + + - + + - + + + +), (+ + + + + + + + - + + + + +)\}.
 \end{aligned}$$

A.7 Hyperplane Arrangement in \mathbb{R}^2 with 20 Hyperplanes

The hyperplane arrangement with 20 hyperplanes used for the numerical observations in Sects. 2.3.2 and 3.6 is:

$$H = \begin{bmatrix} -0.1069 & -0.9943 \\ 0.6363 & -0.7714 \\ -0.8853 & -0.4651 \\ 0.0791 & -0.9969 \\ -0.9957 & 0.0931 \\ 0.1035 & 0.9946 \\ -0.8545 & 0.5195 \\ -0.8676 & -0.4972 \\ -0.1081 & 0.9941 \\ -0.9871 & -0.1601 \\ 0.7955 & 0.6060 \\ -0.0403 & 0.9992 \\ 0.8971 & -0.4418 \\ 0.0675 & 0.9977 \\ -0.6650 & 0.7468 \\ 0.0295 & -0.9996 \\ 0.9414 & -0.3372 \\ 0.9547 & 0.2974 \\ 0.4854 & -0.8743 \\ 0.8924 & 0.4512 \\ -0.0528 & -0.9986 \\ 0.7739 & -0.6333 \\ 0.9537 & 0.3006 \\ 0.2968 & -0.9549 \\ 0.9570 & 0.2901 \\ 0.0816 & -0.9967 \\ 0.9487 & -0.3163 \\ 0.3705 & 0.9288 \\ -0.6812 & 0.7321 \\ -0.8789 & -0.4771 \end{bmatrix}, \quad k = \begin{bmatrix} 10.0882 \\ 2.7396 \\ 8.2881 \\ 4.8767 \\ 8.9032 \\ 4.2338 \\ 7.1722 \\ 0.2417 \\ 10.0049 \\ 7.3457 \\ 6.1603 \\ 5.0181 \\ 3.6768 \\ 9.5919 \\ 3.5357 \\ 3.3916 \\ 9.4935 \\ 9.2727 \\ 2.7050 \\ 4.7929 \\ 9.1356 \\ 11.2074 \\ 5.1821 \\ 7.5297 \\ 0.5360 \\ 1.7437 \\ 2.1479 \\ 2.3465 \\ 2.3203 \\ 1.8710 \end{bmatrix}. \tag{A.10}$$

The associated collection of forbidden tuples of the above hyperplane arrangement is the following:

$$\Sigma^{\bullet,1} = \{
 \begin{aligned}
 & (+ + - - + + + - + - + + + + + - + + + + + + + + - + + + -), (+ + - - \\
 & + + + - + - + + + + + - + + - + + + + + + - + + + -), (+ + - - + + + - + + + + + + + - + + + + + + + \\
 & + - + + - + + + + + + - + + + -), (+ + - - + + + - + + + + + + + - + + + + + + + + \\
 & + - + + + -), (+ + - - + + + - + - + + + + + - + + - + - + + + + - + + + -), (+ - \\
 & + - + + + - + + + + - + + - + + - + + + + - - - + + +), (+ - + - + + + + + + + + \\
 & - + + - + + - + + + + - - - - + + +), (+ - + - + + + + + + + + - + + - - + - + \\
 & + + + - - - - + + +), (+ \\
 & + +), (+ - + + + + +), (+ + + + + + \\
 & + - +), (+ + + + + - - - - + + + - + + - + \\
 & + + + + + + + + + + + - +), (+ + + + + - - - - + + + - + + - + + + + + + + + + + \\
 & + - - -), (+ + + + - - - - + + + - + + - + + + + + + + + + + + - - -), (+ + + + \\
 & + - - - + + + - + + - + + + + + + + + + + + + + - - -), (+ + + + + - + + + + - + + \\
 \end{aligned}$$

Series Editor's Biographies

Tamer Başar is with the University of Illinois at Urbana-Champaign, where he holds the academic positions of Swanlund Endowed Chair, Center for Advanced Study Professor of Electrical and Computer Engineering, Research Professor at the Coordinated Science Laboratory, and Research Professor at the Information Trust Institute. He received the B.S.E.E. degree from Robert College, Istanbul, and the M.S., M.Phil, and Ph.D. degrees from Yale University. He has published extensively in systems, control, communications, and dynamic games, and has current research interests that address fundamental issues in these areas along with applications such as formation in adversarial environments, network security, resilience in cyber-physical systems, and pricing in networks.

In addition to his editorial involvement with these Briefs, Basar is also the Editor-in-Chief of *Automatica*, Editor of two Birkhäuser Series on Systems & Control and Static & Dynamic Game Theory, the Managing Editor of the *Annals of the International Society of Dynamic Games (ISDG)*, and member of editorial and advisory boards of several international journals in control, wireless networks, and applied mathematics. He has received several awards and recognitions over the years, among which are the Medal of Science of Turkey (1993); Bode Lecture Prize (2004) of IEEE CSS; Quazza Medal (2005) of IFAC; Bellman Control Heritage Award (2006) of AACC; and Isaacs Award (2010) of ISDG. He is a member of the US National Academy of Engineering, Fellow of IEEE and IFAC, Council Member of IFAC (2011–2014), a past president of CSS, the founding president of ISDG, and president of AACC (2010–2011).

Antonio Bicchi is Professor of Automatic Control and Robotics at the University of Pisa. He graduated from the University of Bologna in 1988 and was a postdoc scholar at M.I.T. A.I. Lab between 1988 and 1990. His main research interests are in:

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- theory and control of nonlinear systems, in particular hybrid (logic/dynamic, symbol/signal) systems.

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Miroslav Krstic holds the Daniel L. Alspach chair and is the founding director of the Cymer Center for Control Systems and Dynamics at University of California, San Diego. He is a recipient of the PECASE, NSF Career, and ONR Young Investigator Awards, as well as the Axelby and Schuck Paper Prizes. Professor Krstic was the first recipient of the UCSD Research Award in the area of engineering and has held the Russell Severance Springer Distinguished Visiting Professorship at UC Berkeley and the Harold W. Sorenson Distinguished Professorship at UCSD. He is a Fellow of IEEE and IFAC. Professor Krstic serves as Senior Editor for *Automatica and IEEE Transactions on Automatic Control* and as Editor for the Springer series *Communications and Control Engineering*. He has served as Vice President for Technical Activities of the IEEE Control Systems Society. Krstic has co-authored eight books on adaptive, nonlinear, and stochastic control, extremum seeking, control of PDE systems including turbulent flows and control of delay systems.

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