

Glossary

Table A.1 contains a list of frequently used symbols, together with clarifications.

Table A.1 Glossary of frequently used notation

Symbol	Clarification
\mathcal{X}	A topological vector space, $\mathcal{X} = (X, \tau)$
ω	A state
$\ \cdot\ $	A norm
τ	Topology (family of open sets)
\mathcal{B}	Bornology (family of bounded sets)
\mathcal{E}	Configuration space of the theory, $\mathcal{E} = \Gamma(E \rightarrow M)$
$\bar{\mathcal{E}}$	Extended configuration space used in the presence of symmetries, typically $\bar{\mathcal{E}} = \mathcal{E} \oplus \mathfrak{g}[1]$
E^*	Dual bundle of E
\mathcal{E}^*	$\Gamma(E^* \rightarrow M)$
\mathcal{E}'	Topological dual of \mathcal{E} , i.e. $\Gamma'(E \rightarrow M)$
$\mathcal{E}(\Omega)$	$\mathcal{C}^\infty(\Omega, \mathbb{R})$
$\mathcal{D}(\Omega)$	$\mathcal{C}_c^\infty(\Omega, \mathbb{R})$
\mathcal{H}	Typically a Hilbert space, in Sect. 6.5.1 a Hopf algebra
$\mathcal{B}(\mathcal{H})$	The space of bounded operators on the Hilbert space \mathcal{H}
$F^{(n)}(\varphi_0) \equiv \frac{\delta^n F}{\delta \varphi^n}(\varphi_0)$	n th functional derivative of the functional F at point φ_0
$T^*[-1](\cdot)$	Odd cotangent bundle of ... (cotangent bundle with the degree shift on the fibre)

(continued)

Table A.1 (continued)

Symbol	Clarification
$\mathcal{C}^\infty(\cdot)$	Smooth functions on ...
$\mathcal{O}(\cdot)$	Functions on a graded manifold ...
$\mathcal{O}_{\text{loc}}(\cdot)$	Local functions on a graded manifold ...
$\mathcal{O}_{\text{ml}}(\cdot)$	Multilocal functions on a graded manifold ...
\mathcal{V}_{loc}	Multilocal vector fields on \mathcal{E}
\mathcal{V}	Multilocal vector fields on \mathcal{E}
$\bigwedge \mathcal{V}$	Multilocal multi-vector fields on \mathcal{E}
$\mathcal{K}\mathcal{T}$	Underlying algebra of the Koszul–Tate complex, $\mathcal{K}\mathcal{T} \doteq \mathcal{O}_{\text{ml}}(\mathcal{E} \oplus \mathcal{E}^*[1] \oplus \mathfrak{g}^*[2]) = \mathcal{C}_{\text{ml}}^\infty(\mathcal{E}, \Lambda \mathcal{E}^{*'} \widehat{\otimes}_\pi S^\bullet \mathfrak{g}^{*'} \otimes \mathbb{C})$
$\mathcal{C}\mathcal{E}$	Underlying algebra of the Chevalley–Eilenberg complex, $\mathcal{C}\mathcal{E} \doteq \mathcal{O}_{\text{ml}}(\mathcal{E} \oplus \mathfrak{X}[1]) = \mathcal{C}_{\text{ml}}^\infty(\mathcal{E}, \Lambda \mathfrak{g}')$
$\mathcal{B}\mathcal{V}$	Underlying algebra of the BV complex, $\mathcal{B}\mathcal{V} \doteq \mathcal{O}_{\text{ml}}(T^*[-1]\overline{\mathcal{E}}) = \mathcal{C}_{\text{ml}}^\infty(\mathcal{E}, S^\bullet \mathfrak{g}_c \widehat{\otimes}_\pi \Lambda \mathcal{E}_c \widehat{\otimes}_\pi \Lambda \mathfrak{g}' \otimes \mathbb{C})$
$r_{\lambda V}$	Classical Møller operator for the interaction V , see Sect. 4.6
$R_{\lambda V}$	Quantum Møller operator for the interaction V , see Sect. 6.2.4
$\mathcal{S}(\cdot)$ on $\mathfrak{A}_{\text{reg}}[[\lambda]]$	Non-renormalized S-matrix, see Definition 6.3
$\mathcal{S}(\cdot)$ on $\mathfrak{A}_{\text{loc}}[[\lambda]]$	Renormalized S-matrix, see Definition 6.7
<i>Spaces of functionals on \mathcal{E}</i>	
\mathcal{F}_{loc}	The space of local functionals on, see Definition 3.14
\mathcal{F}	The space of multilocal functionals, i.e. the algebraic completion of \mathcal{F}_{loc} with respect to the pointwise product
\mathcal{F}_S	On-shell multilocal functionals for the action S
\mathcal{F}^{inv}	Gauge-invariant multilocal functionals
$\mathcal{F}_S^{\text{inv}}$	Gauge invariant on-shell multilocal functionals for the action S
$\mathcal{F}_{\mu c}$	The space of microcausal local functionals, see Definition 4.9
$\mathcal{F}_{s\mu c}$	The space of strongly microcausal local functionals, see Definition 4.9
$(\mathcal{F}_{\text{loc}})_{\text{pds}}^{\otimes n}$	The subspace of $(\mathcal{F}_{\text{loc}})^{\otimes n}$ spanned by $F_1 \otimes \cdots \otimes F_n$, where $F_1, \dots, F_n \in \mathcal{F}_{\text{loc}}$ have pairwise disjoint supports

(continued)

Table A.1 (continued)

Symbol	Clarification
<i>Products of functionals</i>	
\cdot	Pointwise product, $(F \cdot G)(\varphi) = F(\varphi)G(\varphi)$
\star	Star product, $(F \star G)(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}(\varphi), \left(\frac{i}{2} \Delta_{S_0} \right)^{\otimes n} G^{(n)}(\varphi) \right\rangle$
\star_H	Star product, $(F \star_H G)(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}(\varphi), \left(\Delta_{S_0}^+ \right)^{\otimes n} G^{(n)}(\varphi) \right\rangle$
$\cdot_{\mathcal{T}}$ on \mathcal{F}_{reg}	Non-renormalized time-ordered product, $F \cdot_{\mathcal{T}} G \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}, \left(i \Delta_{S_0}^D \right)^{\otimes n} G^{(n)} \right\rangle = \mathcal{T}(\mathcal{T}^{-1} F \cdot \mathcal{T}^{-1} G)$
$\cdot_{\mathcal{T}}$ on $\mathcal{T}(\mathcal{F})$	Renormalized time-ordered product, $F \cdot_{\mathcal{T}} G = \mathcal{T}(\mathcal{T}^{-1} \cdot \mathcal{T}^{-1} G)$
$\cdot_{\mathcal{T}_H}$ on \mathcal{F}_{reg}	Non-renormalized time-ordered product, $F \cdot_{\mathcal{T}_H} G \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}, \left(\Delta_{S_0}^F \right)^{\otimes n} G^{(n)} \right\rangle = \mathcal{T}^H(\mathcal{T}^{H-1} F \cdot \mathcal{T}^{H-1} G)$
\star_V on \mathcal{F}_{reg}	Interacting star product for the interaction V , $F \star_V G \doteq R_V^{-1}(R_V F \star R_V G)$
<i>Algebras</i>	
\mathfrak{A} in Chap. 2	An algebra
\mathfrak{A} in Chaps. 5–8	The quantum algebra of the free theory, see Definition 5.2
\mathfrak{A}^H	$(\mathcal{F}_{\mu c}[[\hbar]], \star_H)$
$\mathfrak{A}_{\text{reg}}$	$(\mathcal{F}_{\text{reg}}[[\hbar]], \star)$
$\mathfrak{A}_{\text{loc}}$	$\mathcal{T}(\mathcal{F}_{\text{loc}}) \subset \mathfrak{A}$, where $\mathcal{T} \doteq \alpha_H^{-1} \circ \mathcal{T}^H$
$\mathfrak{A}_{\text{loc}}^H$	$\mathcal{T}^H(\mathcal{F}_{\text{loc}}) \subset \mathfrak{A}^H$
<i>Categories</i>	
Loc	Category of spacetimes
FLoc	Category of framed spacetimes
Obs	Category of unital C^* -algebras
Obs_c	The category of locally convex topological Poisson algebras
Obs_p	The category of locally convex topological unital $*$ -algebras
Vec	Category of locally convex topological vector spaces

(continued)

Table A.1 (continued)

Symbol	Clarification
<i>Functors</i>	
$\mathfrak{F}_{\text{loc}}$	Covariant functor of local functionals
\mathfrak{F}	Covariant functor of multilocal functionals
\mathfrak{D}	Covariant functor of test function spaces
\mathfrak{E}	Contravariant functor of configuration spaces
\mathfrak{E}_c	Covariant functor of compactly supported configurations spaces
<i>Propagators</i>	
$\Delta_{S_0}^A$	Advanced Green's function
$\Delta_{S_0}^R$	Retarded Green's function
Δ_{S_0}	Causal propagator $\Delta_{S_0}^R - \Delta_{S_0}^A$
$\Delta_{S_0}^D$	Dirac propagator $\frac{1}{2}(\Delta_{S_0}^R + \Delta_{S_0}^A)$
$\Delta_{S_0}^+$	2-point function $\frac{i}{2}(\Delta_{S_0}^R - \Delta_{S_0}^A) + H$
$\Delta_{S_0}^F$	Feynman propagator $\frac{i}{2}(\Delta_{S_0}^R + \Delta_{S_0}^A) + H$
<i>Functional differential operators</i>	
D_A	$\left\langle \Delta_{S_0}^A, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$, e.g. $D_A(F \otimes G)(\varphi_1, \varphi_2) = \left\langle F^{(1)}(\varphi_1), \Delta_{S_0}^A G^{(1)}(\varphi_2) \right\rangle$
D_R	$\left\langle \Delta_{S_0}^R, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_D	$\left\langle \Delta_{S_0}^D, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_Δ	$\left\langle \Delta_{S_0}, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_+	$\left\langle \Delta_{S_0}^+, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_F	$\left\langle \Delta_{S_0}^F, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_F^{ij}	$\left\langle \Delta_{S_0}^F, \frac{\delta^2}{\delta\varphi_i \delta\varphi_j} \right\rangle$
\mathcal{D}_D	$\left\langle \Delta_{S_0}^D, \frac{\delta^2}{\delta\varphi^2} \right\rangle$, e.g. $\mathcal{D}_D(F)(\varphi) = \left\langle \Delta_{S_0}^D, F^{(2)}(\varphi) \right\rangle$
\mathcal{D}_F	$\left\langle \Delta_{S_0}^F, \frac{\delta^2}{\delta\varphi^2} \right\rangle$
\mathcal{D}_H	$\left\langle H, \frac{\delta^2}{\delta\varphi^2} \right\rangle$
α_H	$e^{\frac{\hbar}{2}\mathcal{D}_H}$
α_H^{-1}	$e^{-\frac{\hbar}{2}\mathcal{D}_H}$
α_w	$\alpha_w F \doteq \lim_{N \rightarrow \infty} \alpha_{w_N} F$, see Eq. (6.5) and the discussion above it
\mathcal{T} on \mathcal{F}_{reg}	$\mathcal{T} = e^{\frac{i\hbar}{2}\mathcal{D}_D}$
\mathcal{T} on \mathcal{F}	$\bigoplus_n \alpha_H^{-1} \circ \mathcal{T}_n^H \circ \alpha_w \circ m^{-1}$, see Definition 6.9
\mathcal{T}^H on \mathcal{F}_{reg}	$\mathcal{T} = e^{\frac{i\hbar}{2}\mathcal{D}_F}$
\mathcal{T}^H on \mathcal{F}_{loc}	α_w

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