Glossary

Table A.1 contains a list of frequently used symbols, together with clarifications.

Table A.1 Glossary of frequently used notation

Symbol	Clarification
$\mathcal{X}$	A topological vector space, $\mathcal{X} = (X, \tau)$
ω	A state
$\|.\|$	A norm
τ	Topology (family of open sets)
$\mathcal{B}$	Bornology (family of bounded sets)
$\mathcal{E}$	Configuration space of the theory, $\mathcal{E} = \Gamma(E \to M)$
$\overline{\mathcal{E}}$	Extended configuration space used in the presence of symmetries, typically $\overline{\mathcal{E}} = \mathcal{E} \oplus \mathfrak{g}[1]$
E^*	Dual bundle of E
$\mathcal{E}^*$	$\Gamma(E^* \to M)$
$\mathcal{E}'$	Topological dual of $\mathcal{E}$, i.e. $\Gamma'(E \to M)$
$\mathscr{E}(\Omega)$	$\mathcal{C}^\infty(\Omega, \mathbb{R})$
$\mathscr{D}(\Omega)$	$\mathcal{C}^\infty_c(\Omega, \mathbb{R})$
$\mathcal{H}$	Typically a Hilbert space, in Sect. 6.5.1 a Hopf algebra
$\mathscr{B}(\mathcal{H})$	The space of bounded operators on the Hilbert space $\mathcal{H}$
$F^{(n)}(\varphi_0) \equiv \frac{\delta^n F}{\delta \varphi^n}(\varphi_0)$	nth functional derivative of the functional F at point φ_0
$T^*[-1](.)$	Odd cotangent bundle of …(cotangent bundle with the degree shift on the fibre)

(continued)

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Table A.1 (continued)

Symbol	Clarification
$\mathcal{C}^\infty(.)$	Smooth functions on …
$\mathcal{O}(.)$	Functions on a graded manifold …
$\mathcal{O}_{\mathrm{loc}}(.)$	Local functions on a graded manifold …
$\mathcal{O}_{\mathrm{ml}}(.)$	Multilocal functions on a graded manifold …
$\mathcal{V}_{\mathrm{loc}}$	Multilocal vector fields on $\mathcal{E}$
$\mathcal{V}$	Multilocal vector fields on $\mathcal{E}$
$\bigwedge\mathcal{V}$	Multilocal multi-vector fields on $\mathcal{E}$
$\mathcal{KT}$	Underlying algebra of the Koszul–Tate complex, $\mathcal{KT} \doteq \mathcal{O}_{\mathrm{ml}}(\mathcal{E}\oplus\mathcal{E}^*[1]\oplus\mathfrak{g}^*[2]) = \mathcal{C}^\infty_{\mathrm{ml}}(\mathcal{E}, \Lambda\mathcal{E}^{*\prime}\widehat{\otimes}_\pi S^\bullet\mathfrak{g}^{*\prime}\otimes\mathbb{C})$
$\mathcal{CE}$	Underlying algebra of the Chevalley–Eilenberg complex, $\mathcal{CE} \doteq \mathcal{O}_{\mathrm{ml}}(\mathcal{E}\oplus\mathfrak{X}[1]) = \mathcal{C}^\infty_{\mathrm{ml}}(\mathcal{E}, \Lambda\mathfrak{g}')$
$\mathcal{BV}$	Underlying algebra of the BV complex, $\mathcal{BV} \doteq \mathcal{O}_{\mathrm{ml}}(T^*[-1]\overline{\mathcal{E}}) = \mathcal{C}^\infty_{\mathrm{ml}}(\mathcal{E}, S^\bullet\mathfrak{g}_c\widehat{\otimes}_\pi\Lambda\mathcal{E}_c\widehat{\otimes}_\pi\Lambda\mathfrak{g}'\otimes\mathbb{C})$
$r_{\lambda V}$	Classical Møller operator for the interaction V, see Sect. 4.6
$R_{\lambda V}$	Quantum Møller operator for the interaction V, see Sect. 6.2.4
$\mathcal{S}(.)$ on $\mathfrak{A}_{\mathrm{reg}}[[\lambda]]$	Non-renormalized S-matrix, see Definition 6.3
$\mathcal{S}(.)$ on $\mathfrak{A}_{\mathrm{loc}}[[\lambda]]$	Renormalized S-matrix, see Definition 6.7
Spaces of functionals on $\mathcal{E}$	
$\mathcal{F}_{\mathrm{loc}}$	The space of local functionals on, see Definition 3.14
$\mathcal{F}$	The space of multilocal functionals, i.e. the algebraic completion of $\mathcal{F}_{\mathrm{loc}}$ with respect to the pointwise product
$\mathcal{F}_S$	On-shell multilocal functionals for the action S
$\mathcal{F}^{\mathrm{inv}}$	Gauge-invariant multilocal functionals
$\mathcal{F}^{\mathrm{inv}}_S$	Gauge invariant on-shell multilocal functionals for the action S
$\mathcal{F}_{\mu c}$	The space of microcausal local functionals, see Definition 4.9
$\mathcal{F}_{s\mu c}$	The space of strongly microcausal local functionals, see Definition 4.9
$(\mathcal{F}_{\mathrm{loc}})^{\otimes n}_{\mathrm{pds}}$	The subspace of $(\mathcal{F}_{\mathrm{loc}})^{\otimes n}$ spanned by $F_1\otimes\cdots\otimes F_n$, where $F_1,\ldots,F_n\in\mathcal{F}_{\mathrm{loc}}$ have pairwise disjoint supports

(continued)

Table A.1 (continued)

Symbol	Clarification
Products of functionals	
$\cdot$	Pointwise product, $(F \cdot G)(\varphi) = F(\varphi)G(\varphi)$
$\star$	Star product, $(F \star G)(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}(\varphi), \left(\frac{i}{2}\Delta_{S_0}\right)^{\otimes n} G^{(n)}(\varphi)\right\rangle$
$\star_H$	Star product, $(F \star_H G)(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}(\varphi), (\Delta^{+}_{S_0})^{\otimes n} G^{(n)}(\varphi)\right\rangle$
$\cdot_{\mathcal{T}}$ on $\mathcal{F}_{\mathrm{reg}}$	Non-renormalized time-ordered product, $F \cdot_{\mathcal{T}} G \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}, \left(i\Delta^{\mathrm{D}}_{S_0}\right)^{\otimes n} G^{(n)}\right\rangle = \mathcal{T}(\mathcal{T}^{-1}F \cdot \mathcal{T}^{-1}G)$
$\cdot_{\mathcal{T}}$ on $\mathcal{T}(\mathcal{F})$	Renormalized time-ordered product, $F \cdot_{\mathcal{T}} G = \mathcal{T}(\mathcal{T}^{-1} \cdot \mathcal{T}^{-1}G)$
$\cdot_{\mathcal{T}_H}$ on $\mathcal{F}_{\mathrm{reg}}$	Non-renormalized time-ordered product, $F \cdot_{\mathcal{T}_H} G \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}, \left(\Delta^{\mathrm{F}}_{S_0}\right)^{\otimes n} G^{(n)}\right\rangle = \mathcal{T}^H(\mathcal{T}^{H-1}F \cdot \mathcal{T}^{H-1}G)$
$\star_V$ on $\mathcal{F}_{\mathrm{reg}}$	Interacting star product for the interaction V, $F \star_V G \doteq R_V^{-1}(R_V F \star R_V G)$
Algebras	
$\mathfrak{A}$ in Chap. 2	An algebra
$\mathfrak{A}$ in Chaps. 5–8	The quantum algebra of the free theory, see Definition 5.2
$\mathfrak{A}^H$	$(\mathcal{F}_{\mu c}[[\hbar]], \star_H)$
$\mathfrak{A}_{\mathrm{reg}}$	$(\mathcal{F}_{\mathrm{reg}}[[\hbar]], \star)$
$\mathfrak{A}_{\mathrm{loc}}$	$\mathcal{T}(\mathcal{F}_{\mathrm{loc}}) \subset \mathfrak{A}$, where $\mathcal{T} \doteq \alpha_H^{-1} \circ \mathcal{T}^H$
$\mathfrak{A}^H_{\mathrm{loc}}$	$\mathcal{T}^H(\mathcal{F}_{\mathrm{loc}}) \subset \mathfrak{A}^H$
Categories	
Loc	Category of spacetimes
FLoc	Category of framed spacetimes
Obs	Category of unital C^*-algebras
$\mathbf{Obs}_c$	The category of locally convex topological Poisson algebras
$\mathbf{Obs}_p$	The category of locally convex topological unital $*$-algebras
Vec	Category of locally convex topological vector spaces

(continued)

Table A.1 (continued)

Symbol	Clarification
Functors	
$\mathfrak{F}_{\mathrm{loc}}$	Covariant functor of local functionals
$\mathfrak{F}$	Covariant functor of multilocal functionals
$\mathfrak{D}$	Covariant functor of test function spaces
$\mathfrak{E}$	Contravariant functor of configuration spaces
$\mathfrak{E}_c$	Covariant functor of compactly supported configurations spaces
Propagators	
$\Delta^{\mathrm{A}}_{S_0}$	Advanced Green's function
$\Delta^{\mathrm{R}}_{S_0}$	Retarded Green's function
Δ_{S_0}	Causal propagator $\Delta^{\mathrm{R}}_{S_0} - \Delta^{\mathrm{A}}_{S_0}$
$\Delta^{\mathrm{D}}_{S_0}$	Dirac propagator $\frac{1}{2}(\Delta^{\mathrm{R}}_{S_0} + \Delta^{\mathrm{A}}_{S_0})$
$\Delta^{+}_{S_0}$	2-point function $\frac{i}{2}(\Delta^{\mathrm{R}}_{S_0} - \Delta^{\mathrm{A}}_{S_0}) + H$
$\Delta^{\mathrm{F}}_{S_0}$	Feynman propagator $\frac{i}{2}(\Delta^{\mathrm{R}}_{S_0} + \Delta^{\mathrm{A}}_{S_0}) + H$
Functional differential operators	
D_{A}	$\left\langle \Delta^{\mathrm{A}}_{S_0}, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$, e.g. $D_{\mathrm{A}}(F \otimes G)(\varphi_1, \varphi_2) = \left\langle F^{(1)}(\varphi_1), \Delta^{\mathrm{A}}_{S_0} G^{(1)}(\varphi_2) \right\rangle$
D_{R}	$\left\langle \Delta^{\mathrm{R}}_{S_0}, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_{D}	$\left\langle \Delta^{\mathrm{D}}_{S_0}, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_{Δ}	$\left\langle \Delta_{S_0}, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_{+}	$\left\langle \Delta^{+}_{S_0}, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D_{F}	$\left\langle \Delta^{\mathrm{F}}_{S_0}, \frac{\delta}{\delta\varphi} \otimes \frac{\delta}{\delta\varphi} \right\rangle$
D^{ij}_{F}	$\langle \Delta^{\mathrm{F}}_{S_0}, \frac{\delta^2}{\delta\varphi_i \delta\varphi_j} \rangle$
$\mathcal{D}_{\mathrm{D}}$	$\left\langle \Delta^{\mathrm{D}}_{S_0}, \frac{\delta^2}{\delta\varphi^2} \right\rangle$, e.g. $\mathcal{D}_{\mathrm{D}}(F)(\varphi) = \left\langle \Delta^{\mathrm{D}}_{S_0}, F^{(2)}(\varphi) \right\rangle$
$\mathcal{D}_{\mathrm{F}}$	$\left\langle \Delta^{\mathrm{F}}_{S_0}, \frac{\delta^2}{\delta\varphi^2} \right\rangle$
$\mathcal{D}_{H}$	$\left\langle H, \frac{\delta^2}{\delta\varphi^2} \right\rangle$
α_H	$e^{\frac{\hbar}{2}\mathcal{D}_H}$
α_H^{-1}	$e^{-\frac{\hbar}{2}\mathcal{D}_H}$
α_w	$\alpha_w F \doteq \lim_{N\to\infty} \alpha_{w_N} F$, see Eq. (6.5) and the discussion above it
$\mathcal{T}$ on $\mathcal{F}_{\mathrm{reg}}$	$\mathcal{T} = e^{\frac{i\hbar}{2}\mathcal{D}_{\mathrm{D}}}$
$\mathcal{T}$ on $\mathcal{F}$	$\bigoplus_n \alpha_H^{-1} \circ \mathcal{T}^H_n \circ \alpha_w \circ m^{-1}$, see Definition 6.9
$\mathcal{T}^H$ on $\mathcal{F}_{\mathrm{reg}}$	$\mathcal{T} = e^{\frac{i\hbar}{2}\mathcal{D}_{\mathrm{F}}}$
$\mathcal{T}^H$ on $\mathcal{F}_{\mathrm{loc}}$	α_w

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