Appendix A
Proofs

A.1 Chapter 4

A.1.1 Proof of Theorem 4.2

The forward difference of the Lyapunov function (4.10) for (4.25) is

$$\begin{aligned}\Delta V(k) &= x^T(k+1)Px(k+1) - x^T(k)Px(k)\\&= (A_K x(k) + BK\varepsilon(k) + w(k))^T P\,(A_K x(k) + BK\varepsilon(k) + w(k)) - x^T(k)Px(k)\\&= -x^T(k)Qx(k) + 2\varepsilon^T(k)(BK)^T PA_K x(k) + \varepsilon^T(k)(BK)^T PBK\varepsilon(k)\\&\quad + 2w^T(k)PA_K x(k) + 2w^T(k)PBK\varepsilon(k) + w^T(k)Pw(k)\\&\leq -\lambda_{min}(Q)\|x(k)\|^2 + 2\|(BK)^T PA_K\|\|\varepsilon(k)\|\|x(k)\| + \|(BK)^T PBK\|\|\varepsilon(k)\|^2\\&\quad + 2\|PBK\|\|w(k)\|\|\varepsilon(k)\| + 2\|PA_K\|\|w(k)\|\|x(k)\| + \lambda_{max}(P)\|w(k)\|^2.\end{aligned} \tag{A.1}$$

The error and the disturbance are bounded by $\|\varepsilon(k)\| \leq 2\delta$ and $\|w(k)\| \leq w_{max}$. Thus, the Lyapunov function decreases if (A.1) is negative. This holds whenever

$$\|x(k)\| \geq \frac{\sigma_b + \sqrt{\sigma_b^2 + 4\sigma_a\sigma_c}}{2\sigma_a} = \sigma_w,$$

where σ_a, σ_b, σ_c and σ_w are defined in (4.27)–(4.30).

The state decreases until it reaches this bound. Let us denote k^* the time instant at which the state enters this region. According to (4.25), the norm of the state at the next step is, in the worst case:

$$\|x(k^*+1)\| \leq \|A_K\|\sigma_w + \|BK\|2\delta + w_{max}.$$

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So if the state leaves the region, the Lyapunov function decreases again. Using the property of the Lyapunov function $\lambda_{min}(P)\|x\|^2 \leq x^T Px \leq \lambda_{max}(P)\|x\|^2$, the state $x(k)$ remains bounded by (4.26) $\forall k \geq k^*$, and this concludes the proof.

A.1.2 Proof of Theorem 4.3

The forward difference of the Lyapunov function (4.42) for (4.41) is

$$\begin{aligned}\Delta V(k) &= \xi^T(k+1)P\xi(k+1) - \xi^T(k)P\xi(k)\\ &= \left(A_{CL}\xi(k) + F(\varepsilon_y(k) + v(k))\right)^T P \left(A_{CL}\xi(k) + F(\varepsilon_y(k) + v(k))\right)\\ &\quad - \xi^T(k)P\xi(k)\\ &= -\xi^T(k)Q\xi(k) + 2(\varepsilon_y^T(k) + v^T(k))F^T PA_{CL}\xi(k)\\ &\quad + (\varepsilon_y^T(k) + v^T(k))F^T PF(\varepsilon_y(k) + v(k))\\ &\leq -\lambda_{min}(Q)\|\xi(k)\|^2 + 2\|F^T PA_{CL}\|\|\varepsilon_y(k) + v(k)\|\|\xi(k)\|\\ &\quad + \|F^T PF\|\|\varepsilon_y(k) + v(k)\|^2. \end{aligned} \tag{A.2}$$

The right-hand side of (A.2) is an algebraic second-order equation in $\|\xi(k)\|$ such that the Lyapunov function decreases whenever

$$\|\xi(k)\| \geq \sigma_\xi \|\varepsilon_y(k) + v(k)\|,$$

where σ_ξ is given in (4.44).

Because the error ε_y is bounded by $2\delta_y$ and the noise by v_{max}, $\Delta V < 0$ in the region $\|\xi(k)\| > \sigma_\xi(2\delta_y + v_{max})$. Thus, the state decreases until it reaches this region. If we denote by k^* the time instant at which the state enters this region and according to (4.41), it follows that

$$\|\xi(k^* + 1)\| \leq (\sigma_\xi \|A_{CL}\| + \|F\|)(2\delta_y + v_{max}).$$

Then the state can leave the region so the Lyapunov function decreases again. If the inequalities $\lambda_{min}(P)\|\xi\|^2 \leq \xi^T P\xi \leq \lambda_{max}(P)\|\xi\|^2$ are used, it is straightforward to see that the state $\xi(k)$ remains bounded by (4.43) $\forall k \geq k^*$, and this concludes the proof.

A.2 Chapter 5

A.2.1 Proof of Proposition 5.2

Consider, first, the system without disturbances ($w(k) \equiv 0$). The forward difference can be calculated as

$$\begin{aligned}\Delta V(k) = {} & x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ & + x^T(k)Z_1x(k) - x^T(k-\tau_{max})Z_1x(k-\tau_{max}) \\ & + x^T(k)Z_2x(k) - x^T(k-\tau_{min})Z_2x(k-\tau_{min}) \\ & + \tau_{max}^2\Delta x^T(k)Z_3\Delta x(k) - \tau_{max}\sum_{j=k-\tau_{max}}^{k-1}\Delta x^T(j)Z_3\Delta x(j) \\ & + \Delta\tau^2\Delta x^T(k)Z_4\Delta x(k) - \Delta\tau\sum_{j=k-\tau_{max}}^{k-\tau_{min}-1}\Delta x^T(j)Z_4\Delta x(j),\end{aligned}$$

where $\Delta\tau = \tau_{max} - \tau_{min}$. The summations can be divided into two parts as follows:

$$\begin{aligned}-\sum_{j=k-\tau_{max}}^{k-1}\Delta x^T(j)Z_3\Delta x(j) &= -\sum_{j=k-\tau_{max}}^{k-\tau(k)-1}\Delta x^T(j)Z_3\Delta x(j) \\ &\quad -\sum_{j=k-\tau(k)}^{k-1}\Delta x^T(j)Z_3\Delta x(j), \\ -\sum_{j=k-\tau_{max}}^{k-\tau_{min}-1}\Delta x^T(j)Z_4\Delta x(j) &= -\sum_{j=k-\tau_{max}}^{k-\tau(k)-1}\Delta x^T(j)Z_4\Delta x(j) \\ &\quad -\sum_{j=k-\tau(k)}^{k-\tau_{min}-1}\Delta x^T(j)Z_4\Delta x(j).\end{aligned}$$

The resulting terms can be bounded using the Jensen inequality:

$$\begin{aligned}-\tau_{max}\sum_{j=k-\tau_{max}}^{k-\tau(k)-1}\Delta x^T(j)Z_3\Delta x(j) &\le -\left[\sum_{j=k-\tau_{max}}^{k-\tau(k)-1}\Delta x(j)\right]^T Z_3\left[\sum_{j=k-\tau_{max}}^{k-\tau(k)-1}\Delta x(j)\right], \\ -\tau_{max}\sum_{j=k-\tau(k)}^{k-1}\Delta x^T(j)Z_3\Delta x(j) &\le -\left[\sum_{j=k-\tau(k)}^{k-1}\Delta x(j)\right]^T Z_3\left[\sum_{j=k-\tau(k)}^{k-1}\Delta x(j)\right],\end{aligned}$$

$$-\Delta\tau \sum_{j=k-\tau_{max}}^{k-\tau(k)-1} \Delta x^T(j) Z_4 \Delta x(j) \le -\left[\sum_{j=k-\tau_{max}}^{k-\tau(k)-1} \Delta x(j)\right]^T Z_4 \left[\sum_{j=k-\tau_{max}}^{k-\tau(k)-1} \Delta x(j)\right],$$

$$-\Delta\tau \sum_{j=k-\tau(k)}^{k-1} \Delta x^T(j) Z_4 \Delta x(j) \le -\left[\sum_{j=k-\tau(k)}^{k-\tau_{min}-1} \Delta x(j)\right]^T Z_4 \left[\sum_{j=k-\tau(k)}^{k-\tau_{min}-1} \Delta x(j)\right].$$

Since the augmented state vector has been defined as $\xi^T(k) = [x^T(k)\ x^T(k-\tau(k))\ x^T(k-\tau_{min})\ x^T(k-\tau_{max})]$, the forward difference of the functional can be written as

$$\Delta V(k) = \xi^T(k)\left(M + \tilde{A}^T P \tilde{A} + (\tilde{A}-\tilde{I})^T(\tau_{max}^2 Z_3 + \Delta\tau^2 Z_4)(\tilde{A}-\tilde{I})\right)\xi(k), \quad \text{(A.3)}$$

satisfying Assumption 5.2. for $w \equiv 0$.

Consider now the case with disturbances. Including $w(k)$ into the augmented state vector, the forward difference (A.3) can be written now as

$$\Delta V(k) = \begin{bmatrix} \xi(k) \\ w(k) \end{bmatrix}^T \left(\begin{bmatrix} M & 0 \\ * & 0 \end{bmatrix} + \begin{bmatrix} \tilde{A}^T \\ B_w^T \end{bmatrix} P \begin{bmatrix} \tilde{A} & B_w \end{bmatrix} + \begin{bmatrix} (\tilde{A}-\tilde{I})^T \\ B_w^T \end{bmatrix} (\tau_{max}^2 Z_3 + \Delta\tau^2 Z_4) \begin{bmatrix} (\tilde{A}-\tilde{I}) & B_w \end{bmatrix} \right) \begin{bmatrix} \xi(k) \\ w(k) \end{bmatrix}. \quad \text{(A.4)}$$

Now some null terms are added to the forward difference:

$$\Delta V(k) = \Delta V(k) + z_\infty^T(k) z_\infty(k) - \gamma^2 w^T(k) w(k) + \gamma^2 w^T(k) w(k) - z_\infty^T(k) z_\infty(k).$$

The first two terms are included in the quadratic product (A.4) taking into account that $z_\infty^T(k) z_\infty(k) = \xi^T(k)\tilde{C}_\infty^T \tilde{C}_\infty \xi(k)$, where $\tilde{C}_\infty = [C_\infty\ \ D_\infty K\ \ 0\ \ 0]$. Thus,

$$\Delta V(k) = \begin{bmatrix} \xi(k) \\ w(k) \end{bmatrix}^T \left(\begin{bmatrix} M + \tilde{C}_\infty^T \tilde{C}_\infty & 0 \\ * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \tilde{A}^T \\ B_w^T \end{bmatrix} P \begin{bmatrix} \tilde{A} & B_w \end{bmatrix} + \begin{bmatrix} (\tilde{A}-\tilde{I})^T \\ B_w^T \end{bmatrix} (\tau_{max}^2 Z_3 + \Delta\tau^2 Z_4) \begin{bmatrix} (\tilde{A}-\tilde{I}) & B_w \end{bmatrix} \right) \begin{bmatrix} \xi(k) \\ w(k) \end{bmatrix} + \gamma^2 w^T(k) w(k) - z_\infty^T(k) z_\infty(k). \quad \text{(A.5)}$$

Equation (A.5) has the same structure as (5.10), so the proposed functional satisfies all the conditions given in Assumption 5.2.

Finally, matrix Ψ can be obtained after some careful mathematical manipulations.

A.3 Chapter 7

A.3.1 Proof of Theorem 7.1

The analytical solution of (7.22) is

$$x(t) = e^{(A_K+\Delta)t}x(0) + \int_0^t e^{(A_K+\Delta)(t-s)}BK\varepsilon(s)ds. \tag{A.6}$$

From Assumption 7.1, the matrix A_K is diagonalizable and

$$\|e^{A_K t}\| \leq \kappa(V)e^{-|\alpha_{max}(A_K)|t}.$$

Thus, (7.10) can be used to bound $e^{(A_K+\Delta)t}$ as

$$\|e^{(A_K+\Delta)t}\| \leq \kappa(V)e^{-(|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|)t}.$$

Note that the exponent is negative since $|\alpha_{max}(A_K)| - \kappa(V)\|\Delta\| > 0$ from Assumption 7.2. Let us denote $\alpha_\Delta = |\alpha_{max}(A_K)| - \kappa(V)\|\Delta\|$.

Consequently, the state can be bounded by

$$\|x(t)\| \leq \kappa(V)\left(e^{-\alpha_\Delta t}\|x(0)\| + \int_0^t e^{-\alpha_\Delta(t-s)}\|BK\|\|\varepsilon(s)\|ds\right).$$

The overall system error is bounded by

$$\|\varepsilon(t)\| \leq \sqrt{N_a}\left(\delta_0 + \delta_1 e^{-\beta t}\right).$$

This yields

$$\begin{aligned}\|x(t)\| &\leq \kappa(V)\left(e^{-\alpha_\Delta t}\|x(0)\| + \int_0^t \sqrt{N_a}e^{-\alpha_\Delta(t-s)}\|BK\|(\delta_0 + \delta_1 e^{-\beta s})\right)\\ &= \kappa(V)\left(e^{-\alpha_\Delta t}\|x(0)\| + \frac{\|BK\|\sqrt{N_a}\delta_0}{\alpha_\Delta}\left(1 - e^{-\lambda_\Delta t}\right)\right.\\ &\quad \left. + \frac{\|BK\|\sqrt{N_a}\delta_1}{\alpha_\Delta - \beta}\left(e^{-\beta t} - e^{-\alpha_\Delta t}\right)\right),\end{aligned}$$

which by reordering terms and restoring $\alpha_\Delta = |\alpha_{max}(A_K)| - \kappa(V)\|\Delta\|$ yield (7.28), proving the first part of the theorem. Note that the previous expression can be upper bounded by

$$\|x(t)\| \le \kappa(V)\left(\|x(0)\|e^{-\alpha_\Delta t} + \frac{\|BK\|\sqrt{N_a}\delta_0}{\alpha_\Delta} + \frac{\|BK\|\sqrt{N_a}\delta_1}{\alpha_\Delta - \alpha}e^{-\beta t}\right), \tag{A.7}$$

by omitting the negative terms.

We next show that broadcasting period is lower bounded. If t^* refers to the last event time occurrence, $\|\varepsilon_i(t^*)\| = 0$, and $F_{e,i}(t^*) = -\delta_0 - \delta_1 e^{-\beta t^*} < 0$.

Because $\dot{\varepsilon}_i(t) = -\dot{x}_i(t)$ and $\|\varepsilon_i(t)\| \le \int_{t^*}^{t} \|\dot{x}_i(t)\| \le \int_{t^*}^{t} \|\dot{x}(t)\|$ hold, and from (7.22) we derive

$$\|\dot{x}(t)\| \le \|A_K + \Delta\|\|x(t)\| + \|BK\|\sqrt{N_a}\left(\delta_0 + \delta_1 e^{-\beta t^*}\right).$$

If the last event occurred at time $t^* > 0$

$$\|\varepsilon_i(t)\| \le \int_{t^*}^{t} \|\dot{x}(t)\| \le \int_{t^*}^{t} (\|A_K + \Delta\|\|x(s)\| + \|BK\|\varepsilon(s))\,ds,$$

and $\|x(t)\| \le \|x(t^*)\|$ holds in (A.7). Thus, defining the following constants:

$$\begin{aligned} k_1 &= \kappa(V)\|A_K + \Delta\|\|x(0)\| \\ k_2 &= \|BK\|\sqrt{N_a}\delta_1\left(\frac{\kappa(V)\|A_K + \Delta\|}{\alpha_\Delta - \beta} + 1\right) \\ k_3 &= \|BK\|\sqrt{N_a}\delta_0\left(\frac{\kappa(V)\|A_K + \Delta\|}{\alpha_\Delta} + 1\right), \end{aligned}$$

the error can be bounded as

$$\|\varepsilon_i(t)\| \le \int_{t^*}^{t} (k_1 + k_2 + k_3)ds = (k_1 + k_2 + k_3)(t - t^*).$$

The next event will not be triggered before $\|\varepsilon_i(t)\| = \delta_0 + \delta_1 e^{-\beta t} \ge \delta_0$. Thus a lower bound on the inter-events time is given by

$$T_{min} = \frac{\delta_0}{k_1 + k_2 + k_3}, \tag{A.8}$$

which is a positive quantity.

A.3.2 Pure Exponential Functions

Let us consider the case when $\delta = 0$ and, for simplicity, $\|\Delta\| = 0$. The error is bounded by $\|\varepsilon_i(t)\| \leq \delta_1 e^{-\beta t}$, and so the error goes to zero when times goes to infinity. The state bound (A.7) can be particularized for pure exponential trigger functions as follows:

$$\|x(t)\| \leq \kappa(V)\left(\|x(0)\|e^{-|\alpha_{max}(A_K)|t} + \frac{\|BK\|\sqrt{N_a}\delta_1}{|\alpha_{max}(A_K)|-\beta}e^{-\beta t}\right). \tag{A.9}$$

In order to prove that the Zeno behavior is excluded, we consider that the error $\|\varepsilon_i(t)\|$ is upper bounded by

$$\|\varepsilon_i(t)\| \leq \left(k_1 e^{-|\alpha_{max}(A_K)|t^*} + k_2 e^{-\beta t^*}\right)T.$$

Note that $k_3 = 0$ since $\delta_0 = 0$.

The next event is not triggered before $\|\varepsilon_i(t)\| = \delta_1 e^{-\beta t}$. Thus, a lower bound on the inter-event intervals is given by

$$\left(\frac{k_1}{\delta_1}e^{(\beta-|\alpha_{max}(A_K)|)t^*} + \frac{k_2}{\delta_1}\right)T = e^{-\beta T}. \tag{A.10}$$

The right-hand side of (A.10) is always positive. Moreover, for $\beta < |\alpha_{max}(A_K)|$ the left-hand side is strictly positive as well, and the term in brackets is upper bounded by $\frac{k_2+k_1}{\delta_1}$ and lower bounded by k_2/δ_1, and this yields to a positive value of T for all $t^* \geq 0$.

A.3.3 Proof of Theorem 7.2

Let us denote $F = A_K + \Delta$. The analytical solution of (7.34) is

$$x(\ell) = F^\ell x(0) + \sum_{j=0}^{\ell-1} F^{\ell-1-j}BK\varepsilon(j). \tag{A.11}$$

This can be bounded as

$$\|x(\ell)\| \leq \|F^\ell\|\|x(0)\| + \sum_{j=0}^{\ell-1}\|F^{\ell-1-j}\|\|BK\|\|\varepsilon(j)\|. \tag{A.12}$$

Let us assume that $\Delta^j \approx \mathbf{0}, \forall j \geq 2$. According to (7.11), $\|F^\ell\|$ can be bounded as

$$\|F^\ell\| = \|(A_K + \Delta)^\ell\| \leq \|A_K^\ell\| + \|\sum_{j=0}^{\ell-1} A_K^{\ell-1-j} \Delta A_K^j\| + \mathscr{O}(\|\Delta\|^2)$$
$$\approx \|A_K^\ell\| + \|\sum_{j=0}^{\ell-1} A_K^{\ell-1-j} \Delta A_K^j\|,$$

since $\|\Delta\|^2 \approx 0$.

Moreover, as in the continuous time case, we assume that A_K is diagonalizable, and hence, $A_K = VDV^{-1}$. It also holds that $\|D\| = |\lambda_{max}(D)| = |\lambda_{max}(A_K)|$, where $\lambda_{max}(A_{dK})$ is the eigenvalue with the closer magnitude to 1. Thus,

$$\|A_K^\ell\| = \|VD^\ell V^{-1}\| \leq \kappa(V)|\lambda_{max}(A_K)|^\ell,$$

where $\kappa(V) = \|V\|\|V^{-1}\|$.

Similarly, the following bound can be computed for the sum:

$$\|\sum_{j=0}^{\ell-1} A_K^{\ell-1-j} \Delta A_K^j\| \leq \kappa^2(V) \sum_{j=0}^{\ell-1} |\lambda_{max}(A_K)|^{\ell-1-j} \|\Delta\| |\lambda_{max}(A_K)|^j$$
$$= \kappa^2(V)|\lambda_{max}(A_K)|^{\ell-1}\|\Delta\|\ell.$$

Thus, $\|F^\ell\|$ is bounded by

$$\|F^\ell\| \leq \kappa(V)|\lambda_{max}(A_K)|^\ell \left(1 + \ell \frac{\kappa(V)\|\Delta\|}{|\lambda_{max}(A_K)|}\right). \tag{A.13}$$

If we consider the bound (A.13) in (A.12), it holds that

$$\|x(\ell)\| \leq \kappa(V)|\lambda_{max}(A_K)|^\ell \left(1 + \ell \frac{\kappa(V)\|\Delta\|}{|\lambda_{max}(A_K)|}\right) \|x(0)\| \tag{A.14}$$
$$+ \sum_{j=0}^{\ell-1} \left(\kappa(V)|\lambda_{max}(A_K)|^{\ell-1-j} \left(1 + (\ell - 1 - j) \frac{\kappa(V)\|\Delta\|}{|\lambda_{max}(A_K)|}\right) \|BK\|\|\varepsilon(j)\|\right).$$

Moreover, from (7.39), the error can be bounded as $\|\varepsilon(j)\| \leq \sqrt{N_a}(\delta_0 + \delta_1 \beta^j)$.

The sum in (A.14) can be computed taking into account that

$$\sum_{j=0}^{\ell-1} r^{\ell-1-j} = \frac{1-r^{\ell}}{1-r}$$
$$\sum_{j=0}^{\ell-1} (\ell-1-j) r^{\ell-1-j} = r \cdot \left(\frac{1-r^{\ell}}{(1-r)^2} - \frac{\ell r^{\ell-1}}{1-r} \right),$$

where r can be $|\lambda_{max}(A_K)|$ or $\frac{\beta}{|\lambda_{max}(A_K)|}$.

Thus, it yields that

$$\begin{aligned}\|x(\ell)\| \leq \kappa(V) \Bigg(& \tfrac{\|BK\|\sqrt{N_a}\delta_0}{1-|\lambda_{max}(A_K)|} \left(1 + \tfrac{\kappa(V)\|\Delta\|}{1-|\lambda_{max}(A_K)|}\right) + |\lambda_{max}(A_K)|^{\ell} \Bigg(\|x(0)\| \\ & - \tfrac{\|BK\|\sqrt{N_a}\delta_0}{1-|\lambda_{max}(A_K)|} \left(1 + \tfrac{\kappa(V)\|\Delta\|}{1-|\lambda_{max}(A_K)|}\right) - \tfrac{\|BK\|\sqrt{N_a}\delta_1}{\beta-|\lambda_{max}(A_K)|} \left(1 + \tfrac{\kappa(V)\|\Delta\|}{\beta-|\lambda_{max}(A_K)|}\right) \\ & + \tfrac{\kappa(V)\|\Delta\|}{|\lambda_{max}(A_K)|} \ell \left(\|x(0)\| - \tfrac{\|BK\|\sqrt{N_a}\delta_0}{1-|\lambda_{max}(A_K)|} - \tfrac{\|BK\|\sqrt{N_a}\delta_1}{\beta-|\lambda_{max}(A_K)|} \right) \Bigg) \\ & + \beta^{\ell} \tfrac{\|BK\|\sqrt{N_a}\delta_1}{\beta-|\lambda_{max}(A_K)|} \left(1 + \tfrac{\kappa(V)\|\Delta\|}{\beta-|\lambda_{max}(A_K)|}\right) \Bigg). \end{aligned} \tag{A.15}$$

Defining γ_0, γ_1 as in (7.41)–(7.42), it yields to (7.40), which concludes the proof.

A.3.4 Proof of Lemma 7.4

Assume that the last broadcasting event on the subsystem i occurred at $t = t_k^{\bar{\mathcal{N}_i}}$, meaning that its own events and the neighbors' are included. If this last event does not yield a control update it means that $\|\varepsilon_{u,i}(t_k^{\bar{\mathcal{N}_i}})\| < \delta_u$. Assume that at $t = t_{k+1}^{\bar{\mathcal{N}_i}}$ there is a new broadcast in $\bar{\mathcal{N}_i}$ which triggers a control event. There are two possibilities:

- The subsystem i triggers the event. Thus,

$$\begin{aligned}\|\varepsilon_{u,i}\left(t_{k+1}^{\bar{\mathcal{N}_i}}\right)\| &= \|\varepsilon_{u,i}\left(t_k^{\bar{\mathcal{N}_i}}\right) + u_i\left(t_k^{\bar{\mathcal{N}_i}}\right) - u_i\left(t_{k+1}^{\bar{\mathcal{N}_i}}\right)\| \\ &= \|\varepsilon_{u,i}\left(t_k^{\bar{N}_i}\right) + K_i\left(x_{b,i}\left(t_k^{\bar{N}_i}\right) - x_{b,i}\left(t_{k+1}^{\bar{N}_i}\right)\right)\| \\ &\leq \|\varepsilon_{u,i}\left(t_k^{\bar{\mathcal{N}_i}}\right)\| + \|K_i\|\|x_{b,i}\left(t_k^{\bar{\mathcal{N}_i}}\right) - x_{b,i}\left(t_{k+1}^{\bar{\mathcal{N}_i}}\right)\|,\end{aligned}$$

  that is upper bounded by

$$\|\varepsilon_{u,i}\left(t_{k+1}^{\bar{\mathcal{N}_i}}\right)\| \leq \delta_u + \|K_i\|\left(\delta_{x,0} + \delta_{x,1} e^{-\beta t_{k+1}^{\bar{\mathcal{N}_i}}}\right).$$

- The event has been triggered for any neighbor $j \in \mathscr{N}_i$, it yields

$$\|\varepsilon_{u,i}\left(t_{k+1}^{\bar{\mathscr{N}_i}}\right)\| = \|\varepsilon_{u,i}\left(t_k^{\bar{\mathscr{N}_i}}\right) + L_{ij}\left(x_{b,j}\left(t_k^{\bar{\mathscr{N}_i}}\right) - x_{b,j}\left(t_{k+1}^{\bar{\mathscr{N}_i}}\right)\right)\|$$
$$\leq \delta_u + \|L_{ij}\|\left(\delta_{x,0} + \delta_{x,1}e^{-\beta t_{k+1}^{\bar{\mathscr{N}_i}}}\right).$$

Since this holds for all t, and if the worst case is considered, it yields (7.52). Moreover, from (7.49), it follows that

$$\|\varepsilon_u(t)\| \leq \sqrt{\sum_{i=1}^{N_a} \|\varepsilon_{u,i}\|^2(t)} \leq \sqrt{\sum_{i=1}^{N_a} \bar{\delta}_{u,i}^2(t)} \leq \sqrt{N_a(\max\{\bar{\delta}_{u,i}(t)\})^2},$$

which is equivalent to (7.53).

A.3.5 Proof of Theorem 7.3

The state of the system at any time is given by

$$x(t) = e^{(A_K+\Delta)t}x(0) + \int_0^t e^{(A_K+\Delta)(t-s)}(BK\varepsilon_x(s) + B\varepsilon_u(s))ds.$$

The error ε_x is bounded by $\sqrt{N_a}(\delta_{x,0} + \delta_{x,1}e^{-\beta t})$ and the bound on ε_u is derived in Lemma 7.4. Moreover, as already proved, it holds that

$$\|e^{(A_K+\Delta)t}\| \leq \kappa(V)e^{-(|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|)t}.$$

With these considerations, the bound on $x(t)$ can be calculated following the used methodology in the previous proofs to derive (7.55), showing that the system is globally ultimately bounded. Furthermore, (7.55) is upper bounded by

$$\|x(t)\| \leq \sigma_1 + \kappa(V)\|x(0)\|e^{-(|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|)t} + \sigma_2 e^{-\beta t}, \tag{A.16}$$

if the negative terms are omitted.

The Zeno behavior exclusion in the broadcasting and, as a consequence, in the control update, can also be proved similar to the previous results. Note that in the inter-event times $\|\dot{\varepsilon}_i(t)\| \leq \|\dot{x}_i(t)\| \leq \|\dot{x}(t)\|$, and $\|\dot{x}(t)\|$ can be bounded according to (7.50). Thus,

$$\|\varepsilon_i(t)\| \leq \int_{t^*}^t (\|A_K + \Delta\|\|x(s)\| + \|BK\|\|\varepsilon_x(s)\| + \|B\|\|\varepsilon_u(s)\|)ds.$$

If $x(t)$ is bounded according to (A.16), and the corresponding bounds on ε_x and ε_u are considered, it leads to the following lower bound for the inter-event time

$$T_{x,min} = \frac{\delta_{x,0}}{\gamma_1 + \sqrt{N_a}(\gamma_2 + \gamma_3 + \gamma_4)},$$

where

$$\begin{aligned}
\gamma_1 &= \kappa(V)\|x(0)\|\|A_K + \Delta\| \\
\gamma_2 &= (\|BK\| + \|B\|\|\mu(K)\|_{max})\delta_{x,0}\left(1 + \frac{\kappa(V)\|A_K+\Delta\|}{|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|}\right) \\
\gamma_3 &= (\|BK\| + \|B\|\|\mu(K)\|_{max})\delta_{x,1}\left(1 + \frac{\kappa(V)\|A_K+\Delta\|}{|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|-\beta}\right) \\
\gamma_4 &= \|B\|\delta_u\left(1 + \frac{\kappa(V)\|A_K+\Delta\|}{|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|}\right).
\end{aligned}$$

A.3.6 Proof of Theorem 7.4

Define the overall system state estimation as $x_m = (x_{m,1}^T, \ldots, x_{m,N_a}^T)^T$. Let us prove that the bound for the inter-events time is larger in the model-based approach.

If the last event occurred at t^*, the error in the inter-event time is $\|\varepsilon_i(t)\| \leq \int_{t^*}^{t} \|\dot{\varepsilon}_i(s)\| ds$. In this interval, it also holds that

$$\|\dot{\varepsilon}_i(t)\| = \|\dot{x}_{m,i}(t) - \dot{x}_i(t)\| \leq \|\dot{x}_m(t) - \dot{x}(t)\|.$$

Observe that

$$\begin{aligned}
\dot{x}_m(t) - \dot{x}(t) &= A_{mK}x_m(t) - ((A_K + \Delta)x(t) + BK\varepsilon(t)) \\
&= (\delta A_K - \Delta)x(t) + (A_{mK} - BK)\varepsilon(t).
\end{aligned}$$

Then

$$\begin{aligned}
\|\dot{\varepsilon}_i(t)\| &\leq \|\delta A_K - \Delta\|\|x(t)\| + \|A_{mK} - BK\|\|\varepsilon(t)\| \\
&\leq \|\delta A_K - \Delta\|\|x(t)\| + \|A_{mK} - BK\|\sqrt{N_a}(\delta_0 + \delta_1 e^{-\beta t}). \qquad \text{(A.17)}
\end{aligned}$$

Assume that $\delta_0, \delta_1 \neq 0$. It holds that $\delta_0 + \delta_1 e^{-\beta t} \leq \delta_0 + \delta_1 e^{-\beta t^*}$. As already stated, the bound on the state of Theorem 7.1 holds, and can be upper bounded as

$$\|x(t)\| \leq \kappa(V)\left(\|x(0)\|e^{-\alpha_\Delta t} + \frac{\|BK\|\sqrt{N_a}\delta_0}{\alpha_\Delta} + \frac{\|BK\|\sqrt{N_a}\delta_1}{\alpha_\Delta - \beta}e^{-\beta t}\right).$$

Moreover, it holds that $\|\delta A_K - \Delta\| \leq \|\delta A_K\| + \|\Delta\|$. Thus, the error

$$\|\varepsilon_i(t)\| \leq \int_{t^*}^{t} \|\dot{\varepsilon}(s)\| ds \leq \Bigg((\|\delta A_K\| + \|\Delta\|)\kappa(V) \left(\|x(0)\| e^{-\alpha_\Delta t^*} + \tfrac{\|BK\|\sqrt{N_a}\delta_0}{\alpha_\Delta} + \tfrac{\|BK\|\sqrt{N_a}\delta_1}{\alpha_\Delta - \beta} e^{-\beta t^*} \right) + \|A_{mK} - BK\|\sqrt{N_a} \left(\delta_0 + \delta_1 e^{-\beta t^*} \right) \Bigg) (t - t^*). \tag{A.18}$$

It follows that $\|\varepsilon_i(t)\| \leq (k_{m,1} + k_{m,2} + k_{m,3})(t - t^*)$, where

$$\begin{aligned} k_{m,1} &= \kappa(V)\|x(0)\|(\|\delta A_K\| + \|\Delta\|) \\ k_{m,2} &= \left(\tfrac{\kappa(V)(\|\delta A_K\| + \|\Delta\|)\|BK\|}{\alpha_\Delta - \beta} + \|A_{mK} - BK\| \right) \sqrt{N_a}\delta_1 \\ k_{m,3} &= \left(\tfrac{\kappa(V)(\|A_{mK}\| + \|\Delta\|)\|BK\|}{\alpha_\Delta} + \|A_{mK} - BK\| \right) \sqrt{N_a}\delta_0. \end{aligned} \tag{A.19}$$

The next event will not occur before $\|\varepsilon_i(t)\| = \delta_0 + \delta_1 e^{-\beta t} \geq \delta_0$. This condition gives a lower bound for the broadcasting period

$$T_{m,min} = \frac{\delta_0}{k_{m,1} + k_{m,2} + k_{m,3}}, \tag{A.20}$$

that is larger than the lower bound in (7.29) if $k_{m,1} + k_{m,2} + k_{m,3} < k_1 + k_2 + k_3$, which is equivalent to

$$(\|A_{mK} - BK\| - \|BK\|)\sqrt{N_a}(\delta_0 + \delta_1) < (\|A_K + \Delta\| - \|\delta A_K\| - \|\Delta\|)\Big(\|x(0)\| + \tfrac{\|BK\|\sqrt{N_a}\delta_0}{\alpha_\Delta} + \tfrac{\|BK\|\sqrt{N_a}\delta_1}{\alpha_\Delta - \beta} \Big).$$

After some manipulations

$$\frac{\sqrt{N_a}(\delta_0 + \delta_1)}{\|x(0)\| + \frac{\|BK\|\sqrt{N_a}\delta_0}{\alpha_\Delta} + \frac{\|BK\|\sqrt{N_a}\delta_1}{\alpha_\Delta - \beta}} < \kappa(V)\frac{\|A_K + \Delta\| - \|\delta A_K\| - \|\Delta\|}{\|A_{mK} - BK\| - \|BK\|}. \tag{A.21}$$

The denominator on the right-hand side can be bounded as

$$\|A_{mK} - BK\| - \|BK\| \leq \|A_{mK}\| + \|BK\| - \|BK\| = \|A_{mK}\|.$$

Then if Assumption 7.5 holds, (A.21) is fulfilled. Thus, the lower bound for the broadcasting period is larger for the model-based approach.

A.4 Chapter 9

A.4.1 Proof of Proposition 9.2

The dynamics of a single node is given by (9.6):

$$\hat{x}_i(k+1) = (A+BK)\hat{x}_i(k) + M_i(y_i(k) - C_i\hat{x}_i(k)) + \sum_{j\in\mathcal{N}_i} N_{ij}(\hat{x}_j(k) - \hat{x}_i(k)).$$

And the observation error at instant $k+1$ can be obtained using Proposition 9.1:

$$\begin{aligned} e_i(k+1) &= x(k+1) - \hat{x}_i(k+1) \\ &= (A+BK)x(k) - \sum_{i=1}^{p} B_iK_ie_i(k) - (A+BK)\hat{x}_i(k) - M_i(y_i(k) \\ &\quad - C_i\hat{x}_i(k)) - \sum_{j\in\mathcal{N}_i} N_{ij}(\hat{x}_j(k) - \hat{x}_i(k)). \end{aligned} \tag{A.22}$$

We can write $e_i(k+1) = (tr1) - (tr2)$, where $(tr1)$ are the terms of (A.22) which do not depend on the neighbors and $(tr2)$ are the others. Consider first the terms $(tr1)$.

$$\begin{aligned} (tr1) &= (A+BK)e_i(k) - M_iC_ie_i(k) - \sum_{i=1}^{p} B_iK_ie_i(k) \\ &= (A - M_iC_i + BK)e_i(k) - \sum_{i=1}^{p} B_iK_ie_i(k). \end{aligned} \tag{A.23}$$

Consider now $(tr2)$:

$$\begin{aligned} (tr2) &= \sum_{j\in\mathcal{N}_i} N_{ij}(\hat{x}_j(k) - \hat{x}_i(k)) \\ &= \sum_{j\in\mathcal{N}_i} N_{ij}(e_i(k) - e_j(k)). \end{aligned} \tag{A.24}$$

Using Eqs. (A.23)–(A.24) the observation error at instant $k+1$ can be written as

$$e_i(k+1) = (A - M_iC_i)\, e_i(k) + BKe_i(k) - \sum_{i=1}^{p} B_iK_ie_i(k) - \sum_{j\in\mathcal{N}_i} N_{ij}(e_i(k) - e_j(k)).$$

Finally, since the error vector was defined as $e^T(k) = \begin{bmatrix} e_1^T(k) \ldots e_p^T(k) \end{bmatrix}$, it is immediate that the dynamics of $e(k)$ is (9.12). The proof is completed.

A.5 Chapter 10

A.5.1 Proof of Theorem 10.1

In order to prove the theorem, let us assume that Assumption 10.1 holds.

The analysis will derive an upper bound for the delay which preserves this assumption. The error in the time interval $[t_k^i, t_k^i + \tau_k^i)$ is given by

$$\varepsilon_i(t_k^i + \tau_k^i) - \varepsilon_i(t_k^i) = x_i(t_k^i) - x_i(t_k^i + \tau_k^i),$$

since the broadcast state $x_{b,i}$ is not updated in any agent before the time instance $t_k^i + \tau_k^i$ according to the WfA protocol, so that $x_{b,i}(t_k^i + \tau_k^i) = x_{b,i}(t_k^i) = x_i(t_{k-1}^i)$ holds. This yields

$$\varepsilon_i(t_k^i + \tau_k^i) - \varepsilon_i(t_k^i) = \left(I - e^{A_{K,i}\tau_k^i}\right) x_i(t_k^i) + \int_0^{\tau_k^i} e^{A_{K,i}s} \left(B_i K_i \varepsilon_i(s) + B_i \sum_{j\in\mathcal{N}_i} L_{ij}\varepsilon_j(s)\right) ds,$$

based on which the upper bound for the delay τ_k^i can be derived as

$$\tau_{max,k}^i = \arg\min_{\tau_k^i \geq 0} \left\{ \| \left(I - e^{A_{K,i}\tau_k^i}\right) x_i(t_k^i) + \int_0^{\tau_k^i} e^{A_{K,i}s} \left(B_i K_i \varepsilon_i(s) + B_i \sum_{j\in\mathcal{N}_i} L_{ij}\varepsilon_j(s)\right) ds \| = \delta \right\}.$$

Note that this bound depends on $x_i(t_k^i)$. In order to guarantee the existence of the bound for the delay, we need to find an upper bound of the state for any t_k^i. The state at any time is given by $x_i(t) = e^{A_{K,i}t}x_i(0) + \int_0^t e^{A_{K,i}(t-s)} \left(B_i K_i \varepsilon_i(s) + B_i \sum_{j\in\mathcal{N}_i} L_{ij}\varepsilon_j(s)\right) ds$. The error is bounded by $\|\varepsilon_i(t)\| < 2\delta, \forall i$ by Proposition 10.1. Thus, a bound on $x_i(t)$ can be calculated following the methodology of Chap. 8 as (10.5).

Note that (10.5) is upper bounded by

$$\|x_i(t)\| \leq \kappa(V_i) \left(\frac{\|B_i K_i\| 2\delta + (\sum_{j\in\mathcal{N}_i} \|B_i L_{ij}\|) 2\delta}{|\alpha_{max}(A_{K,i})|} + \|x_i(0)\| \right), \quad \forall t, \tag{A.25}$$

if the negative terms are omitted, and using that $e^{-|\lambda_{max}(A_{K,i})|t} \leq 1, \forall t \geq 0$.

In order to derive an upper bound for the delay for any t, we recall that

$$\dot{\varepsilon}_i(t) = -A_{K,i}x_i(t) - B_iK_i\varepsilon_i(t) - \sum_{j\in\mathcal{N}_i} B_iL_{ij}\varepsilon_j(t)$$

holds in the interval $t \in [t^i_{k-1}+\tau^i_{k-1}, t^i_k+\tau^i_k)$ for any two consecutive events t^i_{k-1}, t^i_k, and, in particular, it holds in the subinterval $[t^i_k, t^i_k + \tau^i_k) \subset [t^i_{k-1} + \tau^i_{k-1}, t^i_k + \tau^i_k)$. Hence, $\dot{\varepsilon}_i(t)$ can be bounded as

$$\begin{aligned}\|\dot{\varepsilon}_i(t)\| &= \|A_{K,i}x_i(t) + B_iK_i\varepsilon_i(t) + \sum_{j\in\mathcal{N}_i} B_iL_{ij}\varepsilon_j(t)\| \\ &\le \|A_{K,i}\|\|x_i(t)\| + \|B_iK_i\|\|\varepsilon_i(t)\| + \sum_{j\in\mathcal{N}_i}\|B_iL_{ij}\|\|\varepsilon_j(t)\|. \end{aligned} \tag{A.26}$$

The state $x_i(t)$ can be bounded according to (A.25), and for the error it holds that $\|\varepsilon_i(t)\| < 2\delta$ (see Proposition 10.1). Thus, (A.26) can be integrated straightforward in the interval $[t^i_k, t^i_k + \tau^i_k)$, and it yields

$$\begin{aligned}&\|\varepsilon_i(t^i_k + \tau^i_k) - \varepsilon_i(t^i_k)\| \\ &\le \left(\|A_{K,i}\|\kappa(V_i)\left(\|x_i(0)\| + \frac{(\|B_iK_i\|+\sum_{j\in\mathcal{N}_i}\|B_iL_{ij}\|)2\delta}{|\alpha_{max}(A_{K,i})|}\right) \right.\\ &\left.\quad + (\|B_iK_i\|+\textstyle\sum_{j\in\mathcal{N}_i}\|B_iL_{ij}\|)2\delta\right)\tau^i_k.\end{aligned}$$

Thus, the delay bound (10.5) for agent i ensures that Assumption 10.1 is not violated, and this concludes the proof.

A.5.2 Proof of Corollary 10.1

Assuming that an event was triggered at time t^i_k. The accumulated error after n^i_p consecutive packet losses and a transmission delay $\tau^i_k \le \bar{\tau}^i$ is

$$\begin{aligned}&\underbrace{\left(\varepsilon_i\left(t^i_k + T^i_W\right) - \varepsilon_i\left(t^i_k\right)\right) + \left(\varepsilon_i\left(t^i_k + 2T^i_W\right) - \varepsilon_i\left(t^i_k + T^i_W\right)\right) + \cdots}_{n_{p^i}\ \text{times}} \\ &+ \left(\varepsilon_i\left(t^i_k + n^i_pT^i_W + \tau^i_k\right) - \varepsilon_i\left(t^i_k + n^i_pT^i_W\right)\right) \\ &= \varepsilon_i\left(t^i_k + n^i_pT^i_W + \tau^i_k\right) - \varepsilon_i\left(t^i_k\right).\end{aligned} \tag{A.27}$$

Since $n_p^i T_W^i + \tau_k^i \leq n_p^i T_W^i + \bar{\tau}^i = (n_p^i + 1)\bar{\tau}^i = \tau_{max}^i$, and τ_{max}^i is also the minimum inter-event time for the system, this implies that $\|\varepsilon_i(t_k^i + n_p^i T_W^i + \tau_k^i) - \varepsilon_i(t_k^i)\| < \delta$. Hence, $\|\varepsilon_i(t)\| < 2\delta$ holds and so does the bound (10.6).

A.5.3 Proof of Theorem 10.2

According to the UwR protocol, $\|\varepsilon_i(t)\| \leq \delta$ holds and $\varepsilon_i(t) \neq \varepsilon_{ij}(t)$, in general. However, as Assumption 10.1, $\|\varepsilon_{ij}(t_k^{ij}) - \varepsilon_i(t_k^i)\| < \delta$ yields $\|\varepsilon_{ij}(t)\| < 2\delta$.

Thus, a bound on the state can be derived from (10.9) in a similar way as in Theorem 10.1 and (10.12) holds. The proof of the first part of the theorem can be obtained by following the proof of Theorem 10.1, since in the interval $[t_k^i, t_k^{ij})$ the state information $x_{b,ij}$ remains constant in the agent j, so that $\dot{\varepsilon}_{ij}(t) = -\dot{x}_i(t)$ holds. If the error $\varepsilon_{ij}(t)$ is integrated in the interval $[t_k^i, t_k^{ij})$ considering that the state is bounded by (10.12), and that the error is bounded as discussed above, then (10.11) is derived. Finally, (10.10) can be derived as in Corollary 10.1.

A.5.3.1 Proof of Proposition 10.2

Assume that the last event occurred at time t_k^i and that the maximum transmission delay to its neighbors is τ_k^i. From Assumption 10.1, it follows that

$$\| \int_{t_k^i}^{t_k^i + \tau_k^i} \dot{\varepsilon}_i(s) ds \| = \|\varepsilon_i\left(t_k^i + \tau_k^i\right) - \varepsilon_i\left(t_k^i\right)\| < \delta_1 e^{-\beta\left(t_k^i + \tau_k^i\right)}, \tag{A.28}$$

has to be satisfied (see (10.13)) because no event is generated in the time interval $[t_k^i, t_{k+1}^i)$. Since an event has occurred at time t_k^i, $\|\varepsilon_i\left(t_k^i\right)\| = \delta_1 e^{-\beta t_k^i}$ holds and, thus

$$\|\varepsilon_i\left(t_k^i + \bar{\tau}_k^i\right)\| < \delta_1 e^{-\beta t_k^i} + \delta_1 e^{-\beta\left(t_k^i + \tau_k^i\right)} = \delta_1\left(1 + e^{\beta \tau_k^i}\right) e^{-\beta\left(t_k^i + \tau_k^i\right)},$$

must hold. Because this result is valid for any time t and $e^{\beta\tau_k^i} < e^{\beta\tau_{max}}$, $\forall \tau_k^i < \tau_{max}$, it follows that:

$$\|\varepsilon_i(t)\| < \delta_1(1 + e^{\beta\tau_{max}})e^{-\beta t}.$$

A.5.4 Proof of Theorem 10.3

The state at any time is given as

$$x_i(t) = e^{A_{K,i}t} x_i(0) + \int_0^t e^{A_{K,i}(t-s)} \left(B_i K_i \varepsilon_i(s) + B_i \sum_{j \in \mathcal{N}_i} L_{ij} \varepsilon_j(s) \right) ds.$$

According to Proposition 10.2, the error is bounded by $\|\varepsilon_i(t)\| < \delta_1(1+e^{\beta\tau_{max}})e^{-\beta t}$. Thus, a bound on $x_i(t)$ can be calculated following the methodology of Chap. 8 as

$$\|x_i(t)\| \leq \kappa(V_i)\left(\frac{\mu_i\delta_1(1+e^{\beta\tau_{max}})e^{-\beta t}}{|\alpha_{max}(A_{K,i})|-\beta} + e^{-|\alpha_{max}(A_{K,i})|t}\left(\|x_i(0)\| - \frac{\mu_i\delta_1(1+e^{\beta\tau_{max}})e^{-\beta t}}{|\alpha_{max}(A_{K,i})|-\beta}\right)\right),$$

which proves the second part of the theorem.

Note that (10.18) can be upper bounded as

$$\|x_i(t)\| \leq \kappa(V_i)\left(\frac{\mu_i\delta_1(1+e^{\beta\tau_{max}})e^{-\beta t}}{|\alpha_{max}(A_{K,i})|-\beta} + e^{-|\alpha_{max}(A_{K,i})|t}\|x_i(0)\|\right). \tag{A.29}$$

Moreover, in the interval $t \in [t_{k-1}^i + \tau_{k-1}^i, t_k^i + \tau_k^i)$ it holds that

$$\dot{\varepsilon}_i(t) = -A_{K,i}x_i(t) - B_iK_i\varepsilon_i(t) - \sum_{j\in\mathcal{N}_i} B_iL_{ij}\varepsilon_j(t),$$

and this is particularly true in the subinterval $[t_k^i, t_k^i + \tau_k^i)$. Thus

$$\begin{aligned}\|\dot{\varepsilon}_i(t)\| &= \|A_{K,i}x_i(t) + B_iK_i\varepsilon_i(t) + \sum_{j\in\mathcal{N}_i} B_iL_{ij}\varepsilon_j(t)\| \\ &\leq \|A_{K,i}\|\|x_i(t)\| + \|B_iK_i\|\|\varepsilon_i(t)\| + \sum_{j\in\mathcal{N}_i}\|B_iL_{ij}\|\|\varepsilon_j(t)\|.\end{aligned}$$

Therefore, integrating the error in the interval $[t_k^i, t_k^i + \bar{\tau}_k^i)$ and noting that $\|x_i(t)\| \leq \|x_i(t_k^i)\|$ in (A.29) in this interval

$$\begin{aligned}\|\varepsilon_i(t_k^i + \tau_k^i) - \varepsilon_i(t_k^i)\| \leq &\left(\|A_{K,i}\|\kappa(V_i)\left(\frac{\mu_i\delta_1(1+e^{\beta\tau_{max}})e^{-\beta t_k^i}}{|\alpha_{max}(A_{K,i})|-\beta} + e^{-|\alpha_{max}(A_{K,i})|t_k^i}\|x_i(0)\|\right)\right. \\ &\left. + \mu_i\delta_1\left(1+e^{\beta\tau_{max}}\right)e^{-\beta t_k^i}\right)\tau_k^i.\end{aligned}$$

Denote $k_{1,i} = \|A_{K,i}\|\kappa(V_i)\|x_i(0)\|$ and $k_{2,i} = (\|A_{K,i}\|\kappa(V_i)\frac{1}{|\alpha_{max}(A_{K,i})|-\beta} + 1)\mu_i\delta_1$. From (A.28) in Preposition 10.2, it follows that the upper bound on the delay satisfies

$$\left(k_{1,i}e^{-|\alpha_{max}(A_{K,i})|t_k^i} + k_{2,i}\left(1+e^{\beta\tau_{max}}\right)e^{-\beta t_k^i}\right)\tau_k^i = \delta_1 e^{-\beta\left(t_k^i+\tau_k^i\right)}.$$

It yields that

$$\left(\frac{k_{1,i}}{\delta_1}e^{-(|\alpha_{max}(A_{K,i})|-\beta)t_k^i} + \frac{k_{2,i}}{\delta_1}\left(1+e^{\beta\tau_{max}}\right)\right)\tau_k^i = e^{-\beta\tau_k^i}.$$

The right-hand side is always positive and takes values in the interval [0, 1). The left-hand side is also positive and its image is $[0, +\infty)$. Hence, there is a positive solution for the upper bound on the delay. Moreover, the left-hand side is upper bounded by $(\frac{k_{2,i}}{\delta_1} + \frac{k_{2,i}}{\delta_1}(1 + e^{\beta\tau_{max}}))\bar{\tau}_k^i$ for $\beta < |\alpha_{max}(A_{K,i})|$. Hence, the most conservative bound on the delay τ_{max} is given as

$$\tau_{max} = \min\{\tau_{max}^i,\ i = 1, \ldots, N_a\},$$

where τ_{max}^i are the solutions of

$$\left(\frac{k_{1,i}}{\delta_1} + \frac{k_{2,i}}{\delta_1}\left(1 + e^{\beta\tau_{max}^i}\right)\right)\tau_{max}^i = e^{-\beta\tau_{max}^i}.$$

A.5.5 Proof of Theorem 10.4

The state at any time is given as

$$x_i(t) = e^{A_{K,i}t}x_i(0) + \int_0^t e^{A_{K,i}(t-s)}\left(B_iK_i\varepsilon_i(s) + B_i\sum_{j\in N_i} L_{ij}\varepsilon_{ji}(s)\right)ds.$$

Under the UwR protocol, it holds that $\|\varepsilon_i(t)\| \leq \delta_1 e^{-\beta t}$, and $\|\varepsilon_{ji}(t)\| < \delta_1(1 + e^{\beta\tau_{max}})e^{-\beta t}$. Hence, following the same steps as in the proof of Theorem 10.3, it yields

$$\|x_i(t)\| \leq \kappa(V_i)\left(\frac{\bar{\mu}_i(\tau_{max})\delta_1 e^{-\beta t}}{|\alpha_{max}(A_{K,i})|-\beta} + e^{-|\alpha_{max}(A_{K,i})|t}\left(\|x_i(0)\| - \frac{\bar{\mu}_i(\tau_{max})\delta_1 e^{-\beta t}}{|\alpha_{max}(A_{K,i})|-\beta}\right)\right),$$

where $\bar{\mu}_i(\tau_{max}) = \|B_iK_i\| + \sum_{j\in N_i}\|B_iL_{ij}\|(1 + e^{\beta\tau_{max}})$.
In the interval $[t_k^i, t_k^{ij})$, $\dot{\varepsilon}_{ij}(t) = -\dot{x}_i(t)$ holds. Thus, it can be derived easily that

$$\|\varepsilon_{ij}(t_k^{ij}) - \varepsilon_{ij}(t_k^i)\| \leq \left(k_{1,i}e^{-|\alpha_{max}(A_{K,i})|t_k^i} + \left(k_{2,i} + k_{3,i}(1 + e^{\beta\tau_{max}})\right)e^{-\beta t_k^i}\right)\tau_k^{ij},$$

$k_{1,i}$, $k_{2,i}$ and $k_{3,i}$ defined in (10.21)–(10.23).

According to Proposition 10.2, $\|\varepsilon_{ij}(t_k^{ij}) - \varepsilon_{ij}(t_k^i)\| < \delta_1 e^{-\beta t_k^{ij}}$. And the upper bound on the delay is the minimum value of τ_{max}^i which solves

$$\left(\frac{k_{1,i}}{\delta_1} + \frac{k_{2,i}}{\delta_1} + \frac{k_{3,i}}{\delta_1}\left(1 + e^{\beta\tau_{max}^i}\right)\right)\tau_{max}^i = e^{-\beta\tau_{max}^i}.$$

A.5.6 Proof of Theorem 10.5

From 10.31, the state at any time is given as

$$x(t) = e^{(A_K+\Delta)t}x(0) + \int_0^t e^{(A_K+\Delta)(t-s)}\mathrm{M}\overrightarrow{\varepsilon}(s)ds.$$

According to Proposition 10.3, the error $\overrightarrow{\varepsilon}(s)$ is bounded by $\bar{\delta}$. Moreover, since A_K is diagonalizable, $e^{(A_K+\Delta)t}$ can be bounded using (7.10). Thus, it follows that

$$\|x(t)\| \leq \kappa(V)\Big(\|x(0)\|e^{-(|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|)t} + \frac{\|\mathrm{M}\|\bar{\delta}}{|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|}\left(1 - e^{-(|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|)t}\right)\Big).$$

Reordering terms and noting that $\|\mathrm{M}\|$ is bounded by μ_{max} because is a block diagonal matrix, it falls out (10.34).

The upper bound on the delay can be derived easily noting that if the last event occurred at $t = t_k^i$, it holds that

$$\|\varepsilon_{ij}(t_k^{ij}) - \varepsilon_{ij}(t_k^i)\| \leq \int_{t_k^i}^{t_k^{ij}} \|\dot{\varepsilon}_{ij}(s)\|ds \leq \int_{t_k^i}^{t_k^{ij}} \|\dot{x}_i(s)\|ds \leq \int_{t_k^i}^{t_k^{ij}} \|\dot{x}(s)\|ds,$$

since $x_{b,ij}$ remain constant in the interval and $\|\dot{x}_i(s)\| \leq \|\dot{x}(s)\|$.

Because $\|\dot{x}(s)\| \leq \|A_K + \Delta\|\|x(s)\| + \|\mathrm{M}\|\|\overrightarrow{\varepsilon}(s)\|$, it yields

$$\|\varepsilon_{ij}(t_k^{ij}) - \varepsilon_{ij}(t_k^i)\| \leq \Big(\|A_K + \Delta\|\kappa(V)\Big(\|x(0)\| + \frac{\|\mathrm{M}\|\bar{\delta}}{|\alpha_{max}(A_K)|-\kappa(V)\|\Delta\|}\Big) + \|\mathrm{M}\|\bar{\delta}\Big)(t_k^{ij} - t_k^i).$$

According to Assumption 10.1, no event occurs before the broadcast state is successfully received and, therefore the increase of the error in the interval $[t_k^i, t_k^{ij})$ is bounded by δ, giving the upper bound on the delay (10.33).

A.6 Chapter 11

A.6.1 Proof of Theorem 11.1

Choose the following Lyapunov–Krasovskii functional:

$$V(k) = e^T(k)Pe(k) + \sum_{i=k-\tau_{max}}^{k-1} e^T(i)Z_1e(i) + l \times \tau_{max} \sum_{j=-\tau_{max}+1}^{0} \sum_{i=k+j-1}^{k-1} \Delta e^T(i)Z_2\Delta e(i), \tag{A.30}$$

where $\Delta e(k) = e(k+1) - e(k)$. Note that the third term is included l times, one for each communication link. The forward difference can be calculated as

$$\begin{aligned}
\Delta V(k) = {} & e^T(k+1)Pe(k+1) - e^T(k)Pe(k) + \\
& e^T(k)Z_1e(k) - e^T(k-\tau_{max})Z_1e(k-\tau_{max}) + \\
& l \times \tau_{max}^2 \Delta e^T(k)Z_2\Delta e(k) - l \times \tau_{max} \sum_{j=k-\tau_{max}}^{k-1} \Delta e^T(j)Z_2\Delta e(j) \\
= {} & \begin{bmatrix} e^T(k) & d^T(k) \end{bmatrix} \begin{bmatrix} \Phi^T(\mathscr{M}) \\ \Lambda^T(\mathscr{N}) \end{bmatrix} P \begin{bmatrix} \Phi(\mathscr{M}) & \Lambda(\mathscr{N}) \end{bmatrix} \begin{bmatrix} e(k) \\ d(k) \end{bmatrix} + \\
& \begin{bmatrix} e^T(k) & e^T(k-\tau_{max}) \end{bmatrix} \begin{bmatrix} Z_1 - P & 0 \\ 0 & -Z_1 \end{bmatrix} \begin{bmatrix} e(k) \\ e(k-\tau_{max}) \end{bmatrix} + \\
& l \times \tau_{max}^2 \Delta e^T(k)Z_2\Delta e(k) - l \times \tau_{max} \sum_{j=k-\tau_{max}}^{k-1} \Delta e^T(j)Z_2\Delta e(j).
\end{aligned}$$

Defining the augmented state vector

$$\xi(k) = \begin{bmatrix} e(k) \\ e(k-\tau_1(k)) \\ e(k-\tau_2(k)) \\ \vdots \\ e(k-\tau_l(k)) \\ e(k-\tau_{max}) \end{bmatrix} = \begin{bmatrix} e(k) \\ d(k) \\ e(k-\tau_{max}) \end{bmatrix},$$

the forward difference of the Lyapunov–Krasovskii functional can be written using the following quadratic form:

$$\Delta V(k) = \xi^T(k) \left(\begin{bmatrix} Z_1 - P & 0 & 0 \\ * & 0 & 0 \\ * & * & -Z_1 \end{bmatrix} + \begin{bmatrix} \Phi^T(\mathscr{M}) \\ \Lambda^T(\mathscr{N}) \\ 0 \end{bmatrix} P \left[\Phi(\mathscr{M}) \; \Lambda(\mathscr{N}) \; 0 \right] \right.$$
$$\left. + \, l \times \tau_{max}^2 \begin{bmatrix} \Phi^T(\mathscr{M}) - I \\ \Lambda^T(\mathscr{M}) \\ 0 \end{bmatrix} Z_2 \left[(\Phi(\mathscr{M}) - I) \; \Lambda(\mathscr{M}) \; 0 \right] \right) \xi(k)$$
$$-l \times \tau_{max} \sum_{j=k-\tau_{max}}^{k-1} \Delta e^T(j) Z_2 \Delta e(j).$$

In order to take into account the delay of each different communication link ($\tau_r(k), \forall r = 1, \ldots, l$), we split the last term in the above equation (which appears l times) into 2 terms, considering the delay in each specific link:

$$-\tau_{max} \sum_{j=k-\tau_{max}}^{k-1} \Delta e^T(j) Z_2 \Delta e(j) =$$
$$- \tau_{max} \sum_{j=k-\tau_{max}}^{k-\tau_r(k)-1} \Delta e^T(j) Z_2 \Delta e(j) - \tau_{max} \sum_{j=k-\tau_r(k)}^{k-1} \Delta e^T(j) Z_2 \Delta e(j).$$

The resulting terms can be bounded using the Jensen inequality:

$$-\tau_{max} \sum_{j=k-\tau_{max}}^{k-\tau_r(k)-1} \Delta e^T(j) Z_2 \Delta e(j) \le - \left[\sum_{j=k-\tau_{max}}^{k-\tau_r(k)-1} \Delta e(j) \right]^T Z_2 \left[\sum_{j=k-\tau_{max}}^{k-\tau_r(k)-1} \Delta e(j) \right],$$
$$-\tau_{max} \sum_{j=k-\tau_r(k)}^{k-1} \Delta e^T(j) Z_2 \Delta e(j) \le - \left[\sum_{j=k-\tau_r(k)}^{k-1} \Delta e(j) \right]^T Z_2 \left[\sum_{j=k-\tau_r(k)}^{k-1} \Delta e(j) \right].$$

The terms in brackets can be cancelled in pairs, except the first and the last one in the summatory, yielding:

$$-\tau_{max} \sum_{j=k-\tau_{max}}^{k-\tau_r(k)-1} \Delta e^T(j) Z_2 \Delta e(j) \le$$
$$- \left[e(k - \tau_r(k)) - e(k - \tau_{max}) \right]^T Z_2 \left[e(k - \tau_r(k)) - e(k - \tau_{max}) \right],$$

$$-\tau_{max}\sum_{j=k-\tau_r(k)}^{k-1}\Delta e^T(j)Z_2\Delta e(j)\leq$$
$$-\left[e(k)-e(k-\tau_r(k))\right]^T Z_2\left[e(k)-e(k-\tau_r(k))\right].$$

The above terms are also written in the same quadratic manner as

$$-\tau_{max}\sum_{j=k-\tau_{max}}^{k-\tau_r(k)-1}\Delta e^T(j)Z_2\Delta e(j)\leq$$
$$\left[e^T(k-\tau_r(k))\ e^T(k-\tau_{max})\right]\begin{bmatrix}-Z_2 & Z_2\\ * & -Z_2\end{bmatrix}\begin{bmatrix}e(k-\tau_r(k))\\ e(k-\tau_{max})\end{bmatrix},$$
$$-\tau_{max}\sum_{j=k-\tau_r(k)}^{k-1}\Delta e^T(j)Z_2\Delta e(j)\leq$$
$$\left[e^T(k)\ e^T(k-\tau_r(k))\right]\begin{bmatrix}-Z_2 & Z_2\\ * & -Z_2\end{bmatrix}\begin{bmatrix}e(k)\\ e(k-\tau_r(k))\end{bmatrix}.$$

Including all the terms, the forward difference is

$$\Delta V(k)\leq\xi^T(k)L_1\xi(k)$$

where

$$L_1=\xi^T(k)\left(\begin{bmatrix}\Xi & \Theta & 0\\ * & \Upsilon & \Theta^T\\ * & * & \Omega\end{bmatrix}+\begin{bmatrix}\Phi^T(\mathscr{M})\\ \Lambda^T(\mathscr{N})\\ 0\end{bmatrix}P\left[\Phi(\mathscr{M})\ \Lambda(\mathscr{N})\ 0\right]\right.$$
$$\left.+\,l\times\tau_{max}^2\begin{bmatrix}\Phi^T(\mathscr{M})-I\\ \Lambda^T(\mathscr{M})\\ 0\end{bmatrix}Z_2\left[(\Phi(\mathscr{M})-I)\ \Lambda(\mathscr{M})\ 0\right]\right)\xi(k).\quad\text{(A.31)}$$

In order to ensure the error convergence to zero, it will be demonstrated that $\Delta V(k)<0$ for all $\xi(k)\neq 0$ through the negative definiteness of L_1. Applying Schur complements, one can obtain that the previous matrix is negative definite if and only if the following holds:

$$\begin{bmatrix}\Xi & \Theta & 0 & \Phi^T(\mathscr{M}) & (\Phi^T(\mathscr{M})-I)\tau_{max}\\ * & \Upsilon & \Theta^T & \Lambda^T(\mathscr{N}) & \Lambda^T(\mathscr{N})\tau_{max}\\ * & * & \Omega & 0 & 0\\ * & * & * & -P^{-1} & 0\\ * & * & * & * & -\frac{1}{l}Z_2^{-1}\end{bmatrix}<0.$$

Finally, pre- and post-multiplying the previous matrix by $diag\{I, I, I, P, P\}$ and its transpose, this condition is equivalent to the one stated in Theorem 11.1. Therefore, the negative definiteness of this matrix is ensured.

A.6.2 Proof of Theorem 11.2

Consider the Lyapunov–Krasovskii functional (A.30). Including the disturbances due to the asynchronous flow of information, the forward difference takes the form

$$\Delta V(k) \leq \xi^T(k)L_1\xi(k) + 2\varepsilon^T(k)L_2\xi(k) + \varepsilon^T(k)L_3\varepsilon(k).$$

From Theorem 11.1 we can ensure that matrix L_1 is negative definite, so there exists a positive matrix Q such that $L_1 < -Q$. Taking norms, the forward difference can be bounded as follows:

$$\Delta V(k) \leq -\lambda_{min}^{Q} \|\xi(k)\|_\infty^2 + 2\|L_2\|_\infty \|\varepsilon(k)\|_\infty \|\xi(k)\|_\infty + \|L_3\|_\infty \|\varepsilon(k)\|_\infty^2 .$$

The triggering condition (11.12) ensures that $\|\varepsilon(k)| \leq \delta$, in such a way that

$$\Delta V(k) \leq -\lambda_{min}^{Q} \|\xi(k)\|_\infty^2 + 2\|L_2\|_\infty \|\xi(k)\|_\infty \delta + \|L_3\|_\infty \delta^2.$$

We are interested in the values of $\xi(k)$ that achieve that $\Delta V(k) \leq 0$. In order to find a feasible region, the following second-order equation in $\|\xi(k)\|$ can be solved:

$$-\lambda_{min}^{Q} \|\xi(k)\|_\infty^2 + 2\|L_2\|_\infty \|\xi(k)\|_\infty \delta + \|L_3\|_\infty \delta^2 = 0.$$

By solving the previous equation, it can be ensured that $\Delta V(k) \leq 0$ for $\|\xi(k)\|_\infty > D_1\delta$, with $D_1 = \frac{\|L_2\|_\infty + \sqrt{\|L_2\|_\infty^2 + \lambda_{min}^{Q}\|L_3\|_\infty}}{\lambda_{min}^{Q}}$.

For a generic vector x and a positive scalar D, let B_D^x denote the region of the space defined by $\{x : \|x\|_\infty \leq D\}$. Please note that the above result implies that $V(k)$ decreases for every $\xi(k) \notin B_{D_1\delta}^{\xi}$. Hence, it is obvious that there exists a time instant k^* in which $\xi(k^*)$ enters into the region $B_{D_1\delta}^{\xi}$. The augmented state vector $\xi(k)$ includes the observation error $e(k)$, so it turns out that $e(k^*) \in B_{D_1\delta}^{e}$.

As $\xi(k^*) \in B_{D_1}^{\xi}$ for any realization of $\mu_r(k) \in [0, \tau_{max}]$, $r \in \mathscr{L}$, it also holds that $\zeta(k^*) \in B_{D_1}^{\zeta}$.

From instant k^* on, the functional is not necessarily decreasing and the augmented state may jump outside the region, that is, it may occur that $\xi(k^*+1) \notin B_{D_1\delta}^{\xi}$. Using the dynamics of the observation error given in Eq. (11.14), it is possible to bound the error at instant $k^* + 1$ by

$$\begin{aligned}\|e(k^*+1)\|_\infty &< \|\Phi\|_\infty \|e(k^*)\|_\infty + \|\Lambda\|_\infty \|d(k^*)\|_\infty + \|\Gamma\|_\infty \|\varepsilon(k^*)\|_\infty \\ &< (\|\Phi\|_\infty + \|\Lambda\|_\infty) D_1 + \|\Gamma\|_\infty \delta.\end{aligned}$$

Then $e(k^* + 1) \in B^e_{D_2\delta}$, where $D_2 = (\|\Phi\|_\infty + \|\Lambda\|_\infty)D_1 + \|\Gamma\|_\infty$. This way, $\xi(k^* + 1)$, and hence $\zeta(k^* + 1)$, may leave the regions $B^{\xi}_{D_1}$ and $B^{\zeta}_{D_1}$, respectively. In that case, the Lyapunov–Krasovskii functional must be decreasing again, implying that

$$\begin{aligned}\forall k > k^* + 1, \quad V(k) &< \max\{V(k^*+1)\} = \max\{\zeta^T(k^*+1)\Psi\zeta(k^*+1)\} \\ &< \lambda^{\Psi}_{max} \max\{\left\|\zeta(k^*+1)\right\|^2_\infty\} \\ &< \lambda^{\Psi}_{max}(D_2\delta)^2.\end{aligned}$$

Finally, to get the final bound on $e(k)$ for $k > k^* + 1$, note that all the terms of the Lyapunov functional involve positive definite matrices, so

$$e(k)^T P e(k) < V(k) < \lambda^{\Psi}_{max}(D_2\delta)^2, \forall k > k^* + 1.$$

And using well-known properties, it yields

$$\begin{aligned}\lambda^P_{min} \|e(k)\|^2_\infty &< e(k)^T P e(k) < \lambda^{\Psi}_{max}(D_2\delta)^2 \\ &\Rightarrow \|e(k)\|_\infty < \sqrt{\frac{\lambda^{\Psi}_{max}}{\lambda^P_{min}}} D_2\delta.\end{aligned}$$

Appendix B
Dealing with Nonlinear Terms in Matrix Inequalities

Sometimes, when the control problems are posed as matrix inequalities, it is inevitable that some nonlinear terms appears, so the existing methods for LMIs cannot directly be applied. This appendix proposes two different solutions for some nonlinearities that are very common both in this book and in other approaches based in Lyapunov–Krasovskii theorem. By means of appropriate transformations and additional constraints, the nonlinear matrix inequality can be replaced by a problem with linear constraints.

Consider a nonlinear matrix inequality

$$\begin{bmatrix} f_{11}(X_1,\ldots,X_m) & \cdots & f_{1k}(X_1,\ldots,X_m) & \cdots & f_{1p}(X_1,\ldots,X_m) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_{1k}^T(X_1,\ldots,X_m) & \cdots & g_{kk}(X_1,\ldots,X_m) & \cdots & f_{ip}(X_1,\ldots,X_m) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ f_{1p}^T(X_1,\ldots,X_m) & \cdots & f_{kp}^T(X_1,\ldots,X_m) & \cdots & f_{pp}(X_1,\ldots,X_m) \end{bmatrix} < 0, \tag{B.1}$$

where f are affine functions on the decision variables $X_1, \ldots, X_m$ and g are nonlinear functions with the following particular structure:

$$g(X_1,\ldots,X_m) = -X_i X_j^{-1} X_i,\ i \neq j.$$

Note that the nonlinear function appears in the diagonal of the inequality. In the following sections, two solutions are given to deal with the nonlinearity $X_i X_j^{-1} X_i$, $i \neq j$. The first introduces an additional constraint which lets us address the problem by means of a set of linear matrix inequalities. The second solution employs the *cone complementary algorithm* to transform the nonlinear inequality into an iterative optimization problem with linear constraints. Comparing both solutions, the former could be more conservative, but it is computationally more efficient, as the number of constraints and variables is lower.

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B.1 Direct Constraint

Consider the introduction of the following additional constraint:

$$-X_i X_j^{-1} X_i < -\frac{1}{\mu} X_i,$$

being μ a positive design scalar. Note that previous condition is equivalent to $X_j < \mu X_i$. Then, the nonlinear constraint in Eq. (B.1) can be replaced by

$$\begin{cases} \Upsilon(X_1, \ldots, X_m) < 0, \\ X_j < \mu X_i \end{cases} \tag{B.2}$$

where Υ is the matrix required to be negative definite in (B.1), but substituting the terms $g(X_1, \ldots, X_m) = -X_i X_j^{-1} X_i$ by $-\frac{1}{\mu} X_i$.

It is worth comparing the proposed method with that introduced in [275] and used in other papers to handle the same nonlinearity. While in [275] it is directly imposed X_j to be X_i times a given scalar, this method just restricts $X_j < \mu X_i$, which covers a much wider range of possible solutions in the space of positive definite matrices. Therefore, it leads to less conservative solutions.

B.2 Cone Complementary Algorithm

Another possibility consists in using the well-known *cone complementary algorithm*. The idea is the following: first, the nonlinear inequality can be addressed by solving an optimization problem with linear constraints. Then, a solution for this problem can be found with an extended algorithm whose convergence is theoretically ensured.

Following the idea of [184], define a variable T such that

$$X_i X_j^{-1} X_i \geq T > 0, \tag{B.3}$$

which is equivalent to

$$\begin{bmatrix} -T^{-1} & X_i^{-1} \\ X_i^{-1} & -X_j^{-1} \end{bmatrix} \leq 0. \tag{B.4}$$

Now, introducing some new variables,

$$\hat{X}_i = X_i^{-1}, \quad \hat{T} = T^{-1}, \quad \hat{X}_j = X_j^{-1}, \tag{B.5}$$

Equation (B.4) can be rewritten as

$$\begin{bmatrix} -\hat{T} & \hat{X}_i \\ \hat{X}_i & -\hat{X}_j \end{bmatrix} \leq 0. \tag{B.6}$$

Now, instead of using the original nonlinear inequality (B.1), consider the following nonlinear minimization problem involving LMI conditions:

$$\text{Minimize } \mathbf{Tr}\left(\hat{X}_i X_i + \hat{X}_j X_j + \hat{T} T\right) \tag{B.7}$$

subject to

$$\begin{cases} \Upsilon(X_1, \ldots, X_m) < 0, \\ \begin{bmatrix} -\hat{T} & \hat{X}_i \\ * & -\hat{X}_j \end{bmatrix} \leq 0, \begin{bmatrix} X_i & I \\ * & \hat{X}_i \end{bmatrix} \geq 0, \begin{bmatrix} X_j & I \\ * & \hat{X}_j \end{bmatrix} \geq 0, \begin{bmatrix} T & I \\ * & \hat{T} \end{bmatrix} \geq 0, \end{cases} \tag{B.8}$$

where Υ is as before the matrix required to be definite negative in (B.1), but substituting $X_i X_j^{-1} X_i$ by T. From Eqs. (B.3), it is immediate that, if $\Upsilon < 0$, then (B.1) holds. The minimization problem is introduced to force (B.5). When the LMIs in the second row of the restrictions (B.8) saturate, the optimum is reached and (B.1) holds.

In order to solve the aforementioned minimization problem (B.7) the following algorithm introduced in [64] can be implemented:

---

1. Set $k = 0$. Find a feasible solution under the conditions in (B.8):

$$(X_1^0, X_2^0, \ldots, X_m^0, T^0, \hat{X}_i^0, \hat{X}_j^0, \hat{T}^0)$$

If there is no solution, exit.
2. Solve the following optimization problem with LMI constraints with decision variables $(X_1, X_2, \ldots, X_m, T, \hat{X}_i, \hat{X}_j, \hat{T})$

$$\min \quad \mathbf{Tr}\left(\hat{X}_i^k X_i + X_i^k \hat{X}_i + \hat{X}_j^k X_j + X_j^k \hat{X}_j + \hat{T}^k T + T^k \hat{T}\right)$$

subject to LMIs in (B.8)

Set $X_i^{k+1} = X_i$, $\hat{X}_i^{k+1} = \hat{X}_i$, $X_j^{k+1} = X_j$, $\hat{X}_j^{k+1} = \hat{X}_j$, $\hat{T}^{k+1} = \hat{T}$, $T^{k+1} = T$.
3. If the condition (B.1) is satisfied, exit. Otherwise, set $k = k + 1$ and return to Step 2.

The first and second steps of the algorithm are simple LMI problems, and they can be solved efficiently using an appropriate computational software. As it is stated in Theorem 2.1 in [64], the algorithm converges and then $\hat{X}_i X_i = I$, $\hat{X}_j X_j = I$, $\hat{T} T = I$.

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