# Appendix

## *Results of Numerical 3D Simulation of the Thermal Evolution of the Earth and the Moon at the Process of Their Accumulation*

To analyze the temperature distribution in the growing proto-planet we used Safronov's [1] model, the most valid model available today, of the accumulation of the planets of the Earth's group and his equation (Chap. 4, (4.1)), which describes the changing of the proto-planet's mass with the growing velocity. The process of body collision depends on their mass, composition and state and research of that process is a great problem (see, for instance, [2, 3]). In that statement the quantitative description of planetary accumulation from the proto-planet cloud is beyond the realm of current computer engineering. In the Safronov equation the whole of the statistics of the collision process is approximated by a parameter θ—a statistical parameter that takes into account the distribution of the particles by masses and velocities into the planets' "power" zone. Beginning from the papers [4, 5], we take into account the 3-D distribution of falling bodies by defining boundary conditions as modified boundary conditions (4.3) (Chap. 4).

On the outer surface of the spherical layer with a thickness $\Delta r$ during time $dt$ the whole increase of the energy in it is described by a term in the left part of equation (Chap. 4, (4.3)), which depends on the coordinates of the surface points. The solution is discovered in the 1/8 part of the spherical body. Since for further solution of the problem we use the finite-difference method, for each cell of the surface we define the part of energy that is obtained from the whole amount of obtained energy, using a random number generator. In Fig. A.1 we can see the variants of such temperature distribution.

As seen from the results obtained, the temperature distribution, which is responsible for collisions of accumulated bodies and particles, is very heterogeneous and during the time interval required for forming the next layer it does not become smooth. This is clearly seen in Fig. A.2 from [6], which presents a set of variants of temperature sections of the growing Moon with a subsequently growing radius.

The main peculiarities of the temperature distribution in the Moon's model composition are as follows: after reaching the end of the accumulation process,



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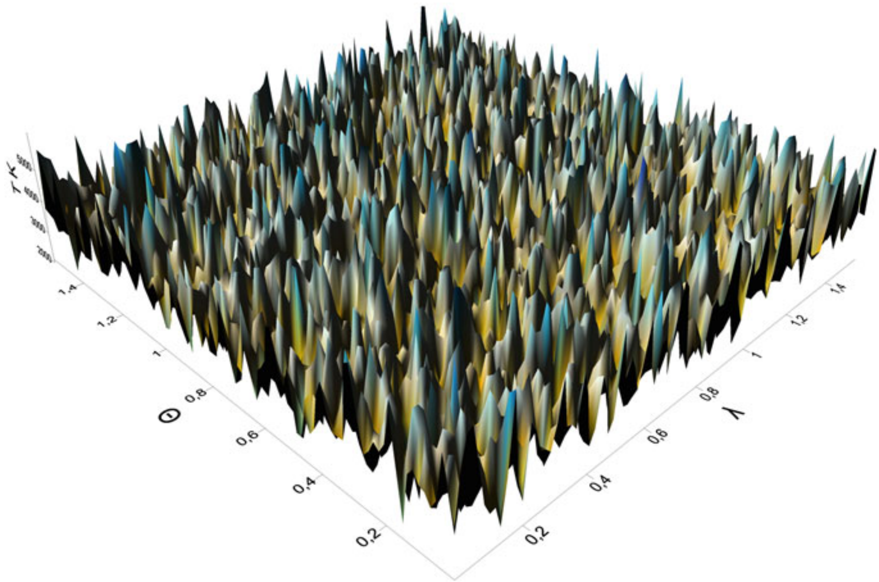


**Fig. A.1** The variant of the temperature distribution, stipulated by the random distribution of the accumulated bodies by their values and kinetic energy on the planet's surface at the radius R = 1000 km [5]

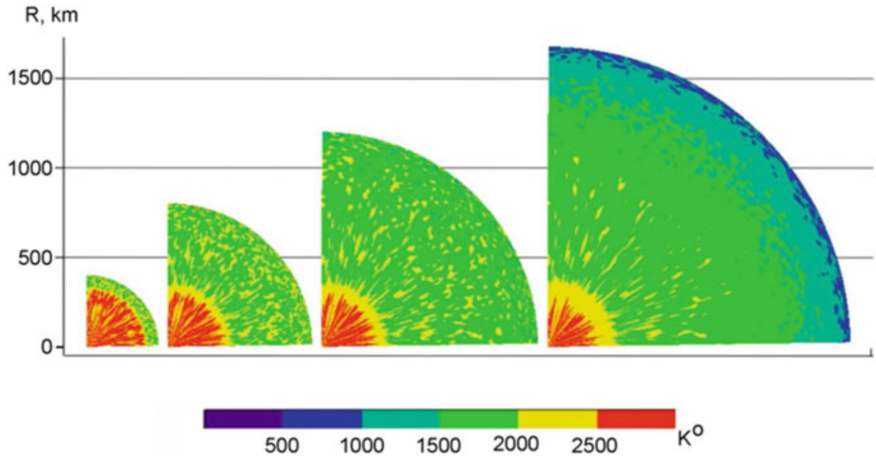


**Fig. A.2** The variant of the temperature distribution for the radial Moon's sections of the successfully increasing radius. The thermal heterogeneities are stipulated by their random location, the variant of which is presented in Fig. A.1 [6] a small size melted inner core is formed, and in the mantle a thick melted and a partially melted layer of matter is formed.

The greatest difficulties are linked with the numerical solution of the Earth's thermal evolution, because we need to take into account the adiabatic compressibility of the matter, which leads to the increase of the matter density from 2310 kg/$m^3$ on the crust–mantle boundary to the density of 7200 kg/$m^3$ on the mantle–core surface.

The numerical solution of the problem of the temperature distribution in the inner parts of the Earth at the stage of accumulation in the model, when taking account of adiabatic compression, includes an additional numerical block. The initial conditions are defined in it. For the presented variant, the initial value of the radius is $R_0$ = 1000 m. The temperature inside and on the surface of the pre-planetary body at the initial time moment is T = 320 K, the density and the module of volume compression of iron in the pre-planetary body are $\rho_0 = 7.6 \times 10^3$ kg/$m^3$, $K = 16 \times 10^{10}$ PA, and the mass of the pre-planetary body is:

$$m_0 = \frac{4}{3} \cdot \pi \cdot R_0^3 \cdot \rho_0$$

The step of the planetary radius growing by passing on to the next time layer is established as const $\Delta R$, while the step for time is variable and it is calculated at each stage of planetary growth from Safronov's equation (Chap. 4, (4.1)) [1]. It is assumed that in each layer with a thickness $\Delta R$ the values of the density, module of compression and pressure remain constant.

In each time layer the planetary radius increases such that: $R_{j+1} = R_j + \Delta R$

Using a 1-D spherical-symmetric model, we obtain a new value for the bodies' mass and the pressure distribution. The lithostatic pressure is defined, using the pressure of the upper layers:

$$p_i = \sum_{i=0}^{i=j} \rho_i \cdot g_i \cdot \Delta R_i \tag{A.1}$$

where $g_i$—acceleration of the gravity force of the sphere with radius $R_i$ is equal to:

$$g_i = \frac{G \cdot m}{R_i^2} \tag{A.2}$$

where $G$—gravitational constant, $m$—mass of the sphere with the radius $R_i$.

The module of compression $K$ according to the accepted definition is equal to:

$$K = \rho \cdot \frac{\Delta p}{\Delta \rho} \tag{A.3}$$

Thus, for the density distribution we obtain:

$$\rho_i = \rho_{i+1} + \rho_{i+1} \cdot \frac{p_i - p_{i+1}}{K_i} \tag{A.4}$$

Using Eqs. (A.1)–(A.3), we can obtain a new value of the bodies' mass and pressure in the layers. After that, we can derive the value of the compression module from (A.3):

$$K = \rho_i \cdot \frac{p_i - p_{i+1}}{\rho_i - \rho_{i+1}} \tag{A.5}$$

After calculating the physical parameters with a newly formed layer, we can calculate the time step from equation (Chap. 3, (3.1)) and the whole time of planetary growth for further numerical solution of the boundary problem equations (Chap. 4, (4.2)–(4.3)). The temperature distribution in the body with increasing radius is obtained from the numerical solution of the boundary problem for the equation of heat conductivity, taking account of the possibility of a melting area appearing without explicit release of the boundary location of the crystallization front and parametrical account of convective heat transfer in the melted area after [7]:

$$c_{ef}\,\rho \frac{\partial T}{\partial t} = \nabla\left(\lambda_{ef}\,\nabla T\right) + Q \tag{A.6}$$

where: $c_{ef}$, $\lambda_{ef}$—effective values of heat capacity, and heat conductivity, which take account of the melting heat in Stefan's problem after [8] and the existence of convective heat transfer; $T$—is the temperature at a point and a moment $t$, $Q$—is the volume capacity of the inner sources of heat. The problem is solved using the method of finite differences together with a whole implicit monotonic, conservative scheme. The dimensional step for the space grid is constant. The step for the time grid is variable and for the given distribution of the density, as a function of the depth, is calculated from equation (Chap. 4, (4.1)). Using that equation for each time step, we calculate the mass of the growing planet and the distribution of lithostatic pressure in the inner areas. For each value of the dimension reached by the growing planet, we calculate the distribution of the melting temperature. In the core the function of the melting temperature, mainly of the iron composition, is calculated after [9]:

$$\begin{gathered}
\rho\,[\frac{\partial \vec{V}}{\partial t} + (\vec{V}\,\nabla)\vec{V}] = -\nabla P + \eta \Delta\,\vec{V} + (\frac{\eta}{3} + \xi)\nabla(\nabla\vec{V}) - \rho\,\nabla W \\
\rho\,T\,[\frac{\partial S}{\partial t} + (\vec{V}\,\nabla)\,S] = \lambda\,\Delta\,T + Q \\
\Delta\,W_1 = -4\pi\gamma\,\rho \quad W = W_1 + W_2 \\
\frac{\partial \rho}{\partial t} + \nabla\,(\rho\,\vec{V}) = 0 \\
L\,\frac{\partial \vec{\psi}}{\partial t} = \vec{q}|_{\xi+0} - \vec{q}|_{\xi-0}
\end{gathered} \tag{A.7}$$

where: $\vec{V}$—fluid velocity, $P$—pressure, $S$—entropy, $W_1$—gravitational potential, $W_2$—centrifugal potential, $\rho$—density, $\eta$ and $\xi$—coefficients of the first and second viscosity, $\lambda$—coefficient of thermal conductivity, $\gamma$—gravitational constant, $Q$—summarized capacity of the inner sources in the volume unit, $L$—heat of the phase transfer, $\frac{\partial \vec{\psi}}{\partial t}$—the velocity of the boundary location of the phase division, $\vec{q}|_{\xi+0}$ *and* $\vec{q}|_{\xi-0}$—density of the heat flow, correspondingly, in front of and behind the phase boundary, ∇ and Δ–"nabla" and Laplace operators.

In the area of the forming mainly silicate mantle, the relation of the temperature with pressure is used after [10]. The zone of complete or partial melting is defined in each time layer by comparison of the calculated temperature distribution with the melting temperature at the given depth. Nevertheless, the large gradients of the density and the temperature in the large areas, and the necessity of taking into account the dependence of the matter viscosity on temperature and pressure, lead to a need to derive the solution in a more comprehensive statement.

The solution of Eqs. (A.7, 1) and (A.7, 5) is obtained using the natural parameters: the vector of velocity and pressure [11, 12].

The solution of the boundary problems of the first equation (A.7, 2) of that system, which is known as the Navier-Stokes equation, presents difficulties. And

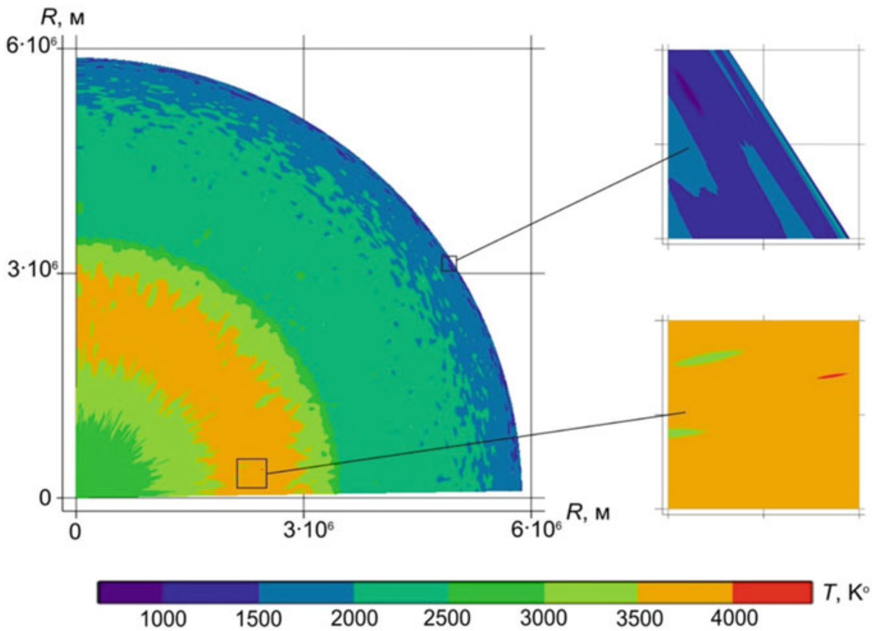


**Fig. A.3** The variant of the temperature distribution for one section of the Earth's model up to the end of accumulation. One of the radial sections is shown. The layer of the silicate melt is formed on the core–mantle boundary. The structure of the thermal heterogeneities is shown in the small pictures [14] for the approximation with constant coefficients of viscosity, as used in (A.7, 3), for a 3-D spherical layer, obtaining a numerical solution represents an essential problem. Besides that, in the frame of equation (A.7, 2) it is hard to describe the forced mixing of convective matter near the surface of the growing body by the falling of some other bodies. Obtaining the solution in the frame of a 3-D model is a complicated and cumbersome problem [13], therefore the mass numerical modelling was made in the approximation (Chap. 4, 4.1–4.3) for the 3-D model.

For the geometrical model of the forming Earth we used the same model for the 1/8 part of the spherical volume as for the Moon model. The problem (4.1)–(4.3) was solved taking account of the adiabatic compression and the heat dissipation by that adiabatic compression. It is interesting to compare the obtained variants of the temperature sections of the Earth and the Moon with the finishing of their accumulation. The quantitative estimations of the temperature of these bodies in their central parts coincide; there they had been controlled by heat dissipation through the decay of short-living elements. But the larger mass of the Earth compared to the Moon explains the melted state of the Earth's outer iron core, as against the solid state of similar areas of the Moon. This may be the reason for the development of the magneto-hydro-dynamic mechanism of the Earth's magnetic field generation and its absence on the Moon (Fig. A.3).

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## References


1. Safronov V (1969) Evolution of the protoplanetary cloud and formation of the Earth and the Planets. Nauka, Moscow (in Russian)
2. Artemieva N, Shuvalov V (2008) Numerical simulation of high-velocity impact ejecta following falls of comets and asteroids onto the Moon. Solar Syst Res 42:329–334 (in Russian)
3. Grae J, Appel P, Hatsukawa Y et al (2009) The Earth-Moon system during the late heavy bom-bardment period—geochemical support for impacts dominated by comets. Icarus 204:368–380
4. Khachay Y, Anfilogov V (2013) Numerical results of Earth's core accumulation 3-D modelling. EGU Geophys Res Abs 15:51
5. Khachay Y, Anfilogov V, Antipin A (2009) 3-D model of the thermal heterogeneity evolution on the Earth's core on its accumulation. In: Proceedings 5th Bulashevich conference geodynamics. Deep structure. The Earth's thermal field. Geophysical fields interpretation, Ekaterinburg, pp 321–323 (in Russian)
6. Khachay Y, Antipin A (2014) The results of numerical 3D simulation of temperature distribution in the lunar envelopes at its accumulation monitoring. Nauka i Technol 1:28–32 (in Russian)
7. Tikhonov A, Liubimova E, Vlasov V (1969) About the evolution of fusion zones in the thermal history of the Earth. Doklady academy of sciences of USSR 188:338–342 (in Russian)
8. Samarsky A, Moiseenko B (1965) The economic scheme of transparent account Stefan problem. J Calc Math 5:816–827 (in Russian)
9. Stacey F (1969) Physics of the Earth. Wiley, New York
10. Kaula W (1971) Dynamical aspects of lunar origin. Geophys Rev Space Phys 9:217–238

11. Belocerkovskiy O (1984) Numerical modelling in the mechanics of a continuous medium. Nauka, Moscow (in Russian)
12. Berkovskiy V, Nogotov G (1976) Numerical methods of investigation of heat exchanging problems. Nauka i Technika, Minsk (in Russian)
13. Khachay Y (2013) MHD process in the layer of gravitating sphere growing radius. Magnetohy-drodynamics 49(1–3):81–86
14. Khachay Y, Antipin A (2014) Numerical modelling of temperature distribution at the stage of Earth's accumulation in the frame of 3-D model and peculiarities of its initial minerageny. Ural Geophys Bull 1:81–85