Appendix 1

Answers and/or Hints to Selected Problems

2.1

Period	3	4	5	6	7	8	9	10
$\hat{a}$	456.7	460.0	468.7	476.0	485.3	492.7	496.0	492.7

2.4 a) Define

$$f_t = \hat{a}_t - c - d \cdot t, g_t = \hat{b}_t - d$$

We obtain

$$f_t = (1-\alpha)\, f_{t-1} + (1-\alpha)\, g_{t-1},$$

$$g_t = -\alpha\beta\, f_{t-1} + (1-\alpha\beta)\, g_{t-1}.$$

It follows that both f_t and g_t will approach zero.

b) Define $h_t = \hat{a}_t - c - d \cdot (t+1)$. We obtain

$$h_t = (1-\alpha)\, h_{t-1} - d = (1-\alpha)\, h_{t-1} + \alpha\, (-d/\alpha),$$

and from our results on exponential smoothing it follows that h_t will approach $-d/\alpha$ in the long run.

2.7 $(1000 + 1 \cdot 10)\hat{F}_1 = 808 \qquad \hat{F}_1 = 0.8$

$(1000 + 2 \cdot 10)\hat{F}_2 = 1020 \qquad \hat{F}_2 = 1.0$

$(1000 + 3 \cdot 10)\hat{F}_3 = 1648 \qquad \hat{F}_3 = 1.6$

$(1000 + 4 \cdot 10)\hat{F}_4 = 624 \qquad \hat{F}_4 = 0.6$

$\hat{a}_{05.1} = 0.8 \cdot (1000 + 10) + 0.2 \cdot (795/0.8) = 1006.75$

$\hat{b}_{05.1} = 0.8 \cdot 10 + 0.2 \cdot (1006.75 - 1000) = 9.35$

$\hat{F}_1' = 0.8 \cdot 0.8 + 0.2 \cdot (795/1006.75) = 0.7979$

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$\bar{T} = \sum_{t=1}^{4} \hat{F}'_t = 0.7979 + 1.0 + 1.6 + 0.6 = 3.9979 \hat{F}_t = \left(4/\bar{T}\right) \hat{F}'_t$

$\hat{F}_1 = 0.79835, \hat{F}_2 = 1.0005, \hat{F}_3 = 1.6008, \hat{F}_4 = 0.6003$

$\hat{a}_{05.2} = 0.8 \cdot (1006.75 + 9.35) + 0.2 \cdot (1023/1.0005) = 1017.38$

$\hat{b}_{05.2} = 0.8 \cdot 9.35 + 0.2 \cdot (1017.38 - 1006.75) = 9.606$

$\hat{F}'_2 = 0.8 \cdot 1.0005 + 0.2 \cdot (1023/1017.38) = 1.0015$

$\bar{T} = 4.001$

$\hat{F}_1 = 0.7981, \hat{F}_2 = 1.0013, \hat{F}_3 = 1.6004, \hat{F}_4 = 0.6002$

$\hat{x}_{05.2,05.3} = (1017.38 + 1 \cdot 9.606) \cdot 1.6004 = 1643.60$

$\hat{x}_{05.2,05.4} = (1017.38 + 2 \cdot 9.606) \cdot 0.6002 = 622.16$

$\hat{x}_{05.2,06.1} = (1017.38 + 3 \cdot 9.606) \cdot 0.7981 = 834.97$

$\hat{x}_{05.2,06.2} = (1017.38 + 4 \cdot 9.606) \cdot 1.0013 = 1057.18$

2.12 Use that

$$\hat{a}_t = x_t - (1 - \alpha)\,\varepsilon_t,$$

$$\hat{b}_t = x_t - x_{t-1} - (1 - \alpha\beta)\,\varepsilon_t + (1 - \alpha)\,\varepsilon_{t-1}.$$

2.13 a)

Month	1	2	3	4	5	6
$\hat{a}$	727.6	731.1	738.3	736.2	746.6	755.9
MAD	18.4	18.2	21.7	19.4	25.9	30.0

b)

Month	1	2	3	4	5	6
$\hat{a}$	727.6	730.7	738.1	737.5	748.5	760.0
$\hat{b}$	– 0.48	0.24	1.68	1.21	3.18	4.83

Forecast for month 5 after month 2: $730.7 + 3 \cdot 0.24 = 731.4$.

2.14 $\hat{a}_5 = 234, \hat{b}_5 = 10.4, MAD_5 = 33.5, \sigma = 41.9$,
Months 6–8: mean = 764.4, standard deviation = 72.53.

4.1 a) $Q_A = 1000$, $Q_B = 939$.
b) The relative cost increase is given by

$$\frac{C}{C^*} = \frac{1}{2}\left(\sqrt{125/200} + \sqrt{200/125}\right) = 1.02774.$$

For product A the correct cost is $C^* = \sqrt{2 \cdot 200 \cdot 48000 \cdot 0.24 \cdot 50} =$ \$15179 so the cost increase is \$421.

4.2

$$\frac{C}{C^*} = \frac{1}{2}\left(\sqrt{0.6} + \sqrt{1/0.6}\right) = 1.0328.$$

4.6

$$C = \frac{hQ}{2}1 + d/p + \frac{Ad}{Q}, Q^* = \sqrt{\frac{2Ad}{h\,(1 + d/p)}}, C^* = \sqrt{2Adh\,(1 + d/p)}.$$

4.11 a) We get $Q^* = 4$ and $x^* = 1/4$.
b) The average waiting time for a customer is $Qx^2/2d = 1/8$.
c) The considered problem is to minimize

$$C\frac{Q(1-x)^2}{2}h + \frac{d}{Q}A,$$

under the constraint $Qx^2/2d \leq 1/8$. Let us first show that the constraint is binding. If not, we get for a given x, $C = \sqrt{2Adh}\,(1 - x)$ and we can see that the costs will decrease with x. Consequently, $Qx^2/2d = 1/8$, or $Q = 1/(4x^2)$. Inserting in C we get

$$C = \frac{1}{8}\left(\frac{1}{x^2} - \frac{2}{x} + 1\right) + 24x^2.$$

It is easy to verify that C is minimized for $x^* = 1/4$. This implies $Q^* = 4$. (An alternative way to see that the original solution solves the new problem is to note that b_1 serves a Lagrange multiplier for the considered constraint.)

4.16 a)

Period t	1	2	3	4	5
d_t	25	40	40	40	90
$k = t$	100	200	240	320	*380*
$k = t + 1$	*140*	240	*280*	410	
$k = t + 2$	220	320			

Deliveries 65 in period 1, 80 in period 3, and 90 in period 5. Cost \$380.
b) Two periods $(100 + 40)/2 = 70 \leq 100$,
Three periods $(100 + 40 + 80)/3 = 73.3 > 70$, delivery in period 3.
Two periods $(100 + 40)/2 = 70 \leq 100$,
Three periods $(100 + 40 + 180)/3 = 106.7 > 70$, delivery in period 5, i.e., the same solution.
c) Two periods $40 \leq 100$,
Three periods $40 + 80 > 100$, delivery in period 3.
Two periods $40 \leq 100$,
Three periods $40 + 180 > 100$, delivery in period 5, i.e., same solution.

4.17 a)

Period t	1	2	3	4	5	6
d_t	10	40	95	70	120	50
$k=t$	100	200	220	315	355	455
$k=t+1$	*120*	247.5	*255*	375	*380*	
$k=t+2$	215	317.5		425		

Deliveries 50 in period 1, 165 in period 3, and 170 in period 5. Cost \$380.

b) Two periods $(100+20)/2=60 \leq 100$,
Three periods $(100+20+95)/3=71.7>60$, delivery in period 3.
Two periods $(100+35)/2=67.5 \leq 100$,
Three periods $(100+35+120)/3=85>67.5$, delivery in period 5,
Two periods $(100+25)/2=62.5 \leq 100$, i.e., same solution.

4.18 a)

Period t	1	2	3	4	5	6	7
d_t	10	24	12	7	5	4	3
$k=t$	*100*	200	296	348	404	464	500
$k=t+1$	196	*248*	324	368	420	476	
$k=t+2$	292	304	364	400	444		
$k=t+3$	376	364	412	*436*			
$k=t+4$	456	428	460				
$k=t+5$	536	488					
$k=t+6$	608						

Deliveries 10 in period 1, 36 in period 2, and 19 in period 4. Cost \$436.

b) Two periods $(100+96)/2=98 \leq 100$,
Three periods $(100+96+96)/3=97.3 \leq 98$,
Four periods $(100+96+96+84)/4=94 \leq 97.3$,
Five periods $(100+96+96+84+80)/5=91.2 \leq 94$,
Six periods $(100+96+96+84+80+80)/6=89.3 \leq 91.2$,
Seven periods $(100+96+96+84+80+80+72)/7=86.9 \leq 89.3$, i.e., just one delivery in period 1. Cost \$608. Note the large difference due to the special demand structure.

5.1 Use the generating function.

5.2 Derive the generating function.
a) 68, 104, and 165.

b) 92.5, 98.3, and 99.2 %.
c) 77.3, and 99.9 %.

5.7 a) $SS = 135 - 100 = 35$. $S_1 = \Phi(0.7) = 75.8\%$. $S_2 = 96.4\,\%$.
b) Let the lead-time demand be uniform on (100 – a, 100 + a). We obtain

$$\left(\sigma'\right)^2 = 2500 = \int\limits_{100-a}^{100+a} \frac{1}{2a}(u-100)^2 du = a^2/3,$$

i.e., $a = \sqrt{7500} = 86.60.S_1 = (135 - -100 + 86.60)/173.20 = 70.2\%$. The average backordered quantity per cycle is

$$E(B) = \int\limits_{135}^{186.6} \frac{(u-135)}{173.20} du = 7.686,$$

and $S_2 = 1 - 7.686/200 = 96.2\%$.

5.8 a) Initially $\mu' = 800$ and $\sigma' = 100$. We get $S_2 = 96.7\,\%$. After reducing the lead-time $\mu' = 400$ and $\sigma' = 50\sqrt{2} = 70.71$. We get $S_2 > 99.99\,\%$. We obtain the average stock on hand as

$$E\left(IL^+\right) = R + Q/2 - \mu' + \frac{\sigma'^2}{Q}\left[H\left(\frac{R-\mu'}{\sigma'}\right) - H\left(\frac{R+Q-\mu'}{\sigma'}\right)\right]$$

The stock on hand is increasing from 351.7 to 750.
b) R can be reduced to 419. The stock on hand is 320.3.

5.11 The inventory position is uniform on 2, 3, 4.
a. We get $P(2) = (e^{-2}/3)(1 + 2 + 2) = 0.226$.
b. Furthermore, $P(1) = 0.241$, $P(3) = 0.135$ and $P(4) = 0.045$. $E(I)^+ = 1.278$. $E(I) = 3 - 2 = 1$. We get $E(I)^- = 1.278 - 1 = 0.278$. The average waiting time is obtained by Little's formula $0.278/2 = 0.139$.

5.12 Determine the average shortage corresponding to Q.

$$B = \int\limits_{R}^{R+Q} (x-R)\frac{1}{m}e^{-\frac{x}{m}}dx + \int\limits_{R+Q}^{\infty} Q\frac{1}{m}e^{-\frac{x}{m}}dx$$

$$= \left[-(x-R)\,e^{-\frac{x}{m}}\right]_R^{R+Q} + \int\limits_{R}^{R+Q} e^{-\frac{x}{m}}dx + \left[-Qe - \frac{x}{m}\right]_{R+Q}^{\infty}$$

$$= m\left(e^{-\frac{R}{m}} - e^{-\frac{R+Q}{m}}\right).$$

$$S_2 = 1 - B/Q = 1 - \frac{m}{Q}\left(e^{-\frac{R}{m}} - e^{-\frac{R+Q}{m}}\right)$$

5.13 a) $S_2 = 0.935$.
b) Set $\mu' = \lambda$ and $\sigma' = \lambda^{1/2}.S_2 = 0.932$.

5.21 The same probabilities 0.128, 0.16 and 0.2.

6.1 It is constant $S_2 = b_1/(h + b_1) = 10/11$. See (5.67).

6.3 We obtain $\mu' = 200$ and $\sigma' = 2^{0.7}\sqrt{\pi/2} \cdot 40 = 81.44$.
a. $SS = 1.28 \cdot 81.44 \approx 104$, and $R = 304$.
b. $S_2 = 92\%, S_2 = 96\%\, and S_2 = 99.7\%$.

6.4 a) $\hat{x}_{t,t+\tau} = \hat{a}_t = 0.8 \cdot \hat{a}_{t-1} + 0.2 \cdot x_t$
$MAD_t = 0.7 \cdot MAD_{t-1} + 0.3 \cdot \left|x_t - \hat{x}_{t-1,t}\right|$

x_t	112	96	84	106	110
$\hat{a}_t$	102.40	101.12	97.70	99.36	101.49
MAD	10.6	9.34	11.67	10.66	10.66

$\sigma^2 \cong (1.25 \cdot MAD)^2$
Expected demand 101.5 units per week with variance 177.6.

b. $Q = \sqrt{2AD/h} = \sqrt{2 \cdot 100 \cdot 101 \cdot 49/1} \cong 142$
$SS = k \cdot \sigma', k = 1.64$
$\sigma' = \sigma \cdot \sqrt{L} = 13.325 \cdot \sqrt{2} = 18.84$ $SS = 1.64 \cdot 18.84 = 30.9$
$R = 30.9 + 101.49 \cdot 2 = 234$

6.5 a) Simple exponential smoothing

Week		16	17	18	19	20
Demand		97	99	100	126	112
$\hat{a}$	100.00	99.40	99.32	99.46	104.77	106.21
MAD	7.00	6.20	5.04	4.17	8.64	8.36

$\hat{a}_{t+1} = (1-\alpha)\hat{a}_t + \alpha \cdot X_{t+1}$
$MAD_{t+1} = (1-\alpha) \cdot MAD_t + \alpha \cdot |\hat{x}_{t,t+1} - X_{t+1}|$
$\sigma \approx 1.25 \cdot 8.36 = 10.45$
Exponential smoothing with trend

Week		16	17	18	19	20
Demand		97	99	100	126	112
$\hat{a}$	100.00	99.40	99.13	99.10	104.35	107.48
$\hat{b}$	0.00	– 0.24	– 0.25	– 0.16	2.00	2.45
MAD	7.00	6.20	4.99	4.22	8.79	8.16

$\hat{a}_{t+1} = (1-\alpha)\cdot(\hat{a}_t + \hat{b}_t) + \alpha \cdot x_{t+1}$
$\hat{b}_{t+1} = (1-\beta)\cdot \hat{b}_t + \beta \cdot (\hat{a}_{t+1} - \hat{a}_t)$
$MAD_{t+1} = (1-\alpha)\cdot MAD_t + \alpha \cdot |\hat{X}_{t,t+1} - \chi_{t+1}|$
$\sigma \approx 1.25 \cdot 8.16 = 10.20$
$Q = (2.2500 \cdot 106.21/10)^{1/2} \approx 230$
$\sigma' = 10.45 \cdot\ 3^{1/2} \approx 18.10$
$G(SS/\sigma') \approx Q \cdot\ (1-S_2)/\ \sigma' = 230 \cdot\ (1\text{–}0.95)/\ 18{,}10 = 0.635$
$SS/\sigma' = -0.4$ (from table)
$SS = -0.4 \cdot 18.10 \approx -7.24$
$R = -7.24 + 3\ \cdot\ 106.21 = 311.39 \approx 312$

b) $(S_1 =\ \Phi(SS/\sigma') \approx 35\,\%$

6.7

Forecast	102.40	101.12	97.70	99.36	101.49
MAD	10.6	9.34	11.67	10.66	10.66

101.5 with variance 177.6, $\sigma = 1.25$ MAD $Q = 142$, $k = 1.64$, $\sigma = \mathrm{SS} = 30.9$, $R = 234$.

7.3 a) Applying (7.12) and (7.13) we obtain $T_1 = 27.037$, $T_2 = 39.528$, $T_3 = 43.759$, and the costs $C_1 = 59.178, C_2 = 25.298, C_3 = 45.705$,. The sum of these costs $C = 130.182$ is a lower bound for the total costs.

b) Assume that the independent solution is feasible and that we start production of product 3 at times t, $t, +43.759$, $t, +2\ 43.759\ldots$ etc. Assume that the first production of product 2 after time t, starts at time $t, +\ \Delta 1$. Clearly $\Delta_1\ \geq\ \sigma_3\ = 15.003$. But the next time the time difference is $\Delta_2 + \mathrm{T2} - \mathrm{T3} = \Delta_1 - 4.231$. The time after that it is $\Delta 3 =\ \Delta 2 - 4.231$, etc. After some time we will obtain for some n, $0 < \Delta_n < \sigma_3$ which means that the solution is infeasible.

c) Applying (7.17)–(7.19) we obtain $T_{min} = 7.292$, and $\hat{T} = 34.462$, i.e., $T_{opt} = \hat{T} = 34.462$. The resulting total costs per unit of time are $C = 133.481$.

d) Starting with $W = 27.037$ we obtain $n_1 = 1, n_2 = n_3 = 2$. Next we get the final solution $W = 23.617$ with the same multipliers and the total costs $C = 131.259$. The solution is feasible if we produce items 2 and 3 in different basic periods. In basic periods where product 1 and 2 are produced, the time required is $\sigma 1 + \sigma 2 = 6.168 + 9.697 = 15.865 < 23.617$. In basic periods where product 1 and 3 are produced, the time required is $\sigma_1 + \sigma_3 = 6.168 + 16.115 = 22.283 < 23.617$.

7.5 a) $T^* = 5.020, T_{min} = 1.806, T_{opt} = 5.020. C = 9960.0$
$Q_{\mathrm{I}} \cong 500,\ Q_{\mathrm{II}} \cong 250,\ Q_{\mathrm{III}} \cong 100$

b) $T_{\mathrm{I}} = 460/100 = 4.6, T_{\mathrm{II}} = 230/50 = 4.6, T_{\mathrm{III}} 184/20 = 9.2, C_{DW} = 9877.7$.

Product I and II are produced in each basic period, III in every second basic period. Feasible schedule. In a basic period with production of all three products the production time is $2.956 < 4.6$.

8.3 a) Change first to the equivalent $R_2^i = 2$ such that $IP_2^{i0} - R_2^i$ is a multiple of Q_1. Next we use (8.4) to obtain $R_1^e = 8$ and $R_2^e = 20$.

b) For both policies we have: Installation 1 order at times 7, 17, 27,. . . , and installation 2 at times 7, 27, 47,.
c) At times 6, 26, 46,. . . . No equivalent installation stock policy exists, since the echelon stock policy is not nested.

8.7

Item A	Period		1	2	3	4	5	6	7	8
Lead-time = 1	Gross requirements		7	9	28	30	18	16	24	13
Order quantity = 30	Scheduled receipts									
	Projected inventory	22	15	6	8	8	20	4	10	27
	Planned orders			30	30	30		30	30	

Item B	Period		1	2	3	4	5	6	7	8
Lead-time = 1	Gross requirements			30	30	30		30	30	
Order quantity = 30	Scheduled receipts									
	Projected inventory	34	34	4	4	4	4	4	4	4
	Planned orders			30	30		30	30		

Item C	Period		1	2	3	4	5	6	7	8
Lead-time = 1	Gross requirements			90	90	60	30	90	60	
Order quantity = 90	Scheduled receipts									
	Projected inventory	12	12	12	12	42	12	12	42	42
	Planned orders		90	90	90		90	90		

8.8 a)

Item A	Period		1	2	3	4	5	6
Lead-time = 1	Gross requirements		5	26		13	12	
Order quantity = 10	Scheduled receipts							
Safety stock = 10	Projected inventory	27	22	16	16	13	11	11
	Planned orders		20		10	10		

Item B	Period		1	2	3	4	5	6
Lead-time = 1	Gross requirements		20		10	10		
Order quantity = 10	Scheduled receipts							
Safety stock = 10	Projected inventory	34	14	14	14	14	14	14
	Planned orders			10	10			

Item C	Period		1	2	3	4	5	6
Lead-time = 1	Gross requirements		40	10	30	20		
Order quantity = 20	Scheduled receipts							
Safety stock = 20	Projected inventory	75	35	25	35	35	35	35
	Planned orders			40	20			

No delayed orders.

b)

Item A	Period		1	2	3	4	5	6
Lead-time = 1	Gross requirements		5	26		13	12	
Order quantity = 10	Scheduled receipts							
Safety time = 1	Projected inventory	27	22	6	16	13	1	1
	Planned orders		10	10	10			

Item B	Period		1	2	3	4	5	6
Lead-time = 1	Gross requirements		10	10	10			
Order quantity = 10	Scheduled receipts							
Safety time = 1	Projected inventory	34	24	14	4	4	4	4
	Planned orders							

Item C	Period		1	2	3	4	5	6
Lead-time = 1	Gross requirements		20	20	20			
Order quantity = 20	Scheduled receipts							
Safety time = 1	Projected inventory	75	55	35	15	15	15	15
	Planned orders							

The order for item A in period 1 is delayed one period.

9.1 Denote the final demand $d = 100$.

$$C = \frac{d}{p_1}\frac{Q}{2}1 + \frac{d}{p_2}\frac{Q\,(1 - p_2/p_1)}{2}2 + \frac{Q\,(1 - d/p_2)}{2}3 + \frac{d}{Q}\,(A_1 + A_2 + A_3).$$

We get $Q^* \approx 526$.

9.6 a) We have $e_1 = e_2 = 1$. Since $A_2/e_2 > A_1/e_1$, the constraint $Q_2 \geq Q_1$ will be satisfied automatically if we optimize the items separately in the relaxed problem. We get $Q_1^* = \sqrt{2A_1 d/e_1} \approx 44.72$, and $Q_2^* = \sqrt{2A_2 d/e_2} = 100$. A lower bound for the costs is $\sqrt{2A_1 de_1} + \sqrt{2A_2 de_2} \approx 144.72$. Without any lack of generality we can assume that $20\sqrt{5}/\sqrt{2} < q \leq 20\sqrt{5} \cdot \sqrt{2}$. This means that $Q_1 = q$. Furthermore, we should choose $Q^2 = 2q$ for $q \geq 50/\sqrt{2}$and $Q_2 = 4q$ otherwise. We obtain

$$C\,(q) = 5\frac{q}{2} + \frac{2250}{q}, \quad 20\sqrt{5}/\sqrt{2} < q < 50/\sqrt{2}$$

$$C\,(q) = 3\frac{q}{2} + \frac{3500}{q}, \quad 50/\sqrt{2} \leq q \leq 20\sqrt{5} \cdot \sqrt{2}$$

We obtain the optimal $q \approx 48.30$, $Q_1 = q$, and $Q_2 = 2q$. The optimal costs are $C \approx 144.91$, i.e., only 0.13 % above the lower bound.

b) From (9.13) we obtain $k^* = \sqrt{5}$. Since $\sqrt{5}/2 \leq 3/\sqrt{5}$ it is optimal to have $k = 2$. Inserting in (9.10) and (9.11) we get the same solution as in a).

9.8 a) Let $Q_2 = k_2 Q_1$ and $Q_3 = k_3 Q_2$. We obtain

$$C = C_1^e + C_2^e + C_3^e = (e_1 + k_2 e_2 + k_3 k_2 e_3)\,\frac{Q_1}{2} + \left(A_1 + \frac{A_2}{k_2} + \frac{A_3}{k_3 k_2}\right)\frac{d}{Q_1},$$

i.e., $A_1{}' = A_1 + \frac{A_2}{k_2} + \frac{A_3}{k_3 k_2}, h_1{}' = e_1 + k_2 e_2 + k_3 k_2 e_3$.

b) $p(2) = 3, p(1) = 2$. We obtain $A_3{}' = A_3, h_3' = e_3$, and

$$A_2' = A_2 + \frac{A_3}{k_3}, h_2' = e_2 + k_3 e_3,$$

and

$$A_1{}' = A_1 + \frac{A_2{}'}{k_2} = A_1 + \frac{A_2}{k_2} + \frac{A_3}{k_3 k_2}, h_1{}' = e_1 + k_2 h_2{}' = e_1 + k_2 e_2 + k_3 k_2 e_3.$$

10.2 Note first that $\mu_2' = \sigma_2' = 0$ and that S_1^e does not change. Consequently (10.9) degenerates to

$$\hat{C}_2\,(y_2) = h_2 y_2 + \hat{C}_1\left(S_1^e\right), \quad y_2 > S_1^e$$

$$\hat{C}_2\,(y_2) = h_2 y_2 + \hat{C}_1\,(y_2), \quad y_2 \leq S_1^e$$

Since the costs for $y_2 > S_1^e$ are increasing with y_2, the optimal y_2 must occur for $y_2 \leq S_1^e$ From (10.6) we obtain

$$\hat{C}_2\,(y_2) = h_1 y_2 - h_1 \mu_1{}'' + (h_1 + b_1)\,\sigma_1{}'' G\left(\frac{y_2 - \mu_1{}''}{\sigma_1{}''}\right)$$

This is the cost for a single-echelon system, since there is no stock at installation 2, and since the possible order-up-to level at installation 1 is bounded by y_2. We obtain the optimal y_2 from the condition

$$\Phi\left(\frac{y_2 - \mu_1{}''}{\sigma_1{}''}\right) = \frac{b_1}{h_1 + b_1} = \frac{10}{11.5} \approx 0.870,$$

i.e., $S_2^e = y_2^* \approx \mu_1{}'' + 1.13\sigma_1{}'' = 60 + 1.13 \cdot 5\sqrt{6} \approx 73.8$ and $\hat{C}_2\left(y_2^*\right) \approx 29.9$.

10.4 a) Using (10.18) we obtain $P(IL_0 = 1) = 0.2240$, $P(IL_0 = 2) = 0.1494$, $P(IL_0 = 3) = 0.0498$. The holding costs are determined from (10.19). For Poisson demand the fill rate is equivalent to the ready rate, i.e., $P(IL_0 > 0) = 0.2240 + 0.1494 + 0.0498 = 42.3\,\%$.

b) From (10.21) and (10.22) we obtain $E(IL_0^-) = 0.672$ and $E(W_0) = 0.672/3 = 0.224$. Consequently, $\bar{L}_1 = \bar{L}_2 = 1.224$. For retailer 1 we obtain $P(IL_1 = 1) = 0.2203$, $P(IL_1 = 2) = 0.3599$, $P(IL_1 = 3) = 0.2941$. The holding costs are 1.822 and the backorder costs 0.463. The fill rate is 87.4 %. For retailer 2 we get $P(IL_2 = 1) = 0.1294$, $P(IL_2 = 2) = 0.2114$, $P(IL_2 = 3) = 0.2591$, $P(IL_2 = 4) = 0.2117$, $P(IL_2 = 5) = 0.0865$. The holding costs are 2.608 and the backorder costs 0.565. The fill rate is 89.8 %.

Appendix 2

Normal Distribution Tables

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \varphi(-x) = \varphi(x). \quad \Phi(x) = \int_{-\infty}^{x} \varphi(v)dv, \Phi(-x) = 1 - \Phi(x).$$

$$G(x) = \int_{x}^{\infty} (v - x)\varphi(v)dv = \varphi(x) - x(1 - \Phi(x)), \quad G(-x) = G(x) + x.$$

$$H(x) = \int_{x}^{\infty} G(v)dv = \frac{1}{2}\Big[(x^2+1)(1 - \Phi(x)) - x\varphi(x)\Big], H(-x) = -H(x) + \frac{1}{2}(x^2+1)$$

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
0.00	0.3989	0.5000	0.3989	0.2500
0.01	0.3989	0.5040	0.3940	0.2460
0.02	0.3989	0.5080	0.3890	0.2421
0.03	0.3988	0.5120	0.3841	0.2383
0.04	0.3986	0.5160	0.3793	0.2344
0.05	0.3984	0.5199	0.3744	0.2307
0.06	0.3982	0.5239	0.3697	0.2269
0.07	0.3980	0.5279	0.3649	0.2233
0.08	0.3977	0.5319	0.3602	0.2197
0.09	0.3973	0.5359	0.3556	0.2161
0.10	0.3970	0.5398	0.3509	0.2125
0.11	0.3965	0.5438	0.3464	0.2091
0.12	0.3961	0.5478	0.3418	0.2056

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
0.13	0.3956	0.5517	0.3373	0.2022
0.14	0.3951	0.5557	0.3328	0.1989
0.15	0.3945	0.5596	0.3284	0.1956
0.16	0.3939	0.5636	0.3240	0.1923
0.17	0.3932	0.5675	0.3197	0.1891
0.18	0.3925	0.5714	0.3154	0.1859
0.19	0.3918	0.5753	0.3111	0.1828
0.20	0.3910	0.5793	0.3069	0.1797
0.21	0.3902	0.5832	0.3027	0.1766
0.22	0.3894	0.5871	0.2986	0.1736
0.23	0.3885	0.5910	0.2944	0.1707
0.24	0.3876	0.5948	0.2904	0.1677
0.25	0.3867	0.5987	0.2863	0.1649

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x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
0.26	0.3857	0.6026	0.2824	0.1620
0.27	0.3847	0.6064	0.2784	0.1592
0.28	0.3836	0.6103	0.2745	0.1564
0.29	0.3825	0.6141	0.2706	0.1537
0.30	0.3814	0.6179	0.2668	0.1510
0.31	0.3802	0.6217	0.2630	0.1484
0.32	0.3790	0.6255	0.2592	0.1458
0.33	0.3778	0.6293	0.2555	0.1432
0.34	0.3765	0.6331	0.2518	0.1407
0.35	0.3752	0.6368	0.2481	0.1382
0.36	0.3739	0.6406	0.2445	0.1357
0.37	0.3725	0.6443	0.2409	0.1333
0.38	0.3712	0.6480	0.2374	0.1309
0.39	0.3697	0.6517	0.2339	0.1285
0.40	0.3683	0.6554	0.2304	0.1262
0.41	0.3668	0.6591	0.2270	0.1239
0.42	0.3653	0.6628	0.2236	0.1217
0.43	0.3637	0.6664	0.2203	0.1194
0.44	0.3621	0.6700	0.2169	0.1173
0.45	0.3605	0.6736	0.2137	0.1151
0.46	0.3589	0.6772	0.2104	0.1130
0.47	0.3572	0.6808	0.2072	0.1109
0.48	0.3555	0.6844	0.2040	0.1088
0.49	0.3538	0.6879	0.2009	0.1068
0.50	0.3521	0.6915	0.1978	0.1048
0.51	0.3503	0.6950	0.1947	0.1029
0.52	0.3485	0.6985	0.1917	0.1009
0.53	0.3467	0.7019	0.1887	0.0990
0.54	0.3448	0.7054	0.1857	0.0972
0.55	0.3429	0.7088	0.1828	0.0953
0.56	0.3410	0.7123	0.1799	0.0935
0.57	0.3391	0.7157	0.1771	0.0917
0.58	0.3372	0.7190	0.1742	0.0900
0.59	0.3352	0.7224	0.1714	0.0882
0.60	0.3332	0.7257	0.1687	0.0865
0.61	0.3312	0.7291	0.1659	0.0849
0.62	0.3292	0.7324	0.1633	0.0832
0.63	0.3271	0.7357	0.1606	0.0816
0.64	0.3251	0.7389	0.1580	0.0800
0.65	0.3230	0.7422	0.1554	0.0784
0.66	0.3209	0.7454	0.1528	0.0769
0.67	0.3187	0.7486	0.1503	0.0754
0.68	0.3166	0.7517	0.1478	0.0739
0.69	0.3144	0.7549	0.1453	0.0724
0.70	0.3123	0.7580	0.1429	0.0710
0.71	0.3101	0.7611	0.1405	0.0696

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
0.72	0.3079	0.7642	0.1381	0.0682
0.73	0.3056	0.7673	0.1358	0.0668
0.74	0.3034	0.7704	0.1334	0.0654
0.75	0.3011	0.7734	0.1312	0.0641
0.76	0.2989	0.7764	0.1289	0.0628
0.77	0.2966	0.7794	0.1267	0.0615
0.78	0.2943	0.7823	0.1245	0.0603
0.79	0.2920	0.7852	0.1223	0.0591
0.80	0.2897	0.7881	0.1202	0.0578
0.81	0.2874	0.7910	0.1181	0.0567
0.82	0.2850	0.7939	0.1160	0.0555
0.83	0.2827	0.7967	0.1140	0.0543
0.84	0.2803	0.7995	0.1120	0.0532
0.85	0.2780	0.8023	0.1100	0.0521
0.86	0.2756	0.8051	0.1080	0.0510
0.87	0.2732	0.8078	0.1061	0.0499
0.88	0.2709	0.8106	0.1042	0.0489
0.89	0.2685	0.8133	0.1023	0.0478
0.90	0.2661	0.8159	0.1004	0.0468
0.91	0.2637	0.8186	0.0986	0.0458
0.92	0.2613	0.8212	0.0968	0.0449
0.93	0.2589	0.8238	0.0950	0.0439
0.94	0.2565	0.8264	0.0933	0.0430
0.95	0.2541	0.8289	0.0916	0.0420
0.96	0.2516	0.8315	0.0899	0.0411
0.97	0.2492	0.8340	0.0882	0.0402
0.98	0.2468	0.8365	0.0865	0.0394
0.99	0.2444	0.8389	0.0849	0.0385
1.00	0.2420	0.8413	0.0833	0.0377
1.01	0.2396	0.8438	0.0817	0.0368
1.02	0.2371	0.8461	0.0802	0.0360
1.03	0.2347	0.8485	0.0787	0.0352
1.04	0.2323	0.8508	0.0772	0.0345
1.05	0.2299	0.8531	0.0757	0.0337
1.06	0.2275	0.8554	0.0742	0.0329
1.07	0.2251	0.8577	0.0728	0.0322
1.08	0.2227	0.8599	0.0714	0.0315
1.09	0.2203	0.8621	0.0700	0.0308
1.10	0.2179	0.8643	0.0686	0.0301
1.11	0.2155	0.8665	0.0673	0.0294
1.12	0.2131	0.8686	0.0659	0.0287
1.13	0.2107	0.8708	0.0646	0.0281
1.14	0.2083	0.8729	0.0634	0.0275
1.15	0.2059	0.8749	0.0621	0.0268
1.16	0.2036	0.8770	0.0609	0.0262
1.17	0.2012	0.8790	0.0596	0.0256

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
1.18	0.1989	0.8810	0.0584	0.0250
1.19	0.1965	0.8830	0.0573	0.0244
1.20	0.1942	0.8849	0.0561	0.0239
1.21	0.1919	0.8869	0.0550	0.0233
1.22	0.1895	0.8888	0.0538	0.0228
1.23	0.1872	0.8907	0.0527	0.0222
1.24	0.1849	0.8925	0.0517	0.0217
1.25	0.1826	0.8944	0.0506	0.0212
1.26	0.1804	0.8962	0.0495	0.0207
1.27	0.1781	0.8980	0.0485	0.0202
1.28	0.1758	0.8997	0.0475	0.0197
1.29	0.1736	0.9015	0.0465	0.0193
1.30	0.1714	0.9032	0.0455	0.0188
1.31	0.1691	0.9049	0.0446	0.0184
1.32	0.1669	0.9066	0.0436	0.0179
1.33	0.1647	0.9082	0.0427	0.0175
1.34	0.1626	0.9099	0.0418	0.0171
1.35	0.1604	0.9115	0.0409	0.0166
1.36	0.1582	0.9131	0.0400	0.0162
1.37	0.1561	0.9147	0.0392	0.0158
1.38	0.1539	0.9162	0.0383	0.0155
1.39	0.1518	0.9177	0.0375	0.0151
1.40	0.1497	0.9192	0.0367	0.0147
1.41	0.1476	0.9207	0.0359	0.0143
1.42	0.1456	0.9222	0.0351	0.0140
1.43	0.1435	0.9236	0.0343	0.0136
1.44	0.1415	0.9251	0.0336	0.0133
1.45	0.1394	0.9265	0.0328	0.0130
1.46	0.1374	0.9279	0.0321	0.0127
1.47	0.1354	0.9292	0.0314	0.0123
1.48	0.1334	0.9306	0.0307	0.0120
1.49	0.1315	0.9319	0.0300	0.0117
1.50	0.1295	0.9332	0.0293	0.0114
1.51	0.1276	0.9345	0.0286	0.0111
1.52	0.1257	0.9357	0.0280	0.0109
1.53	0.1238	0.9370	0.0274	0.0106
1.54	0.1219	0.9382	0.0267	0.0103
1.55	0.1200	0.9394	0.0261	0.0100
1.56	0.1182	0.9406	0.0255	0.0098
1.57	0.1163	0.9418	0.0249	0.0095
1.58	0.1145	0.9429	0.0244	0.0093
1.59	0.1127	0.9441	0.0238	0.0090
1.60	0.1109	0.9452	0.0232	0.0088
1.61	0.1092	0.9463	0.0227	0.0086
1.62	0.1074	0.9474	0.0222	0.0084
1.63	0.1057	0.9484	0.0216	0.0081

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
1.64	0.1040	0.9495	0.0211	0.0079
1.65	0.1023	0.9505	0.0206	0.0077
1.66	0.1006	0.9515	0.0201	0.0075
1.67	0.0989	0.9525	0.0197	0.0073
1.68	0.0973	0.9535	0.0192	0.0071
1.69	0.0957	0.9545	0.0187	0.0069
1.70	0.0940	0.9554	0.0183	0.0067
1.71	0.0925	0.9564	0.0178	0.0066
1.72	0.0909	0.9573	0.0174	0.0064
1.73	0.0893	0.9582	0.0170	0.0062
1.74	0.0878	0.9591	0.0166	0.0060
1.75	0.0863	0.9599	0.0162	0.0059
1.76	0.0848	0.9608	0.0158	0.0057
1.77	0.0833	0.9616	0.0154	0.0056
1.78	0.0818	0.9625	0.0150	0.0054
1.79	0.0804	0.9633	0.0146	0.0053
1.80	0.0790	0.9641	0.0143	0.0051
1.81	0.0775	0.9649	0.0139	0.0050
1.82	0.0761	0.9656	0.0136	0.0048
1.83	0.0748	0.9664	0.0132	0.0047
1.84	0.0734	0.9671	0.0129	0.0046
1.85	0.0721	0.9678	0.0126	0.0044
1.86	0.0707	0.9686	0.0123	0.0043
1.87	0.0694	0.9693	0.0119	0.0042
1.88	0.0681	0.9699	0.0116	0.0041
1.89	0.0669	0.9706	0.0113	0.0040
1.90	0.0656	0.9713	0.0111	0.0039
1.91	0.0644	0.9719	0.0108	0.0037
1.92	0.0632	0.9726	0.0105	0.0036
1.93	0.0620	0.9732	0.0102	0.0035
1.94	0.0608	0.9738	0.0100	0.0034
1.95	0.0596	0.9744	0.0097	0.0033
1.96	0.0584	0.9750	0.0094	0.0032
1.97	0.0573	0.9756	0.0092	0.0031
1.98	0.0562	0.9761	0.0090	0.0031
1.99	0.0551	0.9767	0.0087	0.0030
2.00	0.0540	0.9772	0.0085	0.0029
2.01	0.0529	0.9778	0.0083	0.0028
2.02	0.0519	0.9783	0.0080	0.0027
2.03	0.0508	0.9788	0.0078	0.0026
2.04	0.0498	0.9793	0.0076	0.0026
2.05	0.0488	0.9798	0.0074	0.0025
2.06	0.0478	0.9803	0.0072	0.0024
2.07	0.0468	0.9808	0.0070	0.0023
2.08	0.0459	0.9812	0.0068	0.0023
2.09	0.0449	0.9817	0.0066	0.0022

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
2.10	0.0440	0.9821	0.0065	0.0021
2.11	0.0431	0.9826	0.0063	0.0021
2.12	0.0422	0.9830	0.0061	0.0020
2.13	0.0413	0.9834	0.0060	0.0020
2.14	0.0404	0.9838	0.0058	0.0019
2.15	0.0396	0.9842	0.0056	0.0018
2.16	0.0387	0.9846	0.0055	0.0018
2.17	0.0379	0.9850	0.0053	0.0017
2.18	0.0371	0.9854	0.0052	0.0017
2.19	0.0363	0.9857	0.0050	0.0016
2.20	0.0355	0.9861	0.0049	0.0016
2.21	0.0347	0.9864	0.0047	0.0015
2.22	0.0339	0.9868	0.0046	0.0015
2.23	0.0332	0.9871	0.0045	0.0014
2.24	0.0325	0.9875	0.0044	0.0014
2.25	0.0317	0.9878	0.0042	0.0013
2.26	0.0310	0.9881	0.0041	0.0013
2.27	0.0303	0.9884	0.0040	0.0013
2.28	0.0297	0.9887	0.0039	0.0012
2.29	0.0290	0.9890	0.0038	0.0012
2.30	0.0283	0.9893	0.0037	0.0012
2.31	0.0277	0.9896	0.0036	0.0011
2.32	0.0270	0.9898	0.0035	0.0011
2.33	0.0264	0.9901	0.0034	0.0010
2.34	0.0258	0.9904	0.0033	0.0010
2.35	0.0252	0.9906	0.0032	0.0010
2.36	0.0246	0.9909	0.0031	0.0009
2.37	0.0241	0.9911	0.0030	0.0009
2.38	0.0235	0.9913	0.0029	0.0009
2.39	0.0229	0.9916	0.0028	0.0009
2.40	0.0224	0.9918	0.0027	0.0008
2.41	0.0219	0.9920	0.0026	0.0008
2.42	0.0213	0.9922	0.0026	0.0008
2.43	0.0208	0.9925	0.0025	0.0008
2.44	0.0203	0.9927	0.0024	0.0007
2.45	0.0198	0.9929	0.0023	0.0007
2.46	0.0194	0.9931	0.0023	0.0007
2.47	0.0189	0.9932	0.0022	0.0007
2.48	0.0184	0.9934	0.0021	0.0006
2.49	0.0180	0.9936	0.0021	0.0006
2.50	0.0175	0.9938	0.0020	0.0006
2.51	0.0171	0.9940	0.0019	0.0006
2.52	0.0167	0.9941	0.0019	0.0006
2.53	0.0163	0.9943	0.0018	0.0005
2.54	0.0158	0.9945	0.0018	0.0005
2.55	0.0154	0.9946	0.0017	0.0005

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
2.56	0.0151	0.9948	0.0017	0.0005
2.57	0.0147	0.9949	0.0016	0.0005
2.58	0.0143	0.9951	0.0016	0.0005
2.59	0.0139	0.9952	0.0015	0.0004
2.60	0.0136	0.9953	0.0015	0.0004
2.61	0.0132	0.9955	0.0014	0.0004
2.62	0.0129	0.9956	0.0014	0.0004
2.63	0.0126	0.9957	0.0013	0.0004
2.64	0.0122	0.9959	0.0013	0.0004
2.65	0.0119	0.9960	0.0012	0.0004
2.66	0.0116	0.9961	0.0012	0.0003
2.67	0.0113	0.9962	0.0012	0.0003
2.68	0.0110	0.9963	0.0011	0.0003
2.69	0.0107	0.9964	0.0011	0.0003
2.70	0.0104	0.9965	0.0011	0.0003
2.71	0.0101	0.9966	0.0010	0.0003
2.72	0.0099	0.9967	0.0010	0.0003
2.73	0.0096	0.9968	0.0010	0.0003
2.74	0.0093	0.9969	0.0009	0.0003
2.75	0.0091	0.9970	0.0009	0.0003
2.76	0.0088	0.9971	0.0009	0.0002
2.77	0.0086	0.9972	0.0008	0.0002
2.78	0.0084	0.9973	0.0008	0.0002
2.79	0.0081	0.9974	0.0008	0.0002
2.80	0.0079	0.9974	0.0008	0.0002
2.81	0.0077	0.9975	0.0007	0.0002
2.82	0.0075	0.9976	0.0007	0.0002
2.83	0.0073	0.9977	0.0007	0.0002
2.84	0.0071	0.9977	0.0007	0.0002
2.85	0.0069	0.9978	0.0006	0.0002
2.86	0.0067	0.9979	0.0006	0.0002
2.87	0.0065	0.9979	0.0006	0.0002
2.88	0.0063	0.9980	0.0006	0.0002
2.89	0.0061	0.9981	0.0006	0.0002
2.90	0.0060	0.9981	0.0005	0.0001
2.91	0.0058	0.9982	0.0005	0.0001
2.92	0.0056	0.9982	0.0005	0.0001
2.93	0.0055	0.9983	0.0005	0.0001
2.94	0.0053	0.9984	0.0005	0.0001
2.95	0.0051	0.9984	0.0005	0.0001
2.96	0.0050	0.9985	0.0004	0.0001
2.97	0.0048	0.9985	0.0004	0.0001
2.98	0.0047	0.9986	0.0004	0.0001
2.99	0.0046	0.9986	0.0004	0.0001
3.00	0.0044	0.9987	0.0004	0.0001
3.01	0.0043	0.9987	0.0004	0.0001

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
3.02	0.0042	0.9987	0.0004	0.0001
3.03	0.0040	0.9988	0.0003	0.0001
3.04	0.0039	0.9988	0.0003	0.0001
3.05	0.0038	0.9989	0.0003	0.0001
3.06	0.0037	0.9989	0.0003	0.0001
3.07	0.0036	0.9989	0.0003	0.0001
3.08	0.0035	0.9990	0.0003	0.0001
3.09	0.0034	0.9990	0.0003	0.0001
3.10	0.0033	0.9990	0.0003	0.0001
3.11	0.0032	0.9991	0.0003	0.0001
3.12	0.0031	0.9991	0.0002	0.0001
3.13	0.0030	0.9991	0.0002	0.0001
3.14	0.0029	0.9992	0.0002	0.0001
3.15	0.0028	0.9992	0.0002	0.0001
3.16	0.0027	0.9992	0.0002	0.0001
3.17	0.0026	0.9992	0.0002	0.0001
3.18	0.0025	0.9993	0.0002	0.0001
3.19	0.0025	0.9993	0.0002	0.0000
3.20	0.0024	0.9993	0.0002	0.0000
3.21	0.0023	0.9993	0.0002	0.0000
3.22	0.0022	0.9994	0.0002	0.0000
3.23	0.0022	0.9994	0.0002	0.0000
3.24	0.0021	0.9994	0.0002	0.0000
3.25	0.0020	0.9994	0.0002	0.0000
3.26	0.0020	0.9994	0.0001	0.0000

x	$\varphi(x)$	$\Phi(x)$	$G(x)$	$H(x)$
3.27	0.0019	0.9995	0.0001	0.0000
3.28	0.0018	0.9995	0.0001	0.0000
3.29	0.0018	0.9995	0.0001	0.0000
3.30	0.0017	0.9995	0.0001	0.0000
3.31	0.0017	0.9995	0.0001	0.0000
3.32	0.0016	0.9995	0.0001	0.0000
3.33	0.0016	0.9996	0.0001	0.0000
3.34	0.0015	0.9996	0.0001	0.0000
3.35	0.0015	0.9996	0.0001	0.0000
3.36	0.0014	0.9996	0.0001	0.0000
3.37	0.0014	0.9996	0.0001	0.0000
3.38	0.0013	0.9996	0.0001	0.0000
3.39	0.0013	0.9997	0.0001	0.0000
3.40	0.0012	0.9997	0.0001	0.0000
3.41	0.0012	0.9997	0.0001	0.0000
3.42	0.0012	0.9997	0.0001	0.0000
3.43	0.0011	0.9997	0.0001	0.0000
3.44	0.0011	0.9997	0.0001	0.0000
3.45	0.0010	0.9997	0.0001	0.0000
3.46	0.0010	0.9997	0.0001	0.0000
3.47	0.0010	0.9997	0.0001	0.0000
3.48	0.0009	0.9997	0.0001	0.0000
3.49	0.0009	0.9998	0.0001	0.0000
3.50	0.0009	0.9998	0.0001	0.0000

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