

Index

A

Accessible information, 140, 576, 620, **622**
A/D conversion, 142, 644
Additive noise, 138, 368
Additive noise channel, 620
Adjoint operator, 34, 44
Alphabet, 470, 577
Analog systems, 138
 through digital systems, 140
Annihilation operator, 476, **482**
 interpretation, 479
Anticommutator, 35, 109
APD, 147, 163
A priori probabilities, 139, 184
Arrivals, 155
Astronomical unit (AU), 152
Atypical sequences, 597
Average, 472
Average number of photons, 80

B

Baker–Campbell–Hausdorff (BCH) identity,
 485, 515, 558
Baker–Hausdorff formula, 485
Bandwidth, 138
Basis, 72
 of a vector space, 24, 29
BB84 protocol, 648
BBM92 protocol, 653
Beam splitter, 425, 514, 546–549
 as an entangler, 551
Bell states, 456, 459, 590, 592, 661
Benioff, 8
Bennett, 8, 652
Binary conversion, 142
Binary source, 577

Binary symmetric channel, 202, 606
Bipartite quantum system, 592
Bipartite states, 460
 mixed, 460
 pure, 457
Bit, 574, 581, 594
Bit-flip channel, 616
Black body, 2
Bloch sphere, 122–124, 617
Bogoliubov transformation, 514, 525, 560
 definition, 509
Bohr, 3
Boltzmann’s constant, 85
Boschi, 9
Bosonic operators, 482
Bosonic variables, 465
Bosons, 479
Bouwmeester, 9
BPSK modulation, *see* PSK modulation
 classical, 321
 quantum, 320
Bra, 31, 467
Brassard, 9, 652

C

Cahill, 487
Campbell’s theorems, 159
Canonical variables, 465
Capacity, 140, 576, 605, **607**
 classical, 625
 multiplicity, 625
 of input $c \rightarrow q$ channel, 627
 product state, 629–633
 with entropy exchange, 635
 with input entanglement, 633
Cauchy–Schwarz’s inequality, 109

- Channel coding, 576, 605
 - Chaotic state, 588
 - Characteristic function, 86, 452, 492–498, 505
 - normal-ordered, 500
 - Chernoff bound, 252, 277
 - Cholesky's decomposition, 67
 - Chuang, 10
 - Circulant matrix, 267
 - block, 272
 - EID, 279
 - Circulant vector, 267
 - Classical channel, 605
 - Classical communications system, 143
 - with thermal noise, 373
 - Classical information, 143, 575, 577, 626
 - Classical mechanics, 465
 - Classical optical decision, 373
 - Classical symbols, 287
 - Classical–classical (c→c) mapping, 144
 - Classical–quantum (c→q) mapping, 144, 366, 614
 - Codeword, 605
 - Coherent modulation, 288
 - Coherent states, 80, 183, 479, 282–481, 505, 531
 - multimode, 291
 - Coherent transmission, 148
 - Combinations of projectors, 100
 - Commutable operators, 62, 109
 - Commutation condition, 561
 - Commutation relation, 89
 - Commutativity, 62
 - Commutator, 35, 109
 - Complementary projector, 48
 - Complete positivity, 616
 - Completeness of a vector space, 28
 - Complex envelope, 166–168, 288
 - Complex symbols
 - normalized, 296, 382
 - scaled, 296, 382
 - Composite quantum systems, 111
 - Compressed Hilbert space, 240, 241
 - dimension, 243
 - Compression, *see* state compression
 - Compression rate, 603
 - Compressor, 240
 - Computational complexity
 - in PPM with thermal noise, 402
 - Conjugate transpose, 34
 - Constellation, 622
 - of coherent states, 281, 287, 289, 292–296
 - of complex symbols, 289
 - of squeezed-displaced states, 353
 - of states, 203, 221, 231
 - symmetric, 292
 - Constellation of density operators, 382
 - Continuous observable, 471
 - Continuous quantum variables, 9, 12, 451, 464
 - abstract formulation, 481–484
 - Contour level, 287
 - Convex semidefinite programming, *see* CSP
 - Coordinate system, 30
 - Correct decision probability, 139, 190, 208, 260, 263, 269, 275, 303, 385
 - Covariance matrix, 494
 - Creation operator, 476, 482
 - interpretation, 479
 - Convex semidefinite programming (CSP), 210, 213, 383, 403, 407
 - with GUS, 411
- D**
- D/A conversion, 142
 - Data compression, 575, 595–600
 - De Broglie, 3
 - Decision by geometric method, 198
 - Decision criterion, 184, 189
 - Decision operator, 193, 386
 - Deep space, 152
 - Degree of superposition
 - of coherent states, 285
 - of squeezed states, 351
 - Density operator, 82, 452, 484
 - discretization, 369–373, 415–418
 - nonuniqueness of decomposition, 85, 117–120
 - weighted, 203, 211, 221
 - Depolarizing channel, 617
 - Deutsch, 8
 - Discrete Fourier transform (DFT), 267
 - DFT matrix, 267
 - block, 274
 - Diagonalization, 54, 60
 - simultaneous, 62, 419
 - Digital channel, 187
 - Digital messages, 577
 - Digital revolution, 140
 - Digital systems, 139
 - Dimensions of a vector space, 24
 - Dirac, 464, 469
 - Dirac's delta function, 467, 468
 - Dirac's notation, 31–33

Direct sum, 52
 Discrete observable, 471
 Discrete quantum variables, 8, 451, 464
 Displacement operator, 506, 513, 528
 Distance, 27
 Dolinar's receiver, 439
 Dual theorem, 212
 Dynamical variables, 470

E

Eavesdropping, 649
 Eckert, 9
 Eigendecomposition (EID), 54–62, 225, 279
 of a circulant matrix, 267
 of a density operator, 369
 reduced form, 61, 62, 119, 205, 225, 257
 Eigendecomposition, *see* EID
 Eigenfunction of Fourier transform, 502, 505
 Eigenket, 468
 Eigenvalue, 38, 468
 Eigenvector, 38, 468
 Einstein, 3
 Einstein–Podolsky–Rosen (EPR) states, 568
 Ekert, 653
 Elementary measurement operators, 106
 Elementary operator, 40, 43
 Elementary POVMs, 104
 Elementary projector, 53
 Energy quantization, 478
 Ensemble, 82, 83, 117, 577, 586
 nonunicity, 117–121
 Entangled, 457
 maximally, 457
 Entangled measurements, 626
 Entanglement, 7, 114, 452
 in bipartite states, 460
 in EPR states, 549
 in two-mode states, 549
 Entropy
 in a bipartite system, 582
 of a sequence, 584
 of a symbol, 581
 Entropy rate, 585
 Error probability, 139, 189, 257
 Error probability (plot)
 in 2-PSK, 321
 in BPSK, 388
 in K-PSK, 335
 in OOK, 319
 in PPM, 345–348, 406, 412
 in PSK, 398
 in PSK with squeezed states, 358

 in QAM, 330, 393
 Expander, 240
 Exponential operator, 485

F

Factor of a density operator, 118, 205–207
 minimum, 119–120, 206
 orthonormal, 119–120, 206
 Factorization, 205
 Factorization (of a density operator), 262, 370, 419
 Fermions, 479
 Feynman, 8, 452
 Fock basis, 477
 Fock expansion
 of a two-mode state, 544
 Fock representation, 479, 502
 Fock space, 481
 Fock states, 80, 501, 503, 505
 Fourier coefficients, 30
 Fourier expansion, 30, 43, 458, 460
 Fourier transform, 487–490
 complex, 488
 for the N -mode, 489
 symplectic, 488
 two-dimensional, 488
 Free-space, 151, 154
 FT, *see* Fourier transform
 Functional calculus, 62–65

G

Gaussian approximation, 177
 Gaussian bivariate, 286
 Gaussian channel, 618
 with additive noise, 368
 Gaussian density, 163
 Gaussian noise, 177
 Gaussian state, 286, 367, 452, 465, 495–496, 529–530
 definition, 495
 noisy, 538
 rotated, 553
 the most general, 519, 520
 Gaussian transformation, 453, 465
 definition, 508
 in the single mode, 525–529
 in the two-mode, 540–549
 Gaussian unitary, 508
 combination, 516
 in the N -mode, 513–518
 in the single mode, 527–529
 in the two mode, 540

the most general, 517, 528
 Gaussian, 511
 Generator sets, 23
 Geometric interpretation of optimization, 220
 Geometrical distribution, 364, 507
 Geometrically uniform symmetry, *see* GUS
 Geometry of a state constellation, 221–230
 Giovannetti, 614
 Glauber, 7, 487
 Glauber density operator, 367, 375, 381
 Glauber state, *see* coherent state
 Glauber's inversion formula, 492
 Gram's matrix, 224, 251, 252, 259, 273, 293, 384, 385
 Gram's operator, 224, 251, 252, 258, 272, 384, 385
 Ground state, 80, 283, 367, 479
 Geometrically uniform symmetry (GUS), 230–235, 265–276, 315
 in PPM modulation, 339, 558
 in PSK modulation, 333
 with Gaussian states, 552–558
 with mixed states, 385, 554
 with squeezed states, 353

H

Hamiltonian, 482
 Hamiltonian operator, 478
 Harmonic oscillator, 451, 473–479
 Hausladen, 251
 Heisenberg, 4
 Heisenberg's picture, 87
 Helstrom, 7
 Helstrom's bound, 197
 Helstrom's theory, 192, 386, 403
 Hermitian matrix, 44
 Hermitian operator, 45, 63
 Higg's boson, 479
 Hilbert, 5
 Hilbert space, 10, 29–33
 Holevo, 614
 Holevo bound, 620, 622
 Holevo's theorem, 211, 218, 247
 proof, 244
 Holevo- χ , 622, 625
 Homodyne detection, 643
 Homodyne receiver, 300, 310
 HSW theorem, 628, 631
 HWS

 Holevo–Schumacher–Westmoreland, 576

I

Identity operator, 34, 92
 IID quantum source, 600
 Image of an operator, 37
 Incoherent transmission, 147
 Independent identically distributed (IID) sequence, 596
 Indeterminacy, 6
 Information reconciliation, 655
 Information source, 577
 Inner product, 25, 245
 of coherent states, 284
 of squeezed states, 351
 Inner-product vector space, 25
 Instantaneous current, 162
 Intensity (of a Poisson process), 155
 Inverse square root, 256

J

Jeans, 2
 Jet Propulsion Laboratory, 8

K

Kennedy's receiver, 435
 Kennedy's theorem, 216–221
 generalization, 221
 proof, 247
 Ket, 31, 467
 Kraus representation, 184, 616
 Kronecker product, 69–72
 Kronecker's delta, 467
 Kronecker's symbol, 30
 Kwiat, 9

L

Laguerre distribution, 172, 366, 374, 376
 Laguerre polynomials, 366, 505
 Laser, 7, 147, 424
 Laser radiation, 80
 Lau, 8
 Law of large numbers, 575, 596
 LED, 147
 Lexicographical order, 69, 324
 Linear independence, 23
 Linear operator, 33
 Linear transformation, 228
 Linearity, 6
 Linearity of a Hilbert space, 78
 Linear matrix inequalities (LMI), 213, 407
 Logarithmic law, 580
 Least squares measurements (LSM), 251

M

Manin, 8
 MatLab LMI toolbox, 407
 Matrix
 block circulant, 327
 circulant, 342
 Matrix of the inner products, 224
 Matrix of the outer products, 224
 Matrix representation, 497
 Matrix representation of an operator, 35
 Mattle, 9
 Maxwell, 165
 Mean square value in a measurement, 99
 Mean value in a measurement, 99
 Mean vector, 494
 Measure of information, 574, 579–585
 Measurement alphabet, 185
 Measurement factors, 205
 Measurement matrix, 204, 222, 252, 254, 383
 optimal, 256
 Measurement operator, 190
 global, 191
 optimal, 209
 system of, 187, 194, 211
 Measurement operator system, 106
 Measurement vectors, 94, 106, 197, 216, 259
 Measurements with observables, 98, 101
 Memoryless channel, 139, 605
 Mermin, 652
 Message, 135
 as a random process, 578
 analog, 136
 digital, 136
 with independent symbols, 579
 Mixed product matrix, 263
 Mixed state, 82
 Mixed-product law, 72
 Mixture of states, 82, 83
 Modulation
 scalar, 288
 vector, 289
 Modulator, 427
 Moments, 99, 101
 Momentum, 470
 Momentum (of a particle), 110
 Moore's law, 8
 Moore–Penrose generalized inverse, 257
 Multimode Hilbert space, 483
 Mutual information, 140, 606, 620, 622

N

Neumark's theorem, 105

Nielsen, 10
 No-cloning theorem, 87, 116, 660
 Noise temperature, 152
 Noisy coherent state, 365–367
 as Gaussian state, 367
 Nonnegative operator, 48, 64
 Norm, 27
 Normal matrix, 44
 Normal ordering, 522–525
 Normalization
 in continuous variables, 481
 Null operator, 92
 Number of arrivals, 156
 Number of photons, 80
 global average, 366
 Number of signal photons, 173, 365
 Number of signal photons per bit, 294, 305, 432
 Number of signal photons per symbol, 293–295, 303
 Number of thermal photons, 173, 363, 507
 Number operator, 282, 364, 395, 472, 477, 482
 interpretation, 479
 Number states, 80, 85, 282, 477, 482

O

Observable, 98, 108, 284, 468–470
 continuous, 101
 Oliver, 141
 OOK modulation, 148, 281, 307, 432
 classical, 307, 319
 quantum, 318, 390
 Open system, 614–620
 Operator, 227, 228
 Optical channel, 148
 Optical communications, 421–423
 classical, 133, 146–150, 304–313, 431–433
 quantum, 133
 Optical detection
 semiclassical model, 165
 Optical fiber, 148
 Optical frequency, 146, 152
 Optical power, 146, 151
 amplified, 162
 average, 146, 298
 instantaneous, 146, 161, 296
 Optimization, 11, 133, 190
 Optimization of a binary receiver, 192
 Orthogonal condition
 for projectors, 51

Orthonormal basis, 32, 42
 Orthonormality condition, 30
 Outcome of a quantum measurement, 91, 187
 Outer product, 36, 40

P

PAM modulation, 134, 147
 Partial trace, 454–457
 Pauli's matrices, 124
 Phase space, 286, 465
 in the N -mode, 491–499
 in the single-mode, 499–503
 Photodetector, 429
 Photon counter, 179, 301
 Photon counting, 175–178
 quantum interpretation, 307
 Photons, 479
 Pierce, 141
 Pin diode, 147, 162
 Planck, 2
 Planck's constant, 3, 85, 87, 110
 Poisson distribution, 472, 532
 Poisson process, 155–158, 165
 definition, 155
 doubly stochastic, 160, 166, 429
 filtered, 158–161
 Gaussianity, 163
 marked and filtered, 160
 Poisson random variable, 156, 284
 Poisson's regime, 283
 Polar decomposition, 65, 515, 545
 Position, 470
 Positive definite, 73
 Positive operator, 64
 Positive semidefinite, 478
 Positive semidefinite operator, 48, 64, 83
 Postulate 1 of Quantum Mechanics, 78
 Postulate 2 of Quantum Mechanics, 87
 Postulate 3 of Quantum Mechanics, 92
 reformulation with POVMs, 105
 with density operators, 95
 with elementary projectors, 94
 Postulate 4 of Quantum Mechanics, 111
 POVMs, 102, 190
 as projective measurements, 105
 PPM modulation, 134, 147, 281, 338
 classical, 337, 343, 403, 406
 implementation, 444
 quantum, 338, 341, 402, 404
 with thermal noise, 399
 P-representation, 497

Privacy amplification, 656
 Probability amplitude, 471
 Probability density, 473
 Probability distribution, 82, 92, 473, 577
 Product of operators, 34
 Projective measurements, 91
 Projector, 47
 properties, 51
 unit rank, 53
 Projector system, 52, 92, 98, 106, 191, 194
 PSD, *see* positive semidefinite
 Pseudo-random number generation, 640
 PSK modulation, 134, 148, 281, 332, 628
 classical, 144, 332, 334, 388, 396, 398
 implementation, 443
 quantum, 144, 332, 333, 387, 396
 with squeezed states, 355
 with thermal noise, 395
 Pure states, 81, 83
 Purification, 461–462

Q

QAM modulation, 134, 148, 281
 classical, 323, 328
 implementation, 443
 quantum, 323–328
 with thermal noise, 391
 Quantum key distribution (QKD), 646–659
 entanglement-based protocol, 653
 prepare-and-measure protocol, 648
 with discrete variables, 648
 Quadratic error, 255
 Quadrature operators, 482
 Quanta, 3, 162
 Quantization, 6, 142
 Quantum channel, 614–620
 noiseless, 620
 noisy, 620, 625
 Quantum communications, 9, 10
 digital, 464
 Quantum communications system, 143
 binary, 316, 386, 433–443
 implementation, 134
 multilevel, 282, 443–446
 with coherent states, 281
 with squeezed states, 354
 with thermal noise, 381–386
 Quantum compression, 604. *see also* state compression
 Quantum computer, 9
 Quantum cryptography, 645
 Quantum data compression, 600, 604

Quantum decision theory, 11
 Quantum detection theory, 133
 Quantum entropy, **585–588**
 in a bipartite system, 589
 conditional, 592
 definition, 586
 difference with classical, 601
 join, 612
 Quantum expectation, 102, 472
 Quantum information, 7, 12, 143, 465, 575
 continuous, 451
 discrete, 451
 Quantum information theory, 9, 12, 453
 Quantum key distribution (QKD), 9, 451
 Quantum limit, 307, 432
 standard, 311, 433
 Quantum measurement for decision, 187
 Quantum measurements, 91, 466
 collective, 625
 with continuous observables, 471
 with discrete observables, 471
 Quantum operation, 625
 Quantum random number generation (QRNG), 639–644
 with continuous variables, 642
 with discrete variables, 640
 Quantum random state, 586
 Quantum state
 as sources of information, 585
 Quantum–classical ($c \rightarrow q$) mapping, 614
 Quantum–quantum ($q \rightarrow q$) mapping, 144
 Qubit, 79, 84, 121–126, 451, 575, 594, 628
 measurement, 96
 multiple, 126
 Qubit channel, 626
 Qubits/symbol, 603

R

Radio frequency, 152
 Random process, 578
 Random variable, 91, 95, 110
 discrete, 92
 finite, 577
 Randomness, 5
 Rank, 247
 Rank of a projector, 52
 Rank of an operator, 37, 61
 Rayleigh, 2
 Rayleigh-Jeans Law, 2
 Realizations, 578, 602
 Reconstruction criterion, 372
 Reduced density operators, **454–457**

Reliable transmission, 576
 Repetition of a measurement, 107
 Resolution of the generalized identity, 217
 Resolution of the identity, 42, 52, 92, 468
 Responsivity, 165
 Rice density, 172
 Rotated state, 532
 Rotation operator, 333, **513**, 528
 Rutherford, 3

S

Sampling, 142
 Sasaki–Hirota receiver, 442
 Scale factor in a constellation, **295**, 296
 Schmidt decomposition, 116, 458
 Schrödinger, 4
 Schrödinger’s equation, 87
 Schrödinger’s picture, 87
 Schumacher, 595
 Schumacher’s compression protocol, **603**
 Schwartz’s inequality, 28, 33
 Sensitivity of a receiver, 294
 Separability tests, 550
 Separable (not entangled), 457
 Shannon, 9, 141, 453, 573
 Shannon capacity, 626
 Shannon’s channel coding theorem, **612**
 Shannon’s source coding theorem, **597–600**
 Shape factor, **295**, 304, 326
 Shor, 9
 Shot noise, 152, 155, 186, 281
 Shot noise limit, *see* quantum limit
 Sifting and eavesdropping detection, 650
 Sifting property, 468
 Signal, 135
 at decision point, 301
 Signal-to-noise ratio, 138, 303
 Simultaneous measurements, 108
 Single photon, 158
 Single-mode Hilbert space, **481**
 Single-photon state, 504
 Singular value decomposition, *see* SVD
 Singular values, 65
 Sky noise, 152
 SNR, *see* signal-to-noise ratio
 Source coding, *see* data compression
 Source probabilities, 605
 Spectral decomposition, 54, 224, 469
 Spectrum of an operator, 38, 72, 468
 continuous, 469
 Square root measurements, *see* SRM
 Square root of an operator, 64, 268

- Squeeze operator, **513, 528**
 - in the two-mode, **540**
 - Squeezed state, **281, 348–353, 532, 565, 568**
 - in the two-mode, **568**
 - Squeezed vacuum state, **532**
 - Squeezed–displaced state, **349, 533**
 - Caves–Schumaker, **541**
 - in the two-mode, **541**
 - Square root measurements (SRM), **134, 251–257, 265–276, 315**
 - in QAM, **392**
 - with mixed states, **262, 385**
 - State
 - entangled, **115**
 - separable, **115**
 - weighted, **118, 254**
 - State compression, **238–244, 408–410**
 - difference with quantum compression, **604**
 - with GUS, **242, 410**
 - with SRM, **276**
 - State expansion, **239**
 - State factors, **205**
 - State matrix, **204, 222, 252, 258, 293, 382**
 - with weighted states, **208**
 - State superposition, **78**
 - Statistics
 - on coherent states, **283**
 - on noisy coherent states, **365**
 - on squeezed displaced states, **349**
 - on squeezed states, **349**
 - Suboptimization, **11, 134**
 - Subspace, **23, 196, 216, 217, 227**
 - Super quantum limit, **313**
 - Superdense coding, **9**
 - Superhodyne receiver, **311**
 - Superposition, **6**
 - Superposition degree (quadratic), **197**
 - Superposition principle, **33**
 - Singular value decomposition (SVD), **65, 225**
 - reduced form, **66, 255**
 - Symbol alphabet, **184**
 - Symbol probabilities, **139**
 - Symmetry operator, **231, 235**
 - of PPM, **339, 419**
 - of PSK, **333, 395**
 - of QAM, **327**
 - Symplectic decomposition, **511**
 - Symplectic eigenvalues, **511**
 - Symplectic group, **484**
 - Symplectic matrix, **509, 528, 619**
 - Symplectic transformation, **509, 514, 525, 560**
 - definition, **509**
 - Synthesis of an operator from a basis, **59**
- T**
- Telecommunications system, **135–137**
 - Teleportation, **451, 659**
 - Tensor product, **67, 111, 338**
 - of coherent states, **285, 291, 293**
 - Thermal decomposition, **511, 526**
 - Thermal noise, **11, 85, 134, 152, 170, 186, 187, 363**
 - Thermal state, **363, 507, 588**
 - Trace, **36, 43, 73, 83**
 - cyclic property, **37**
 - from the eigenvalues, **40**
 - Trace criterion, **372**
 - Transition probabilities, **139, 188, 208, 257, 259, 263, 268, 273, 303, 385, 605**
 - global, **190**
 - Transjectors, **229, 255**
 - Typical quantum sequence, **601**
 - Typical sequences, **597**
 - Typical subspace, **601**
- U**
- Ultraviolet catastrophe, **3**
 - Uncertainty principle, **5, 109, 537**
 - Unit vector, **27**
 - Unitary matrix, **44**
 - Unitary operator, **46, 63, 87**
- V**
- Vacuum state, **479, 503**
 - Vector space, **22–25**
 - Vilnrotter, **8**
 - Von Neumann, **91**
- W**
- Wave function, **87**
 - Wave–particle duality, **3**
 - Weighted states, **203**
 - Weinfurter, **9**
 - Weyl operator, **491, 499, 559**
 - Wigner, **492**
 - Wigner function, **86, 286, 452, 492, 498, 505, 643**
 - Williamson’s theorem, **511, 526**

Wolfram, [487](#)

Word

as a random vector, [578](#)

Z

Zeilinger, [9](#)

Zero operator, [34](#)