Appendix

Tables

This chapter contains tables which list the first number fields (ordered by increasing discriminant) of degrees 2, 3 and 4 which admit nonzero singular Jacobi forms. More precisely, we searched all of the Bordeaux number field tables of the PARI group [Bor95] for totally real number fields fulfilling the conditions for admitting nonzero singular Jacobi forms.

For number fields K of degree 2 and 3 over $\mathbb{Q}$ we list the first 30 number fields where we find nonzero singular Jacobi forms (Tables A.1 and A.2). The percentage of number fields of degree 2 and 3 admitting nonzero singular Jacobi forms among all fields of these degrees in the Bordeaux tables is 17.87 % and 4.75 %, respectively. For number fields K of degree 4 over $\mathbb{Q}$ we list *all* number fields of the Bordeaux tables admitting nonzero singular Jacobi forms (Table A.3). The corresponding percentage is in this case 0, 25 %. We searched the Bordeaux tables also for number fields of degrees $n = 5, 6, 7$ which admit nonzero singular forms. However, in the available range we could not find any such fields.

The columns of the tables display from left to right the discriminant D_K of K, the number s of nonzero singular Jacobi forms modulo isomorphism, the minimal polynomial f of K, whether the different $\mathfrak{d}_K$ of K is a square in the narrow ideal class group or not, the number of $\mathfrak{g}$ satisfying the assumption of Theorem 4.2, and the number of prime ideals $\mathfrak{p}$ of degree one above 3, respectively.

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Table A.1 Number fields K with $[K:\mathbb{Q}]=2$

D_K	s	f	∂_K	$\mathfrak{g}$	$\mathfrak{p}$
17	1	x^2-x-4	✓	1	0
41	1	x^2-x-10	✓	1	0
57	2	x^2-x-14		1	1
65	2	x^2-x-16	✓	1	0
73	4	x^2-x-18	✓	4	2
89	1	x^2-x-22	✓	1	0
97	4	x^2-x-24	✓	4	2
113	1	x^2-x-28	✓	1	0
129	2	x^2-x-32		1	1
137	1	x^2-x-34	✓	1	0
145	4	x^2-x-36	✓	2	2
185	2	x^2-x-46	✓	1	0
193	4	x^2-x-48	✓	4	2
201	2	x^2-x-50		1	1
217	4	x^2-x-54		2	2
233	1	x^2-x-58	✓	1	0
241	4	x^2-x-60	✓	4	2
257	1	x^2-x-64	✓	1	0
265	4	x^2-x-66	✓	2	2
273	4	x^2-x-68		1	1
281	1	x^2-x-70	✓	1	0
305	2	x^2-x-76	✓	1	0
313	4	x^2-x-78	✓	4	2
337	4	x^2-x-84	✓	4	2
353	1	x^2-x-88	✓	1	0
377	2	x^2-x-94	✓	1	0
401	1	$x^2-x-100$	✓	1	0
409	4	$x^2-x-102$	✓	4	2
417	2	$x^2-x-104$		1	1
433	4	$x^2-x-108$	✓	4	2

Table A.2 Number fields K with $[K:\mathbb{Q}]=3$

D_K	s	f	∂_K	$\mathfrak{g}$	$\mathfrak{p}$
961	1	$x^3-x^2-10x+8$	✓	1	0
1,849	1	$x^3-x^2-14x-8$	✓	1	0
3,969	2	$x^3-21x-28$	✓	2	1
4,481	2	$x^3-17x-8$		1	1
7,057	1	$x^3-x^2-22x+32$	✓	1	0
7,441	1	$x^3-x^2-22x-16$	✓	1	0
8,281	1	$x^3-x^2-30x+64$	✓	1	0
8,289	2	$x^3-21x-12$	✓	2	1
8,713	1	$x^3-25x-32$	✓	1	0
9,153	2	$x^3-21x-4$	✓	2	1
10,641	4	$x^3-x^2-22x+16$	✓	4	2
11,137	1	$x^3-x^2-22x+8$	✓	1	0
11,665	1	$x^3-x^2-26x+40$	✓	1	0
11,881	1	$x^3-x^2-36x+4$	✓	1	0
13,689	2	$x^3-39x-26$	✓	2	1
14,129	2	$x^3-x^2-26x-16$		1	1
14,609	2	$x^3-x^2-26x+32$		1	1
15,641	2	$x^3-29x-36$	✓	2	1
15,961	1	$x^3-x^2-30x-32$	✓	1	0
16,129	1	$x^3-x^2-42x-80$	✓	1	0
16,369	1	$x^3-x^2-26x-8$	✓	1	0
16,649	2	$x^3-x^2-34x-48$	✓	2	1
16,689	4	$x^3-x^2-26x+24$	✓	4	2
17,689	1	$x^3-x^2-44x+64$	✓	1	0
18,201	4	$x^3-x^2-30x+48$	✓	4	2
19,441	1	$x^3-37x-68$	✓	1	0
20,073	4	$x^3-x^2-30x-24$	✓	4	2
20,385	2	$x^3-33x-48$	✓	2	1
21,281	2	$x^3-x^2-42x+104$	✓	2	1
23,321	2	$x^3-x^2-30x-16$		1	1

Table A.3 Number fields K with $[K:\mathbb{Q}]=4$

D_K	s	f	∂_K	$\mathfrak{g}$	$\mathfrak{p}$
122,825	2	$x^4 - x^3 - 23x^2 + x + 86$	✓	1	0
164,441	1	$x^4 - 2x^3 - 13x^2 + 14x + 32$	✓	1	0
171,377	1	$x^4 - 2x^3 - 19x^2 + 20x + 32$	✓	1	0
274,625	2	$x^4 - x^3 - 24x^2 + 4x + 16$	✓	1	0
282,353	1	$x^4 - x^3 - 35x^2 + 41x + 202$	✓	1	0
310,985	2	$x^4 - 2x^3 - 13x^2 + 14x + 8$	✓	1	0
314,721	2	$x^4 - 25x^2 + 16$	✓	1	0
317,033	1	$x^4 - 2x^3 - 17x^2 + 18x + 64$	✓	1	0
340,857	2	$x^4 - 2x^3 - 13x^2 + 14x + 16$	✓	1	0
356,337	2	$x^4 - x^3 - 37x^2 + 25x + 268$	✓	1	0
379,457	2	$x^4 - 2x^3 - 23x^2 + 24x + 76$	✓	1	0
389,017	1	$x^4 - x^3 - 27x^2 + 41x + 2$	✓	1	0
393,129	2	$x^4 - x^3 - 37x^2 + 97x + 4$	✓	1	0
393,329	1	$x^4 - x^3 - 39x^2 + 9x + 302$	✓	1	0
471,537	2	$x^4 - x^3 - 25x^2 + 25x + 64$	✓	1	0
485,809	1	$x^4 - 29x^2 + 36$	✓	1	0
500,033	1	$x^4 - 2x^3 - 21x^2 - 18x + 8$	✓	1	0
506,617	1	$x^4 - 2x^3 - 21x^2 + 22x + 104$	✓	1	0
532,521	4	$x^4 - x^3 - 27x^2 - 7x + 82$		1	1
624,529	1	$x^4 - 2x^3 - 27x^2 + 28x + 128$	✓	1	0
626,441	1	$x^4 - 21x^2 - 8x + 20$	✓	1	0
663,833	1	$x^4 - x^3 - 51x^2 + 49x + 514$	✓	1	0
668,457	2	$x^4 - 2x^3 - 33x^2 + 34x + 136$	✓	1	0
674,057	1	$x^4 - 23x^2 - 2x + 88$	✓	1	0
704,969	1	$x^4 - x^3 - 33x^2 - 39x + 8$	✓	1	0
751,409	1	$x^4 - 23x^2 - 6x + 80$	✓	1	0
754,769	1	$x^4 - x^3 - 26x^2 + 8x + 64$	✓	1	0
756,313	1	$x^4 - x^3 - 53x^2 + 33x + 596$	✓	1	0
768,713	1	$x^4 - 21x^2 - 4x + 32$	✓	1	0
830,297	4	$x^4 - x^3 - 57x^2 + x + 664$	✓	1	0
860,353	2	$x^4 - 2x^3 - 55x^2 + 56x + 172$	✓	1	0
906,593	1	$x^4 - 2x^3 - 31x^2 + 32x + 188$	✓	1	0
996,761	1	$x^4 - 2x^3 - 29x^2 + 30x + 208$	✓	1	0

Glossary

We list in roughly alphabetical order the basic notations which are used throughout this monograph.

$(\mathfrak{a}, \mathfrak{b})$	The greatest common divisor of the integral $\mathscr{O}_K$-ideals $\mathfrak{a}$ and $\mathfrak{b}$.
$\mathbb{C}^{\infty}(V)$	The space of functions which are differentiable for all degrees of differentiation defined on a $\mathbb{C}$-vector space V.
$\mathfrak{d}_K, \mathfrak{d}$	The different of the number field K.
dR	The principal ideal generated by the element d of the ring R.
$\dim_K V$	The dimension of the K-vector space V over the field K. If $K = \mathbb{C}$, we shortly write $\dim V$.
$e\{c\}$	The value $\exp\big(2\pi i \operatorname{tr}(c)\big)$, where c is an element of $\mathbb{C} \otimes_{\mathbb{Q}} K$ [1] (K a number field).
$\Gamma, \tilde{\Gamma}$	The group $\mathrm{SL}(2, \mathscr{O})$ and its metaplectic cover (see Sect. 3.3), respectively.
$\mathrm{GL}(V)$	The group of all automorphisms of a $\mathbb{C}$-vector space V.
$\mathbb{H}$	The upper half plane.
I	The element $(1, -1)$ in the metaplectic cover of $\mathrm{SL}(2, \mathscr{O})$.
$\mathrm{Hol}(V)$	The space of holomorphic functions of a $\mathbb{C}$-vector space V.
$\mathrm{Im}(\phi)$	The image of the map ϕ.
$\mathrm{Ker}(\phi)$	The kernel of the map ϕ.
μ	The Möbius μ-function, i.e. the multiplicative function μ on the semigroup of integral $\mathscr{O}_K$-ideals which for prime ideal power $\mathfrak{p}^n$ assumes the values $1, -1$ and 0 accordingly as $n = 0$ or $n = 1$ or $n \geq 2$.
$\precneqq_l$	The group of lth roots of unity.
$\precneqq_\infty$	The group of all roots of unity.
$\mathrm{N}_{K/\mathbb{Q}}(a)$	The norm of an element a in the number field K.

[1]For the definition of the trace of an element in $\mathbb{C} \otimes_{\mathbb{Q}} K$, we refer to Sect. 3.2.

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$\mathrm{N}(\mathfrak{a})$	The norm of the ideal $\mathfrak{a}$.
$\mathscr{O}_K$, $\mathscr{O}$	The ring of integers of the number field K.
q^t	The function on $\mathscr{H}$ [2] defined by $e\{t\tau\}$.
R^*	The invertible elements of the ring R under multiplication.
R^n	The R-module of column vectors of length n over the ring R.
S	The matrix $\left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$.
$\mathrm{SL}(n, R)$	The subgroup of elements of $\mathrm{GL}(n, R)$ which have determinant 1, where R is a ring.
$\mathbb{S}^1$	The group of all complex numbers whose absolute value equals one.
$\mathrm{Stab}(x)$	The stabilizer of x under a given group action.
$\sigma_0(\mathfrak{a})$	The number of ideals dividing the integral $\mathscr{O}_K$-ideal $\mathfrak{a}$.
T_b	The matrix $\left(\begin{smallmatrix} 1 & b \\ 0 & 1 \end{smallmatrix}\right)$, where b is an element of the ring R.
$\mathrm{tr}_{K/\mathbb{Q}}(a)$	The trace of an element a in the number field K.
$\sqrt{z}$	The root of $z \in \mathbb{C}^*$ whose argument lies in the interval $(\frac{-\pi}{2}\ \frac{\pi}{2}]$.

[2]Here $\mathscr{H}$ is a the upper half plane in $\mathbb{C} \otimes_{\mathbb{Q}} K$ (K a number field) as defined in Sect. 3.2.

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