Appendix
Method of Harmonic Linearization

General information A brief description of this method is restricted, only to the applied side of the issue. Let the restoring force and resistance be described by the nonlinear function $-P(q, \dot{q})$. Then the differential equation, describing the oscillations of the system, with one degree of freedom, in case of the action of the harmonic driving force, takes the form

$$a\ddot{q} + P(q, \dot{q}) = F_0 + F_1 \sin \omega t \tag{A.1}$$

or after dividing by the factor of inertia a

$$\ddot{q} + U(q, \dot{q}) = W_0 + W_1 \sin \omega t, \tag{A.2}$$

where $U = P/a$; $W_0 = F_0/a$; $W_1 = F_1/a$.

By analogy to the linear oscillatory system an approximate solution of Eq. (A.2) will be sought in the form

$$q^0 = A_0 + A \sin(\omega t - \gamma). \tag{A.3}$$

After substitution of (A.3) in $U(q, \dot{q})$, this function appears to be periodic, and therefore it can be represented as the Fourier series

$$U(q^0, \dot{q}^0) = U_0 + U_c \cos \varphi + U_s \sin \varphi + \text{higher harmonic}, \tag{A.4}$$

where $\varphi = \omega t - \gamma$.

We assume that the higher harmonics (A.4) have little influence on the formation of the approximate solutions (A.3), i.e. on A_0, A_1. This assumption is realized if the oscillations are close to harmonic. In this case the judgment, about the validity of this assumption, should be based on the physical background, for example, on the experiment, operating experience, etc.

Let us note here that in the problems of nonlinear mechanics, the performance of the approximate analytical methods largely depends on how correctly the form of the approximate solutions is "guessed".

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We will further introduce the formal linear differential equation

$$\ddot{q}^0 + 2n\dot{q}^0 + k^2 q^0 + S = W_0 + W_1 \sin \omega t, \tag{A.5}$$

where, $n,\ k^2$ are some unknown functions, and S is the unknown constant.

On the other hand, Eq. (A.2), taking into account (A.3), (A.4) and the above assumptions, can be written as

$$\ddot{q}^0 \ + \ U_c \cos \varphi + U_s \sin \varphi + U_0 = W_0 + W_1 \sin \omega t. \tag{A.6}$$

Comparing (A.6) and (A.5), we obtain

$$2n\dot{q}^0 + k^2 q^0 + S = U_c \cos \varphi + U_s \sin \varphi + U_0.$$

Then, taking into account (A.3), after the equalization of the coefficients at $\cos \varphi$, $\sin \varphi$ and free terms, we get $k^2 A_0 + S = U_0$; $2nA\omega = U_c$; $k^2 A = U_s$.

Using formulae for determining the Fourier coefficients, we finally obtain

$$\left.\begin{aligned} S(A, A_0) &= \frac{1}{2\pi} \int_0^{2\pi} U(q^0, \dot{q}^0) d\varphi - k^2(A, A_0) A_0; \\ 2n(A, A_0) &= \frac{1}{\pi A \omega} \int_0^{2\pi} U(q^0, \dot{q}^0) \cos \varphi \ d\varphi; \\ k^2(A, A_0) &= \frac{1}{\pi A} \int_0^{2\pi} U(q^0, \dot{q}^0) \sin \varphi \ d\varphi. \end{aligned}\right\} \tag{A.7}$$

These functions are called the *harmonic linearization coefficients.* So, using the accepted assumptions, the nonlinear differential equation (A.2), in case of using the accepted approximate solutions, is formally equivalent to the linear differential equation (A.5), the coefficients of which n, k^2, S, are the known functions of the unknown parameters of solution A, A_0.

For small values of n, function k corresponds to the frequency of free oscillations, which now, in contrast to the linear case, depends on the level of amplitude.

Coefficient $2n$, as in case of the linear oscillations, characterizes the dissipative properties of the system, which, as already noted, in the engineering problems, are usually evaluated by the dissipation coefficient ψ or by the logarithmic decrement ϑ. For the formal linear equation (A.5), the valid relationship is $n/k = \delta = \vartheta/(2\pi)$.

At the same time

$$2n(A, A_0) = \vartheta k(A, A_0)/\pi. \tag{A.8}$$

It can be shown that (A.8), when taking into account the approximate solutions (A.3), is energetically equivalent to the corresponding expressions in (A.7), however, quite often, it is more convenient, while solving engineering problems, since an analytical description of the dissipative forces, in many cases, is not possible or is very difficult. Meanwhile, the definition of parameters ϑ or ψ generally does not cause difficulties, even in those cases, where these parameters are dependent on the amplitude levels of A, A_0.

Strictly speaking, the real dissipative forces, arising in case of machine and mechanism oscillations, are always nonlinear, so their inclusion in previous chapters, devoted to the linear oscillations, essentially also meets the harmonic linearization of these forces. In this case, according to (A.8), when $k = \text{const}$ and $\lambda = \text{const}$, we have $n = \text{const}$, which allows us to use the linear differential equations to analyze system oscillations (see Chap. 6).

Let us mention the following important property of the linearization coefficients, which allows us to reduce the complex non-linear functions to the combination of more simple ones.

If the nonlinear function U can be represented as a sum $U = \sum_{i=1}^{s} U_i$ *then the coefficients of harmonic linearization are defined as the sum of the corresponding partial values*

$$n = \sum_{i=1}^{s} n_i;\ k^2 = \sum_{i=1}^{s} k_i^2;\ U = \sum_{i=1}^{s} U_{0i}.$$

No limiting requirements are required from the form of the function $U(q, \dot{q})$. In particular, these functions can be composed of individual segments and have discontinuities of the first kind. We will emphasize that harmonic linearization, in contrast to the usual linearization, when the nonlinear function is replaced by the linear one, does not require that q and $\dot{q}$ should be quite small. The only limitation here is the proximity to the harmonic oscillations.

Forced nonlinear oscillations For greater clarity, we will consider the widespread case $U_0 = 0$ and $A_0 = 0$ that means no constant component in the driving force $(F_0 = 0)$ and an odd function $P(q)$, described in Eq. (A.1) the nonlinear restoring force $(P(q) = -P(-q))$.

To solve Eq. (A.1), obtained by harmonic linearization method, we use the solution for the linear system with one degree of freedom (see Sect. 4.1.2)

$$A = \frac{W_1}{\sqrt{[k^2(A) - \omega^2]^2 + 4n^2(A)\omega^2}}; \tag{A.9}$$

$$\tan\gamma = \frac{2n(A)\omega}{k^2(A) - \omega^2}. \tag{A.10}$$

However, the performed linearization leads to the fact that the coefficients of harmonic linearization k^2 and n are the functions of unknown amplitude of forced oscillation A. Therefore if for the linear system, formula (A.9) is the final calculation expression, then now it appears to be the equation with respect to A.

To construct the frequency response $A(\omega)$ it is convenient to use the following method. Take square from both sides of the Eq. (A.9) and write it as follows:

$$A^2\{[k^2(A) - \omega^2]^2 + 4n^2(A)\omega^2\} = W_1^2. \tag{A.11}$$

Relative to ω, the Eq. (A.11) is the biquadratic one. So it allows us to determine ω as the appropriate values of this equation's roots. The resonance mode corresponds to the condition

$$k(A_*) = \omega, \tag{A.12}$$

where A_* is the amplitude of resonance.

Then according to (A.9) $2n(A_*)A_*\omega = W_1$, or with reference to (A.8), (A.12)

$$\vartheta A_* k^2(A_*)/\pi = W_1. \tag{A.13}$$

Thus, the resonant amplitude A_* can be determined as the root of the Eq. (A.13); herewith the phase shift γ according to (A.10) is equal to $\pi/2$.

We will consider the characteristics of nonlinear forced oscillations, using the example of the cubic characteristic of the restoring force $-c_0 q(1 + \xi q^2)$. With $\xi > 0$ the characteristic is termed as "hard", with $\xi < 0$ the one—as "soft", and when $\xi = 0$ it is linear. For given example Eq. (A.1) has the form of Duffing equation

$$a\ddot{q} + b_0\dot{q} + c_0 q(1 + \xi q^2) = F_1 \sin \omega t,$$

and after dividing by the coefficient a

$$\ddot{q} + 2n_0\dot{q} + k_0^2 q(1 + \xi q^2) = W_1 \sin \omega t, \tag{A.14}$$

where $2n_0 = b_0/a$; $k_0^2 = c_0/a$.

So with the account of the above mentioned, about the symmetry of nonlinearity and the oscillatory process

$$\left.\begin{array}{l} U(q^0, \dot{q}^0) = 2n_0 A\omega\cos\varphi + k_0^2 A\sin\varphi(1 + \xi A^2\sin^2\varphi); \\ q^0 = A\sin\varphi;\ \varphi = \omega t - \gamma. \end{array}\right\}.$$

According to (A.1)

$$2n(A) = (\pi A\omega)^{-1}\int_0^{2\pi}[2n_0 A\omega\cos\varphi + k_0^2(A\sin\varphi + \xi A^3\sin^3\varphi)]\cos\varphi d\varphi;$$

$$k^2(A) = (\pi A)^{-1}\int_0^{2\pi}[2n_0 A\omega\cos\varphi + k_0^2(A\sin\varphi + \xi A^3\sin^3\varphi)]\sin\varphi d\varphi.$$

After integration, we have $2n = 2n_0$ and

$$k^2(A) = (1 + 0.75\xi A^2)k_0^2. \tag{A.15}$$

Dependence $\omega = k_0\sqrt{1 + 0.75\xi A^2}$ obtained from (A.15), when $k = \omega$, defines the so-called skeleton curve shown in the graphs $A(\omega)$, with hatch-dotted line (Fig. A.1). To construct the graph of the amplitude-frequency characteristic, we substitute (A.15) into Eq. (A.11) and convert it into the biquadratic equation relative to ω:

$$\omega^4 - 2[k^2(A) - 2n^2(A)]\omega^2 + k^4(A) - W_1^2/A^2 = 0.$$

After solving the given equation we get

$$\omega = \sqrt{k^2(A) - 2n^2(A) \pm \sqrt{W_1^2/A^2 - 4n^2(A)[k^2(A) - n^2(A)]}}. \tag{A.16}$$

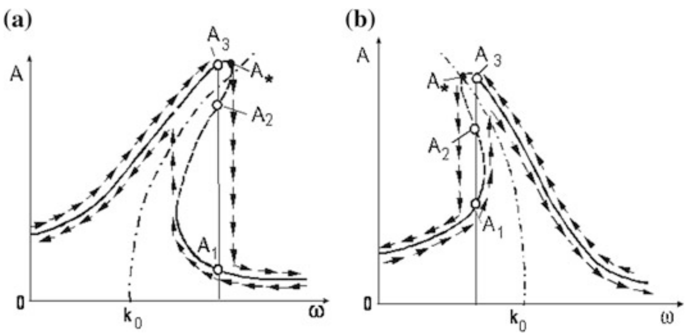

Fig. A.1 Amplitude-frequency characteristic (AFC)

When $\xi > 0$ ("hard" characteristic), the resonance peak inclines to the right (Fig. A.1a) and when $\xi < 0$ ("soft" characteristic), to the left (Fig. A.1b).

Because of the inclination of AFC *several values of the amplitudes of forced vibrations (A_1, A_2, A_3) correspond to one frequency in the certain fixed frequency range* limited by the inflection points. In addition to that, the zone of increased amplitudes now covers a much larger frequency range, as compared to the frequency response of the linear system. The resonant amplitude A_* is determined from the Eq. (A.13) in the reference to (A.15):

$$A_*(1 + 0.75\xi A_*^2)\vartheta k_0^2 - \pi W_1 = 0. \tag{A.17}$$

Here ϑ is logarithmic decrement.

The change of amplitudes, caused by the smooth increase or decrease in frequency ω, is represented on AFC, using arrows. In the vicinity of inflection points the hopping from one branch of the characteristic to another occurs, which is accompanied by an abrupt change in the amplitude of forced oscillations. Thus, the implementation of a mode may depend on the conditions of entrance into this mode.

In general, when $U_0 \neq 0$, we have $A_0 \neq 0$, and the values A and A_0 are determined by the system of equations obtained on the basis of (A.7). Then

$$k^2 = k_0^2[1 + \xi(3A_0^2 + 0.75A^2)].$$

Stability of the modes of forced oscillations We will use the energy method, for the determination of the stability of the identified periodic modes. In case of forced oscillations the introduced energy is defined as $\Delta E_+ = \pi A F_1 \sin\gamma$ and the withdrawn one as $\Delta E_- = 2\pi n\omega A^2 a$.

The indicative graph $\Delta E = \Delta E_+ - \Delta E_-$ for the fixed value of ω, to which the three modes with amplitudes A_1, A_2, A_3 correspond, is represented in Fig. A.2.

Obviously when $\Delta E > 0$ the amplitude increases, otherwise, it decreases, when $\Delta E < 0$. When $\Delta E = 0$, the regime is stationary. However for practical implementation of this mode, it is necessary for it to be stable. In other words, in case of deviations $\Delta A_i = A - A_i\,(i = 1, 2, 3)$, the condition $\Delta A_i \to 0;\ A \to A_i$ is satisfied. Using these simple considerations, we can easily verify that amplitudes A_1 and A_3 correspond to the stable regimes and amplitude A_2 to the unstable regime. The stability condition can be written as follows:

$$\partial(\Delta E)/\partial A < 0 \quad (A = A_i) \tag{A.18}$$

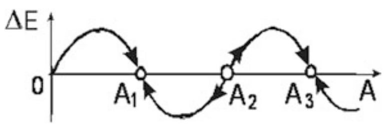

Fig. A.2 To the definition of the conditions of stability of the forced oscillations

It turns out that the stable regimes in the zone, before resonance, have a positive AFC slope, and in the zone after resonance they have the negative slope of the response curve. The branches of the response (AFC), shown in the Fig. A.1, with hatched lines, correspond to the unstable regimes. Unstable branch of the frequency response is limited by the inflection points, at which the tangent to the graph $A(\omega)$ is vertical. Despite the fact that the unstable modes can not practically be implemented, the corresponding, to these regimes, branch of the response is of particular interest.

Suppose, for example, the system oscillated in the steady state regime with amplitude A_1, and then as a result of an accidental disturbance (e.g. impact) it experiences amplitude increment $\Delta A > 0$. Obviously if $A_1 + \Delta A < A_2$, then $\Delta E < 0$ and therefore, we return to the mode $A = A_1$. However, if $A_1 + \Delta A > A_2$, then $\Delta E > 0$ and further system oscillations occur with amplitude A_3.

The dependence of the regime of the implemented forced oscillations, from the initial conditions, is detected similarly.

If the initial amplitude A_0 is smaller than A_2, then $A \rightarrow A_1$ and when $A_0 > A_2$, we have $A \rightarrow A_3$. Thus, the unstable regime in the first approximation serves as a "dividing line" between the two stable states.

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