

# Collection of Essential Equations

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Small load approximation (SLA)	$\frac{\tilde{f}}{f_0} = \frac{i}{\pi Z_q} \tilde{Z}_L = \frac{i}{\pi Z_q} \frac{\langle \tilde{\sigma}_S \rangle_{area}}{V_S}$
Gordon-Kanazawa-Mason result	$\frac{\tilde{f}}{f_0} = \frac{-1+i}{\sqrt{2\pi Z_q}} \sqrt{\tilde{\omega}} \sqrt{\rho_{liq} \eta_{liq}}$
Viscoelastic medium	$\tilde{G}(\omega) \rho = - \left( \pi Z_q \frac{\tilde{f}}{f_0} \right)^2$
Thin rigid film (Sauerbrey Equation)	$\frac{\tilde{f}}{f_0} \approx - \frac{2n f_0}{Z_q} m_f = -n \frac{\langle m_f \rangle_{area}}{m_q}$
Viscoelastic film in air	$\frac{\tilde{f}}{f_0} = \frac{-1}{\pi Z_q} \tilde{Z}_f \tan(\tilde{k}_f d_f)$
Thin viscoelastic film in air	$\frac{\tilde{f}}{f_0} = \frac{-\omega m_f}{\pi Z_q} \left[ 1 + \frac{(n\pi)^2}{3} \left( \frac{\tilde{J}_f(\omega)}{\rho_f} Z_q^2 - 1 \right) \left( \frac{m_f}{m_q} \right)^2 \right]$
Viscoelastic film in liquid	$\frac{\tilde{f}}{f_0} = \frac{-\tilde{Z}_f \tilde{Z}_f \tan(\tilde{k}_f d_f) - i \tilde{Z}_{liq}}{\pi Z_q \tilde{Z}_f + i \tilde{Z}_{liq} \tan(\tilde{k}_f d_f)}$
Thin film in liquid	$\frac{\tilde{f}}{f_0} \approx \frac{-\omega m_f}{\pi Z_q} \left[ 1 - 2\pi i n \frac{\tilde{J}_f(\omega)}{\rho_f} f_0 \rho_{liq} \eta_{liq} \right]$
Thin adsorbate in liquid	$\frac{\tilde{f}}{f_0} \approx - \frac{\rho \omega}{\pi Z_q} \int_0^\infty \left[ \frac{\tilde{G}(z) - \tilde{G}_{liq}}{\tilde{G}(z)} \right] dz$
Contact stiffness	$\Delta f^c = \frac{1}{n} \frac{N_p}{2\pi^2 Z_q} \kappa_P$
Time averaging	$\frac{\tilde{f}}{f_0} = \frac{i}{\pi Z_q} \frac{2 \langle \sigma_S(t) \exp(i\omega t) \rangle_{area,time}}{V_S}$
SLA in tensor form	$\tilde{\Delta f} \approx \frac{i}{4\pi \rho_q} \frac{\sum_{ijk} \int_{Surface} \dot{u}_{0,S} \eta_j \tilde{Z}_{L,ijk}(\mathbf{r}_S) \dot{u}_{0,S,k} d^2 \mathbf{r}_S}{\sum_k \int_{Volume} \dot{u}_{0,k} \dot{u}_{0,k} d^3 \mathbf{r}}$
Piezoelectric stiffening	$\frac{\tilde{\Delta f}_{PE}}{f_0} = \frac{i}{A\pi Z_q} 4\phi^2 \left( (i\omega C_0 + \tilde{Z}_{ex}^{-1})^{-1} - (i\omega C_0 + \tilde{Z}_{ex,ref}^{-1})^{-1} \right)$

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