

Bibliography

- [B] Bari, N., *A treatise on trigonometric series*, Pergamon Press, New York, 1964.
- [BA] Ben-Artzi, A., *Traces of compact operators*, Integr. Equat. Oper. Th., **7** (1984), 310–324.
- [Bo] Bobr, St., *Eine Verallgemeinerung des von Koch'schen Satzes über die absolute Konvergenz der unendlichen Determinanten*, Math. Z. **10** (1921), 1–11.
- [Ca1] Carleman, T., *Über die Fourierkoeffizienten einer steigen Function*, Acta Mathematica, **41** (1918), 377–384.
- [Ca2] Carleman, T., *Zur Theorie der Linearen Integralgleichungen* Math. Zeit., **9** (1921), 196–217.
- [C1] Cohen, L., W., *A note on a system of equations with infinitely many unknowns*, Bull. A.M.S. **36** (2), (1930), 563–572.
- [C2] Cohen, L., W., *Transformations on spaces of infinitely many dimensions*, Annals of Math. **37** (2), (1936), 326–335.
- [D] Denk, R., *Infinite determinants of periodic ODE-systems*, Preprint, to appear
- [DS] Dunford, N., Schwartz, J., *Linear Operators*, Vol. I, II. Spectral Theory, Interscience, New York, 1963.
- [E] Enflo, P., *A counterexample to the approximation problem in Banach spaces*, Acta Math., **130**, (1973), 309–317.
- [FS] Fenyö, S., Stolle, H., *Theorie und Praxis der Linearen Integralgleichungen*, Vol. I–IV, Birkhäuser, Basel, 1982–1984.
- [Fr] Fredholm, I., *Sur une Classe d'Equation Fonctionnelles*, Acta Math. **27** (1903), 365–390.
- [GB] Greenberg, L., Babuska, I., *A continuous analogue of Sturm sequences in the context of Sturm-Liouville equation*, SIAM J. Numer. Anal., **26**, N 4 (1989), 920–945.
- [GG] Gohberg, I., Goldberg, S., *Basic operator theory*, Birkhäuser, Boston, 1981.
- [GG1] Gohberg, I., Goldberg, S., *Counting negative eigenvalues of a Hilbert-Schmidt operator via sign changes of a determinant*, Integr. Equat. Oper. Th., **14** (1991), 92–104.

- [GGKa] Gohberg, I., Goldberg, S., Kaashoek, M., A., *Classes of Linear Operators*, Vol.I, Birkhäuser, Basel, 1990.
- [GGK1] Gohberg, I., Goldberg, S., Krupnik, N., *Traces and Determinants of Linear Operators*, Integr. Equat. Oper. Th, **26** (1996), 136–187.
- [GGK2] Gohberg, I., Goldberg, S., Krupnik, N., *Hilbert-Carleman and Regularized Determinants for Linear Operators*, Integr. Equat. Oper. Th., **27** (1997), 10–47.
- [GKa] Gohberg, I., Kaashoek, M., A., *Asymptotic formulas of Szegő-Kac-Achiezer type*, Asymptotic Analysis **5** (1992), North-Holland, Amsterdam, 187–220.
- [GK1] Gohberg, I., Krein, M., *Introduction to the Theory of Linear Non-Self Adjoint Operators*, Translations of the Amer. Math. Soc., **18** (1969).
- [GK2] Gohberg, I., Krein, M., *New inequalities for the eigenvalues of integral equations with smooth kernels*, Math. Issled., **5**, N1(15), (1970) 22–39 (Russian).
- [GKr] Gohberg, I., Krupnik, N., *Banach Algebras generated by Singular Integral Operators*, Colloquia Math. Soc. Janos Bolyai **5**, Hilbert space operators, Tihany (Hungary), (1970), 237–267 (Russian).
- [GS] Gohberg, I., Sigal, E., *An operator generalization of the logarithmic residue theorem and Roushé's theorem*, Math. USSR, Sb., **13** (1971) 603–625.
- [Gu] Goursat, M., *Sur un cas élémentaire de l'équation de Fredholm*, Bulletin de la Société mathématique, **XXXV**, fask. III, (1907), 163–173.
- [Gr1] Grothendieck, A., *La Theory de Fredholm*, Bull. Soc. Math. France **84**, (1956), 319–384.
- [Gr2] Grothendieck, A., *Produits Tensoriels Topologiques et Espaces Nucléaires*, Memoirs of the Amer. Math. Soc., Number **16**, (1966).
- [H] Hilbert, D., *Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen I*, Nach. Wiss. Math. Phys. Gott. (1904), 49–91.
- [Hi] Hill, G., *On the part of the motion of lunar perigee which is a function of the mean motions of the sun and the moon*, Acta Math. **8**, (1886), 1–3.
- [HT] Hille, E., Tamarkin, J., *On the theory of linear integral equations*, Ann. of Math., **35** (1934), 445–455.
- [Ho] Hoffman, K., *Banach spaces of analytic functions*, Prentice Hall, Englewood Cliffs, N.J., 1962.
- [It] Itskovich, I., *On Fredholm series*, Doklady AN SSSR, **LIX**, N3, (1948), 423–426.
- [KaV] Kaashoek, M., A., and M.P.A. van de Ven, *A linearization for operator polynomials with coefficients in certain operator ideals*, Ann. Math. Pura Appl., **IV**, CXXV, (1980), 329–336.
- [K1] Von Koch, H., *Sur la Convergence des Déterminants Infinis* Rend. Circ. Mat. Palermo **28** (1909), 255–266.
- [K2] Von Koch, H., *Sur les Déterminants Infinis et les Équations Différentielles Linéaires*, Acta Math. **16**, (1892), 271–295.

- [Kö1] König, H., *s-numbers, eigenvalues and trace theorem in Banach spaces*, *Studia Math.*, **LXVII**, (1980), 157–172.
- [Kö2] König, H., *A Fredholm Determinant Theory for p -Summing Maps in Banach Spaces*, *Math. Ann.* **247**, (1980), 255–274.
- [Kö3] König, H., *A trace Theorem and Linearization Method for Operator Polynomials*, *Integr. Equat. Oper. Th.*, **5**, (1982), 828–849.
- [Kö4] König, H., *Eigenvalue distribution of compact operators*, Birkhäuser, Basel, 1986.
- [KZPS] Krasnoselskii, M., Zabreiko, P., Pustyl'nik, E., Sobolevskii, P., *Integral operators in spaces of summable functions*, Noordhoff, Leyden, 1976, 13 (1953), 244–276.
- [L] Levin, B., *Distribution of Zeros of Entire Functions* AMS, Providence, Rhode Island, 1980.
- [Le] Lezanski, T., *The Fredholm Theory of Linear Equations in Banach Spaces*, *Studia Math.*, **13** (1953), 244–276.
- [LS] Liusternik, L., Sobolev, J., *Elements of functional analysis*, 3rd English translation, Hindustan Publ., Delhi (John Wiley New York), 1974.
- [LT] Lindenstrauss, J., Tzafriri, L., *Classical Banach spaces*, Vol.I,II, Springer-Verlag, New York, 1977/79.
- [M] Markus, A., *Introduction to the spectral theory of polynomial operator pencils*, Amer. Math. Soc., Providence, Rhode Island, 1988.
- [MM] Markus, A., Matsaev, V., *Analogs of Weyl Inequalities and the trace theorem in Banach space*, *Math. USSR Sbornik* **15**, N2, (1971), 299–312.
- [Mi] Mikhlin, S., *On the convergence of Fredholm series*, *Doklady AN SSSR*, **XLII**, N9, (1944), 374–377.
- [MW] Mennicken, R., Wagenführer, E., *Über die Konvergenz verallgemeinerter Hillscher Determinanten*. *Math. Nachr.* **72** (1976), 21–49.
- [MWi] Magnus, W., Winkler S., *Hill's Equations*, Wiley, New York, 1996.
- [Pe] Perelson, A., *Generalized Traces and Determinants for Compact Operators*, Thesis submitted for the degree “Doctor of Philosophy”, Tel-Aviv University, 1987.
- [Pi1] Pietsch, A., *Eigenvalues and s-numbers*, Cambridge Studies in Advanced Math. Vol.13, Cambridge Univ. Press, Cambridge, UK, 1987.
- [Pi2] Pietsch, A., *s-numbers of operators in Banach spaces*, *Studia Math.* **51** (1974), 201–223.
- [Ple] Plemelj, J., *Zur Theorie der Fredholmschen Functionalgleichung*, *Monat. für Math. und Phys.*, **15** (1907), 93–128.
- [P1] Poincaré, H., *Sur les déterminants d'ordre infini* *Bull. Soc. Math. France* **14** (1886), 77–90.
- [P2] Poincaré, H., *Remarques diverses sur l'équation de Fredholm*, *Acta Math.* **33** (1910), 57–86.

- [R] Riesz, F., *Les Systèmes d'équations Linéaires à une infinité d'inconnues*, Collection de monographies sur la théorie des fonctions, Gauthier-Villars, Paris, 1913.
- [Ru1] Ruston, A.F., On the Fredholm theory of integral equations for operators belonging to the trace class of a general Banach space, Proc. London Math. Soc. (2) **53**, (1951), 109–124.
- [Ru2] Ruston, A.F., *Fredholm theory in Banach spaces*, Cambridge University Press, Cambridge, UK, 1986, 141–198.
- [Si] Sikorski, R., *The Determinant Theory in Banach spaces*, Colloq. Math. **8**, (1961), 141–198.
- [S1] Simon, B., *Trace ideals and their applications*, Lecture Note Series **35**, London, 1979.
- [S2] Simon, B., *Notes on Infinite Determinants of Hilbert Space Operators*, Adv. Math. **24** (1977), 244–273.
- [SS] Seiler, E., Simon, B., *An Inequality for Determinants*, Proc. Nat. Acad. Sci., **72** (1975), 3277–3278.
- [Sm] Smithies, F., *Integral Equations*, Cambridge University Press, Cambridge, UK, 1965.
- [WW] Whittaker, E., Watson, G., *A Course of Modern Analysis*, Cambridge University Press, Cambridge, UK, 1927.
- [Z] Zygmund, A., *Trigonometric Series, Vol.II*, Cambridge University Press, Cambridge, UK, (1959).

Index

- algebraic multiplicity, 47
- algebras without the approximation property, 243
- analyticity of the determinant, 29, 166, 192
- approximation numbers, 51
- approximation property;
 - of an algebra, 26
 - of a Banach space, 91
- asymptotic behavior of eigenvalues;
 - of nuclear operators, 106
 - of “ r -nuclear” operators, 108
- biorthogonal system, 3
- boundary value matrices, 217, 219
- canonical boundary value matrix, 219
- cascade connection of systems, 219, 220
- characteristic number;
 - of a pencil, 17, 83
- multiplicity of, 17, 83
- characteristic property;
 - of the determinant, 11
 - of the trace, 10
- classes \mathcal{S}_p , 87
- continuous chain of projections, 236
- determinant;
 - diagonally modified (d -modified), 121
 - extension of, 26
 - Fredholm form, 14
 - Hilbert-Carleman, 160, 169, 180
 - m -regularized, 187
 - of $I +$ finite rank operator, 6, 9
 - of infinite matrices, 35
 - of operator $I + A$ in algebra, 28
 - possible values of, 137–139
 - regularized, 68, 160
 - von-Koch form of, 12, 35
- $D(l_1)$ algebra, 45
- embedded subalgebra, 26
- examples of nuclear operators;
 - in l_1 , 97
 - in $C([a,b])$, 99
 - in $L_1(T, \Sigma, \mu)$, 100, 126
- extension of the;
 - determinant, 26
- m -regularized, 187, 188
- regularized, 162, 163
 - trace, 26
- explicit formula for resolvent, 145, 147
- finite rank operator, 3
 - determinant of $I + F$, 6
 - trace of, 6
- Floquet exponent, 117, 209
- Fredholm determinant, 111, 159, 227
- Fredholm form of a determinant, 14
 - diagonally modified (d -modified), 121
- fundamental matrix, 214, 217
- generalized eigenvectors, 47
- Green’s function, 241
- Grothendieck trace theorem, 102
- Hadamard’s inequality, 72

- Hermitian non-negative operator,
 74, 75
 Hilbert-Carleman determinant, 160,
 169
 diagonally modified (d-modified)
 of, 180, 182
 for integral operators, 162, 172
 for matrices, 183
 Hilbert-Schmidt operator, 66, 176
 Hilbert-Schmidt norm, 67
 Hill's differential equations, 40, 117,
 209
 Hill's method, 40, 41
 revisited, 115, 117
 for systems, 209
 homogeneous integral equations, 152
 general solution to, 153, 154

 ideal of trace class operators, 59
 indicator, 224
 input-output operator, 216
 representation of, 216
 input-output of a system, 216
 inverse systems, 222
 inversion formulas, 20, 143, 201
 integral equations 127, 205
 system of, 127
 homogeneous, 152
 integral operators;
 from algebra D_Ω , 130
 with a jump discontinuity on the
 diagonal, 246
 continuous kernel, 111
 degenerate kernel, 14
 Hermitian non-negative kernel, 74
 Hilbert-Schmidt kernel, 70, 77
 piecewise continuous kernel, 114
 semi-separable kernel, 213, 247
 smooth kernel, 78

 Jordan chain, 47
 jump discontinuity, 244, 246

 Lidskii trace theorem, 63

 linearization of a pencil, 18, 83
 Lipschitz conditions for
 determinant, 31, 69, 95, 113, 116,
 194

 Mennicken-Wagenführer algebra, 43,
 184
 Mercer's expansion, 79
 Mikhlín-Itskovich algebra, 177–179,
 208

 minors (Fredholm) of order n , 153
 nuclear operator, 59, 92
 “nuclear 2/3” operator, 103

 Perelson algebra, 133
 piecewise continuous kernels, 114
 Plemelj-Smithies formulas, 15, 17,
 29, 166, 192
 modification of, 124
 Poincaré determinant, 39
 polynomial operator pencils, 17, 66,
 195
 linearization of, 18, 83

 regular integral operator, 172, 205
 regularized determinant, 68, 160
 extension of, 162, 163
 of order m , 187
 resolvent of integral operators, 145
 Riesz projection, 47
 representation of the;
 determinant $\det(I + F)$, 6, 9
 trace $\text{tr } F$, 6, 8
 Ruston-Grothendieck algebra, 91

 Schmidt representation of a compact
 operator, 49
 Schur's lemma, 48
 semi-separable kernel, 213
 sign changes of a function;
 simple, 236
 double, 242
 singular numbers (s -numbers), 49

- Sturm-Liouville differential
 - operator, 241
- systems;
 - time invariant, 232
 - inverse, 222
 - cascade connections of, 220
- test of nuclearity for integral
 - operators, 70, 75
- time invariant systems, 232, 233
- trace;
 - extension of, 26, 28
 - of finite rank operators, 6, 7
 - of linear operator in algebra, 28
 - possible values of, 137–139
- trace class operators, 59, 92
- trace class norm, 59, 92
- trace oriented subalgebra, 137

- von Koch form of a determinant, 12, 42
- von Koch-Riesz algebra, 42

- well-posed boundary conditions, 219

- zeros of determinant, 34

List of Symbols

- $|A|$, 176
 $A^\times(t)$, 215
 $\tilde{\mathcal{B}}$, 129
 $C_m(A)$, 167
 $\det(I + F)$, 6
 $\det_D(I + A)$, 28
 $\det_\Phi(I + A)$, 112
 $\det_2(I + A)$, 68
 $\det_2(I + A)$, 161
 $\det_2(I + A)$, 162
 $\det_{2,D}(I + A)$, 166
 $\text{Det}(I + A)$, 160
 $\text{Det}(I + A)$, 169
 $\det_m(I + F)$, 187
 $\text{Det}_h(I + A)$, 180
 $\det_{D,\phi}(I + A)$, 243
 $\det^\alpha(I + K)$, 246
 $D(n)$, 195
 $D(F, \lambda)$, 21
 $D_\Omega^r(\mathcal{B})$, 35
 $D_\Omega(H)$, 115
 $D(M, W, l_p)$, 43
 $D(\lambda)$, 152
 $D^{N \times N}$, 127
 $D(l_1)$, 45
 $D \left(\begin{array}{c|c} x_1 & x_2 \cdots x_n \\ y_1 & y_2 \cdots y_n \end{array} \middle| \lambda \right)$, 153
 $\Delta_n(F)$, 12
 $\delta(G)$, 78
 E_A , 48
 \mathcal{E}_1 , 179
 \mathcal{E}_p , 177
 F' , 6
 $\mathcal{F}(\mathcal{B})$, 5
 \mathcal{F} , 26
 \mathcal{F}_D , 26
 $F|M$, 7
 $\Phi_m(A)$, 112
 $\Phi_n(A, g)$, 122
 K^+ , 224
 $k \left(\begin{array}{c} t_1, t_2, \dots, t_n \\ t_1, t_2, \dots, t_n \end{array} \right)$, 152
 $k_g \left(\begin{array}{c} t, t_1, t_2, \dots, t_n \\ s, t_1, t_2, \dots, t_n \end{array} \right)$, 146
 $k^*(t, s)$, 177
 $\Psi_n(F)$, 170
 $\Psi_n(A; h)$, 181
 $L(\mathcal{B}) = \mathcal{L}(\mathcal{B})$, 5
 $L(\mathcal{B}) = \mathcal{L}(\mathcal{B})$, 25
 $L(\lambda)$, 17
 $L(\lambda)$, 83
 $L_{1,\infty}(T \times T)$, 100
 $L_{1,\infty}(T \times T)$, 126
 $\mathcal{L}_{1,\infty}(T, \Sigma, \mu)$, 100
 $Lip\alpha$, 71
 \tilde{l}_2 , 118
 $L_p(T, \Sigma, \mu)$, 14
 N_1 , 215
 N_2 , 215
 $\|\cdot\|_1$, 59
 $\|\cdot\|_2$, 67
 $\|\cdot\|_p$, 87
 $\|\cdot\|_{\mathcal{E}_p}$, 177
 $\|\cdot\|_{p,q}$, 177
 $\nu(A)$, 49
 P , 214
 P^\times , 223
 $P_{\{\lambda_0\}}$, 47
 $P(\{X(r)\}, \{Y(r)\})$, 133
 $PC([a, d])$, 114

$[p]$, 194
 R_A , 68
 $R(t, s, \lambda)$, 151
 S_m , 15
 S_1 , 59
 S_2 , 67
 S_2 , 176
 S_p , 87
 $S_h(f)$, 75
 $s_j(A)$, 49
 $\sigma(A)$, 47
 $\sigma_h f$, 134
 $\text{tr } F$, 6
 $\text{tr}_D(A)$, 28
 $\Theta = (A(t), B(t), C(t); N_1, N_2)$, 217
 $\Theta = (A(t), B(t), C(t), I; N_1, N_2)$, 220
 $\Theta_1 \Theta_2$, 220
 Θ^\times , 222
 $U(t)$, 214
 $U^\times(t)$, 215
 V_Θ , 225
 W_2^1 , 209
 $\varphi \otimes g$, 6