

# Bibliography

- [ADN59] S. Agmon, A. Douglis, and L. Nirenberg. Estimates near the boundary for solutions of elliptic partial differential equations satisfying general boundary conditions. I. *Comm. Pure Appl. Math.*, 12:623–727, 1959.
- [Ahl38] L. V. Ahlfors. An extension of Schwarz’s lemma. *Trans. Amer. Math. Soc.*, 43:359–364, 1938.
- [AM85] P. Aviles and R. C. McOwen. Conformal deformations of complete manifolds with negative curvature. *J. Differential Geom.*, 21(2):269–281, 1985.
- [AM88] P. Aviles and R. C. McOwen. Conformal deformation to constant negative scalar curvature on noncompact Riemannian manifolds. *J. Differential Geom.*, 27(2):225–239, 1988.
- [Ama76] H. Amann. Supersolutions, monotone iterations, and stability. *J. Differential Equations*, 21(2):363–377, 1976.
- [Aub76] T. Aubin. Équations différentielles non linéaires et problème de Yamabe concernant la courbure scalaire. *J. Math. Pures Appl. (9)*, 55(3):269–296, 1976.
- [Aub98] T. Aubin. *Some nonlinear problems in Riemannian geometry*. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 1998.
- [BE87] J.P. Bourguignon and J.P. Ezin. Scalar curvature functions in a conformal class of metrics and conformal transformations. *Trans. Amer. Math. Soc.*, 301:723–736, 1987.
- [Bes08] A. L. Besse. *Einstein manifolds*. Classics in Mathematics. Springer-Verlag, Berlin, 2008. Reprint of the 1987 edition.
- [Bis77] R.L. Bishop. Decomposition of cut loci. *Proc. Amer. Math. Soc.*, 65:133–137, 1977.
- [BMR09] B. Bianchini, L. Mari, and M. Rigoli. Spectral radius, index estimates for Schrödinger operators and geometric applications. *J. Funct. Anal.*, 256:1769–1820, 2009.

- [BRS98] L. Brandolini, M. Rigoli, and A. G. Setti. Positive solutions of Yamabe type equations on complete manifolds and applications. *J. Funct. Anal.*, 160:176–222, 1998.
- [BVV91] M.-F. Bidaut-Véron and L. Véron. Nonlinear elliptic equations on compact Riemannian manifolds and asymptotics of Emden equations. *Invent. Math.*, 106:489–539, 1991.
- [Cal57] E. Calabi. An extension of Hopf’s maximum principle with an application to Riemannian geometry. *Duke Math. J.*, 25:45–56, 1957.
- [Car88] É. Cartan. *Leçons sur la géométrie des espaces de Riemann*. Les Grands Classiques Gauthier-Villars. [Gauthier-Villars Great Classics]. Éditions Jacques Gabay, Sceaux, 1988. Reprint of the second (1946) edition.
- [CGT82] J. Cheeger, M. Gromov, and M. Taylor. Finite propagation speed, kernel estimates for functions of the Laplace operator, and the geometry of complete Riemannian manifolds. *J. Differential Geom.*, 17(1):15–53, 1982.
- [Cha84] I. Chavel. *Eigenvalues in Riemannian geometry*, volume 115 of *Pure and Applied Mathematics*. Academic Press Inc., Orlando, FL, 1984.
- [Cha06] I. Chavel. *Riemannian geometry*, volume 98 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, second edition, 2006. A modern introduction.
- [Che55] S.S. Chern. An elementary proof of the existence of isothermal parameters on a surface. *Proc. Amer. Math. Soc.*, 6:771–782, 1955.
- [CL55] E. A. Coddington and N. Levinson. *Theory of ordinary differential equations*. McGraw-Hill, New York, 1955.
- [CL87] K. S. Cheng and J.-T. Lin. On the elliptic equations  $\Delta u = K(x)u^\sigma$  and  $\Delta u = K(x)e^{2u}$ . *Trans. Amer. Math. Soc.*, 304(2):639–668, 1987.
- [CL95] W. X. Chen and C. Li. A note on the Kazdan-Warner type conditions. *J. Differential Geom.*, 41(2):259–268, 1995.
- [CN92] K.-S. Cheng and W.-M. Ni. On the structure of the conformal scalar curvature equation on  $\mathbf{R}^n$ . *Indiana Univ. Math. J.*, 41(1):261–278, 1992.
- [Dav95] E. B. Davies. *Spectral theory and differential operators*, volume 42 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1995.
- [dC92] M. P. do Carmo. *Riemannian geometry*. Mathematics: Theory & Applications. Birkhäuser Boston Inc., Boston, MA, 1992. Translated from the second Portuguese edition by Francis Flaherty.

- [Eis49] L. P. Eisenhart. *Riemannian Geometry*. Princeton University Press, Princeton, N. J., 1949. 2d printing.
- [EL78] J. Eells and L. Lemaire. A report on harmonic maps. *Bull. London Math. Soc.*, 10:1–68, 1978.
- [Esc87] J. Escobar. Positive solutions for some semilinear elliptic equations with critical Sobolev exponents. *Comm. Pure. Appl. Math.*, 40:623–657, 1987.
- [Esc90] J. Escobar. Uniqueness theorems on conformal deformations of metrics, Sobolev inequalities, and an eigenvalue estimate. *Comm. Pure. Appl. Math.*, 43:857–883, 1990.
- [FC85] D. Fischer-Colbrie. On complete minimal surfaces with finite Morse index in three-manifolds. *Invent. Math.*, 82(1):121–132, 1985.
- [FCS80] D. Fischer-Colbrie and R. Schoen. The structure of complete stable minimal surfaces in 3-manifolds of non-negative scalar curvature. *Comm. Pure. Appl. Math.*, 33:199–211, 1980.
- [Fed69] H. Federer. *Geometric measure theory*. Die Grundlehren der mathematischen Wissenschaften, Band 153. Springer-Verlag New York Inc., New York, 1969.
- [For91] O. Forster. *Lectures on Riemann surfaces*, volume 81 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1991. Translated from the 1977 German original by Bruce Gilligan, Reprint of the 1981 English translation.
- [GHV72] W. Greub, S. Halperin, and R. Vanstone. *Connections, curvature, and cohomology. Vol. I: De Rham cohomology of manifolds and vector bundles*. Academic Press, New York, 1972. Pure and Applied Mathematics, Vol. 47.
- [Gri76] P. A. Griffiths. *Entire holomorphic mappings in one and several complex variables*. Princeton University Press, Princeton, N. J., 1976. The fifth set of Hermann Weyl Lectures, given at the Institute for Advanced Study, Princeton, N. J., October and November 1974, *Annals of Mathematics Studies*, No. 85.
- [Gri99] A. Grigor'yan. Analytic and geometric background of recurrence and non-explosion of the Brownian motion on Riemannian manifolds. *Bull. Amer. Math. Soc. (N.S.)*, 36(2):135–249, 1999.
- [GT01] D. Gilbarg and N. Trudinger. *Elliptic partial differential equations of second order*. Classics in Mathematics. Springer-Verlag, Berlin, 2001. Reprint of the 1998 edition.

- [GW79] R. E. Greene and H. Wu. *Function theory on manifolds which possess a pole*, volume 699 of *Lecture Notes in Mathematics*. Springer, Berlin, 1979.
- [HW53] P. Hartman and A. Wintner. On the existence of Riemannian manifolds which cannot carry non-constant analytic or harmonic functions in the small. *Amer. J. Math.*, 75:260–276, 1953.
- [Ili96] S. Ilias. Inégalités de Sobolev et résultats d’isolement pour les applications harmoniques. *J. Funct. Anal.*, 139(1):182–195, 1996.
- [Jin88] Z. R. Jin. A counterexample to the Yamabe problem for complete non-compact manifolds. In *Partial differential equations (Tianjin, 1986)*, volume 1306 of *Lecture Notes in Math.*, pages 93–101. Springer, Berlin, 1988.
- [Kor14] A. Korn. Zwei Anwendungen der Methode der sukzessiven Annäherungen. *Schwarz-Festschr.*, pages 215–229, 1914.
- [Kui49] N. Kuiper. On conformally flat spaces in the large. *Ann. of Math.*, 50:916–924, 1949.
- [KW74a] J. L. Kazdan and F. W. Warner. Curvature functions for compact 2-manifolds. *Ann. of Math. (2)*, 99:14–47, 1974.
- [KW74b] J. L. Kazdan and F. W. Warner. Curvature functions for open 2-manifolds. *Ann. of Math. (2)*, 99:203–219, 1974.
- [KW75a] J. L. Kazdan and F. W. Warner. Existence and conformal deformation of metrics with prescribed Gaussian and scalar curvatures. *Ann. of Math. (2)*, 101:317–331, 1975.
- [KW75b] J. L. Kazdan and F. W. Warner. Scalar curvature and conformal deformation of Riemannian structure. *J. Differential Geometry*, 10:113–134, 1975.
- [Le98] V. K. Le. On some equivalent properties of sub- and supersolutions in quasilinear elliptic equations. *Hiroshima Math. J.*, 28:373–380, 1998.
- [Lee97] J. M. Lee. *Riemannian manifolds*, volume 176 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1997. An introduction to curvature.
- [Lee03] J. M. Lee. *Introduction to smooth manifolds*, volume 218 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2003.
- [Lee11] J. M. Lee. *Introduction to topological manifolds*, volume 202 of *Graduate Texts in Mathematics*. Springer, New York, second edition, 2011.
- [Li90] P. Li. On the structure of complete Kähler manifolds with nonnegative curvature near infinity. *Invent. Math.*, 99(3):579–600, 1990.

- [Lic16] L. Lichtenstein. Zur Theorie der konformen Abbildung nichtanalytischer, singularitätenfreier Flächenstücke auf ebene Gebiete. *Krak. Anz.*, 1916:192–217, 1916.
- [Lic58] A. Lichnerowicz. *Geométrie des groupes des transformation*. Travaux et Recherches Mathématiques, III. Dunod, Paris, 1958.
- [LP87] J. M. Lee and T. H. Parker. The Yamabe problem. *Bull. Amer. Math. Soc.*, 17:37–91, 1987.
- [LR96] P. Li and M. Ramachandran. Kähler manifolds with almost nonnegative Ricci curvature. *Amer. J. Math.*, 118(2):341–353, 1996.
- [LTY98] P. Li, L. F. Tam, and D. Yang. On the elliptic equation  $\Delta u + ku - Ku^p = 0$  on complete Riemannian manifolds and their geometric applications. *Trans. Amer. Math. Soc.*, 350(3):1045–1078, 1998.
- [LY90] P. Li and S.-T. Yau. Curvature and holomorphic mappings of complete Kähler manifolds. *Compositio Math.*, 73(2):125–144, 1990.
- [Mil63] J. Milnor. *Morse theory*. Based on lecture notes by M. Spivak and R. Wells. Annals of Mathematics Studies, No. 51. Princeton University Press, Princeton, N.J., 1963.
- [MR10] P. Mastrolia and M. Rigoli. Diffusion-type operators, Liouville theorems and gradient estimates on complete manifolds. *Nonlinear Anal.*, 72:3767–3785, 2010.
- [MRS10] L. Mari, M. Rigoli, and A. G. Setti. Keller-Osserman conditions for diffusion-type operators on Riemannian manifolds. *J. Funct. Anal.*, 258:665–712, 2010.
- [MRV] P. Mastrolia, M. Rimoldi, and G. Veronelli. Myers-Type Theorems and Some Related Oscillation Results. *J. Geom. Anal.*, pages 1–17. 10.1007/s12220-011-9213-0.
- [MS10] O. Munteanu and N. Sesum. The Poisson equation on complete manifolds with positive spectrum and applications. *Adv. Math.*, 223:198–219, 2010.
- [Ni82] W. M. Ni. On the elliptic equation  $\Delta u + K(x)u^{(n+2)/(n-2)} = 0$ , its generalizations, and applications in geometry. *Indiana Univ. Math. J.*, 31(4):493–529, 1982.
- [NST01] L. Ni, Y. Shi, and L.-F. Tam. Poisson equation, Poincaré-Lelong equation and curvature decay on complete Kähler manifolds. *J. Differential Geom.*, 57:339–388, 2001.
- [Oba62a] M. Obata. Certain conditions for a Riemannian manifold to be isometric with a sphere. *J. Math. Soc. Japan*, 14:333–340, 1962.

- [Oba62b] M. Obata. Conformal transformations of compact Riemannian manifolds. *Illinois J. Math*, 6:292–295, 1962.
- [Pet06a] P. Petersen. *Riemannian geometry*, volume 171 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2006.
- [Pet06b] P. Petersen. *Riemannian geometry*, volume 171 of *Graduate Texts in Mathematics*. Springer, New York, second edition, 2006.
- [PR] S. Pigola and M. Rimoldi. Characterizations of Model Manifolds by Means of Certain Differential Systems. *Canad. Math. Bull.*, pages 1–14. doi:10.4153/CMB-2011-134-0.
- [PRS03a] S. Pigola, M. Rigoli, and A. G. Setti. Some applications of integral formulas in Riemannian geometry and PDE’s. *Milan J. Math.*, 71:219–281, 2003.
- [PRS03b] S. Pigola, M. Rigoli, and A. G. Setti. Volume growth, “a priori” estimates, and geometric applications. *Geom. Funct. Anal.*, 13(6):1302–1328, 2003.
- [PRS05a] S. Pigola, M. Rigoli, and A. G. Setti. A Liouville-type result for quasilinear elliptic equations on complete Riemannian manifolds. *J. Funct. Anal.*, 219:400–432, 2005.
- [PRS05b] S. Pigola, M. Rigoli, and A. G. Setti. Maximum principles on Riemannian manifolds and applications. *Mem. Amer. Math. Soc.*, 174(822):x+99, 2005.
- [PRS05c] S. Pigola, M. Rigoli, and A. G. Setti. Vanishing theorems on Riemannian manifolds, and geometric applications. *J. Funct. Anal.*, 229:424–461, 2005.
- [PRS07] S. Pigola, M. Rigoli, and A. G. Setti. Some characterizations of spaceforms. *Trans. Amer. Math. Soc.*, 359(4):1817–1828 (electronic), 2007.
- [PRS08] S. Pigola, M. Rigoli, and A. G. Setti. *Vanishing and finiteness result in geometric analysis*, volume 266 of *Progress in Mathematics*. Birkhauser Verlag Ag, 2008.
- [PRS10] S. Pigola, M. Rigoli, and A. G. Setti. Existence and non-existence results for a logistic-type equation on manifolds. *Trans. Amer. Math. Soc.*, 362:1907–1936, 2010.
- [PW67] M. H. Protter and H. F. Weinberger. *Maximum principles in differential equations*. Englewood Cliffs Prentice-Hall, 1967.
- [RRS95] A. Ratto, M. Rigoli, and A.G. Setti. On the Omori-Yau maximum principle and its application to differential equations and geometry. *J. Funct. Anal.*, 134:486–510, 1995.

- [RRV94a] A. Ratto, M. Rigoli, and Véron. Conformal immersions of complete Riemannian manifolds and extensions of the Schwarz lemma. *Duke Math. J.*, 74(1):223–236, 1994.
- [RRV94b] A. Ratto, M. Rigoli, and L. Véron. Scalar curvature and conformal deformations of hyperbolic space. *J. Funct. Anal.*, 121:15–77, 1994.
- [RRV97] A. Ratto, M. Rigoli, and L. Veron. Scalar curvature and conformal deformations of noncompact riemannian manifolds. *Math. Z.*, 225:395–426, 1997.
- [RS01] M. Rigoli and A. G. Setti. Liouville type theorems for  $\phi$ -subharmonic functions. *Rev. Mat. Iberoamericana*, 17:471–520, 2001.
- [RSV05] M. Rigoli, M. Salvatori, and M. Vignati. Some remarks on the weak maximum principle. *Rev. Mat. Iberoamericana*, 21(2):459–481, 2005.
- [RZ07] M. Rigoli and S. Zamperlin. *A priori* estimates, uniqueness and existence of positive solutions of Yamabe type equations on complete manifolds. *J. Funct. Anal.*, 245:144–176, 2007.
- [Sat73] D. H. Sattinger. *Topics in stability and bifurcation theory*. Lecture Notes in Mathematics, Vol. 309. Springer-Verlag, Berlin, 1973.
- [Sch84] R. Schoen. Conformal deformation of a Riemannian metric to constant scalar curvature. *J. Differential Geom.*, 20(2):479–495, 1984.
- [Spi79] M. Spivak. *A comprehensive introduction to differential geometry*. Vol. II. Publish or Perish Inc., Wilmington, Del., second edition, 1979.
- [Swa68] C. A. Swanson. *Comparison and oscillation theory of linear differential equations*. Academic Press, New York, 1968. Mathematics in Science and Engineering, Vol. 48.
- [Swa75] C. A. Swanson. Picone’s identity. *Rend. Mat. (6)*, 8(2):373–397, 1975. Collection of articles dedicated to Mauro Picone on the occasion of his ninetieth birthday, II.
- [SY94] R. Schoen and S.-T. Yau. *Lectures on differential geometry*. Conference Proceedings and Lecture Notes in Geometry and Topology, I. International Press, Cambridge, 1994.
- [Tru68] N. S. Trudinger. Remarks concerning the conformal deformation of Riemannian structures on compact manifolds. *Ann. Scuola Norm. Sup. Pisa (3)*, 22:265–274, 1968.
- [Wat66] G.N. Watson. *A treatise on the theory of Bessel functions*. Cambridge University Press, 1966.
- [Yam60] H. Yamabe. On a deformation of Riemannian structures on compact manifolds. *Osaka Math. J.*, 12:21–37, 1960.

- [Yau73] S.T. Yau. Remarks on conformal transformations. *J. Diff. Geom.*, 3:369–381, 1973.
- [YN59] K. Yano and T. Nagano. Einstein spaces admitting a one-parameter group of conformal transformations. *Ann. of Math.*, 69:451–461, 1959.



# List of Symbols

$[X, Y]$	Lie bracket of two vector fields $X$ and $Y$ , page 12
$ \text{Hess}(u) $	norm of $\text{Hess}(u)$ , page 19
$ X $	norm of the vector field $X$ , page 19
$ x $	distance of the point $x \in \mathbb{R}^m$ from the origin 0, page 211
$\Lambda^2(U)$	space of skew-symmetric 2-forms, page 14
$C^\infty(M)$	set of smooth functions defined on $M$ , page 16
$(U, \varphi)$	local chart, page 8
$\Delta u$	Laplacian of the function $u$ , page 18
$\delta_i^j$	suggestive version of the Kronecker symbol, page 8
$\delta_{ij}$	Kronecker symbol, page 8
$\dot{\gamma}$	tangent vector of the curve $\gamma$ , page 22
$x^1, \dots, x^m$	coordinate functions, page 8
$\frac{\partial u}{\partial \nu}$	directional derivative of the function $u$ in the direction of $\nu$ , page 62
$\text{Hess}(u)$	Hessian of the function $u$ , page 18
$\lambda_1^{L_H}(\Omega)$	first eigenvalue of the operator $L_H$ on the bounded domain $\Omega$ , page 74
$\lambda_1^{L_H}(M)$	first eigenvalue of the operator $L_H$ on the Riemannian manifold $M$ , page 74
$\mathcal{L}_X \langle , \rangle$	Lie derivative of the metric $\langle , \rangle$ in the direction of $X$ , page 13

$\mathcal{L}_X\omega$	Lie derivative of the 1-form $\omega$ in the direction of $X$ , page 13
$\mathcal{L}_Xf$	Lie derivative of the function $f$ in the direction of $X$ , page 13
$\mathcal{L}_XY$	Lie derivative of the vector field $Y$ in the direction of $X$ , page 13
$\mathbb{B}^m$	unit ball of $\mathbb{R}^m$ , page 71
$\mathbb{B}_R^m(0)$	open disk of radius $R$ centered at the origin in $T_oM \approx \mathbb{R}^m$ , page 54
$\mathbb{H}^m$	standard hyperbolic space of dimension $m$ , page 47
$\mathbb{H}_{-H^2}^m$	hyperbolic space of constant sectional curvature $-H^2$ , page 75
$\mathbb{S}^m$	standard sphere of dimension $m$ , page 47
$\mathbb{S}_+^m$	standard upper hemisphere, page 53
$\mathbb{S}_+^m(\sqrt{k})$	upper hemisphere of radius $k^{-1/2}$ , page 63
$\mathbb{S}_{k^2}^m$	sphere of constant sectional curvature $k^2$ , page 61
$\mathfrak{X}(M)$	set of smooth vector fields on $M$ , page 10
$\text{Ann}_{\eta,1}$	annulus $B_1 \setminus \overline{B}_\eta$ , page 214
$\text{cut}(o)$	cut locus of the point $o$ , page 21
$\text{II}$	second fundamental tensor, page 33
$\langle \cdot, \cdot \rangle_{ij}$	(local) components of the metric, page 8
$\nabla\omega$	covariant derivative of the 1-form $\omega$ , page 11
$\nabla f$	gradient of the function $f$ , page 11
$\nabla X$	covariant derivative of the vector field $X$ , page 10
$\nabla_Y\omega$	covariant derivative of $\omega$ in the direction of $Y$ , page 11
$\nabla_Y X$	covariant derivative of $X$ in the direction of $Y$ , page 10
$\ f\ _\infty$	$L^\infty$ -norm of the function $f$ , page 153
$\omega_m$	volume of the unit sphere in $\mathbb{R}^m$ , page 28
$\omega_{ik}$	covariant derivative of the coefficient $\omega_i$ , page 11

$\text{Conf}(M)$	group of conformal diffeomorphisms on $M$ , page 146
$\dim(M)$	dimension of the manifold $M$ , page 8
$\text{div } X$	divergence of a vector field $X$ , page 10
$\text{hess}(u)$	(1, 1) version of $\text{Hess}(u)$ , page 20
$\text{Id}$	identity matrix, page 23
$\text{Iso}(M)$	group of isometries of $M$ , page 146
$\text{Lip}_0(M)$	set of Lipschitz functions on $M$ with compact support, page 29
$\text{ric}$	(1, 1) version of $\text{Ric}$ , page 20
$\text{Riem}$	Riemann curvature tensor of type (1, 3), page 15
$\text{Sect}(u \wedge v)$	sectional curvature of the plane $\pi$ spanned by $u$ and $v$ , page 16
$\text{supp } \varphi$	support of the function $\varphi$ , page 77
$\text{Tor}$	torsion tensor, page 12
$\text{tr}$	trace, page 10
$\text{vol } \partial B_R(o)$	volume of the boundary of the geodesic ball $B_R(o)$ , page 28
$\text{vol } B_R(o)$	volume of the geodesic ball $B_R(o)$ , page 28
$\text{W}$	Weyl tensor, page 45
$\otimes$	tensor product, page 10
$\overline{\mathbb{B}^m}$	closed unit ball of $\mathbb{R}^m$ , page 71
$\otimes$	Kulkarni-Nomizu product, page 46
$\frac{\partial \cdot}{\partial r}$	derivative in the radial direction, page 28
$\partial B_R(o)$	boundary of the geodesic ball centered at $o$ with radius $R$ , page 21
$(\varphi_t)_*$	push-forward of the flow, page 14
$\rho$	Riemannian distance from the origin of $\mathbb{B}^m$ with respect to the hyperbolic metric, page 212
$\text{Ric}$	Ricci tensor, page 16

$\text{Ric}(\nabla r, \nabla r)$	radial Ricci curvature, page 23
$(M, \langle \cdot, \cdot \rangle)$	Riemannian manifold with metric $\langle \cdot, \cdot \rangle$ , page 7
$\{\theta^i\}$	local orthonormal coframe, page 8
$\{e_i\}$	local orthonormal frame, page 8
$\text{sgn}(\cdot)$	signum function, page 110
$\sharp$	sharp map, page 11
$\sqrt{g}$	square root of the determinant of the metric in polar geodesic coordinates, page 23
$\Theta_j^i$	curvature forms, page 14
$\theta_j^i$	Levi-Civita connection forms, page 8
$\varphi_t$	local flow of a vector field, page 14
$\wedge$	wedge product, page 8
$\widetilde{\langle \cdot, \cdot \rangle}$	conformally deformed metric, page 37
$A$	Schouten tensor, page 45
$a_+$	positive part of the function $a$ , page 89
$A_t^f$	$t$ -level set of a function $f$ , page 29
$B_R(o)$	geodesic ball centered at $o$ with radius $R$ , page 21
$C$	Cotton tensor, page 45
$C^\infty(U)$	set of smooth functions defined on the open set $U$ , page 9
$C^{0,\alpha}(M)$	space of locally Hölder continuous functions on $M$ with exponent $\alpha$ , page 165
$C_0^\infty(M)$	set of smooth function with compact support on $M$ , page 27
$c_m$	constant appearing in the (geometric) Yamabe equation ( $m \geq 3$ ), page 39
$d\omega$	exterior differential of the 1-form $\omega$ , page 12
$df$	exterior differential of the function $f$ , page 11

$dx^i$	differential of the coordinate function $x^i$ , page 8
$exp_o$	exponential map of $M$ at $o$ , page 21
$f^*$	pull-back <i>via</i> the map $f$ , page 32
$f_i$	local components of the differential $df$ , page 11
$G(x, y)$	Green kernel, page 95
$H$	mean curvature vector field, page 34
$h_{ijk}^\alpha$	coefficients of the covariant derivative of II, page 35
$h_{ij}^\alpha$	coefficients of the second fundamental tensor II, page 33
$h^\nu$	mean curvature in the direction of $\nu$ , page 34
$K_p(\pi)$	sectional curvature of the 2-plane $\pi$ , page 16
$K_{rad}$	radial sectional curvature, page 31
$R$	curvature tensor of type $(0, 4)$ , page 15
$r$	Euclidean distance from the origin in $\mathbb{R}^m$ , page 212
$r(x)$	Riemannian distance function, page 21
$R_{ijkt}^i$	(local) components of the Riemann curvature tensor of type $(1, 3)$ , page 15
$R_{ijkt,l}$	covariant derivatives of the (local) components of the curvature tensor, page 16
$R_{ijkt}$	(local) components of the curvature tensor of type $(0, 4)$ , page 15
$R_{ij}$	(local) components of the Ricci tensor, page 16
$S$	scalar curvature, page 16
$T$	traceless Ricci tensor, page 18
$T_p^*M$	cotangent space at $p$ , page 10
$T_r^s(M)$	set of tensor fields of type $(r, s)$ , page 10
$T_pM$	tangent space at $p$ , page 10

$T_{ij}$	(local) components of the traceless Ricci tensor, page 18
$u^*$	supremum of the function $u$ , page 102
$u_*$	infimum of the function $u$ , page 148
$u_{ijkl}$	fourth derivatives of the function $u$ , page 20
$u_{ijk}$	third derivatives of the function $u$ , page 19
$u_{ij}$	(local) components of $\text{Hess}(u)$ , page 18
$W^{1,1}(M)$	Sobolev space of functions in $L^1(M)$ with (weak) gradient in $L^1(M)$ , page 29
$W^{1,2}(M)$	Sobolev space of functions in $L^2(M)$ with (weak) gradient in $L^2(M)$ , page 143
$X_k^i$	covariant derivative of the coefficient $X^i$ , page 10

# Index

- $L^\infty$  *a priori* estimate, 149
- $L_H$ , 74
- a priori* estimates
  - from above, 113
  - from below, 106
- Bessel function, 235
- Bianchi identities
  - first, 15
  - second, 16
- Bishop-Gromov comparison theorem, 28
- Bochner-Weitzenböck formula, 19
- boundary point lemma, 66
- Cartan's lemma, 33
- co-area formula, 29
- Codazzi equations, 35
- Codazzi tensor, 46
- comparison result, 140
- conformal
  - deformation of the metric, 37
  - diffeomorphism, 146
  - vector field, 41, 207
- coordinate functions, 8
- cotangent space, 10
- Cotton tensor, 45
- covariant derivative
  - of a 1-form, 11
  - of a function, 11
  - of a generic tensor field, 11
  - of a vector field, 10
  - of the metric, 12
- curvature forms, 14
- curvature tensor
  - of type (0, 4), 15
  - Riemann, 15
- cut locus, 21
- cut point, 21
  - ordinary, 21
  - singular, 21
- Darboux
  - coframe, 32
  - frame, 33
  - frames along  $f$  preserving orientations, 34
- de Rham cohomology groups, 47
- decomposition of the curvature tensor
  - using the Schouten tensor, 46
- differential
  - of a coordinate function, 8
- Dirichlet problem
  - for the operator  $L_H$ , 74
- divergence
  - of a vector field, 10
- dual orthonormal frame, 8
- Einstein manifold, 16
- Einstein summation convention, 8
- entire
  - subsolution, 166
- estimate
  - from above, 112
  - from below, 105
- exponential map, 21
- exterior differential
  - of a 1-form, 12
- Fatou's lemma, 218

- first Bianchi identities, 15
- first eigenvalue
  - negativity, 170
  - nonnegativity, 75
- first eigenvalue of the operator  $L_H$ 
  - on  $M$ , 74
  - on bounded domains, 74
- first structure equations, 8
- fundamental theorem of Riemannian geometry, 13
  
- Gauss equations, 35
- Gauss lemma, 22
- Gram-Schmidt orthonormalization process, 8
- Green kernel, 94
- Gronwall inequality, 221
- ground state, 103
- group
  - of conformal diffeomorphisms, 146
  - of isometries, 146
  
- Hessian, 18
  - (1, 1) version, 20
- Hessian comparison theorem, 31
- Hopf classification theorem, 56
- Hopf-Rinow theorem, 124
- hypersurface, 34
  
- immersion, 32
  - isometric, 32
  - minimal, 34
  - totally geodesic, 34
  - totally umbilical, 34
- index
  - of a Schrödinger operator, 173
- integral curve, 22
- interior of a set, 74
- isothermic coordinates, 46, 48
  
- Künneth formula, 47
- Kazdan-Warner obstruction, 18, 41
- Kronecker symbol, 8
- Kulkarni-Nomizu product, 46
  
- Laplace-Beltrami operator, 18
  - transformation law, 99
- Laplacian, 18
- Laplacian comparison theorem, 25
- Levi-Civita connection forms, 8
- Lie bracket
  - of two vector fields, 12
- Lie derivative
  - geometric meaning, 14
  - of a 1-form, 13
  - of a function, 13
  - of a vector field, 13
- local chart, 8
- local orthonormal coframe, 8
- locally conformally flat manifold, 46
- lowering indices, 15
  
- Mayer-Vietoris argument, 47
- mean curvature
  - in the direction of a unit normal vector field, 34
  - of an immersed hypersurface, 34
  - vector field, 34
- metric
  - induced by an immersion, 32
  - parallelism of the, 12
  - torsion-free, 12
- minimizing geodesic, 22
- model manifold, 115
- Monotone iteration scheme, 191
- monotonicity of the first eigenvalue, 74
- Morse lemma, 56
  
- negative part, 125
- Newton inequality, 60
- Newton's inequality, 23
- nonparabolic manifold, 94
- normal bundle, 35
  
- Obata type vector field, 57
- Obata's theorem, 53
- oscillating solution, 171
  
- parabolic manifold, 94



- parallel translation, 22
- Picone's identity, 74
- Poincaré model
  - of the hyperbolic space, 211
- Poisson equation, 94
- pole, 29, 202
- positive part, 112
- positive part of a function, 89
- radial Ricci curvature, 23, 26
- reference point, 22
- Rellich-Pohozaev formula, 207
- Rellich-Pohozaev identity, 208
- Riccati differential inequalities, 25
- Ricci equations, 36
- Ricci identities, 19
- Ricci tensor, 16
  - (1, 1) version, 20
  - traceless, 18, 40
- Riemann theorem, 46
- Riemann-Köbe uniformization theorem, 48
- Riemannian
  - manifold, 7
  - metric, 7
- Riemannian distance function, 21
- Riemannian product
  - of locally conformally flat manifolds, 47
- scalar curvature, 16
- Schouten tensor, 45
- Schur's theorem, 17
- second fundamental tensor, 33
- second Green formula, 27
- second structure equations, 14
  - pull-back, 34
- sectional curvature, 16
  - radial, 31
- Seifert-Van Kampen theorem, 56
- sharp map, 11
- smallness
  - in a spectral sense, 159
- solution
  - maximal, 165, 166
  - minimal, 166
- spectral theory
  - of Schrödinger operators, 158
- sphere
  - $m$ -dimensional, 43
- spherical mean, 170, 173
- stretching factor, 146
- subsolution, 111, 191
- supersolution, 111, 191
  - of a boundary value problem, 191
- support of a function, 77
- symmetries
  - of the Riemann curvature tensor, 15
  - of the second derivatives, 19
- tangent space, 10
- tangent vector of a curve, 22
- tensor field
  - of type  $(r, s)$ , 10
- third derivatives, 19
- torsion tensor, 12
- trace, 10
- transformation law
  - for a local o.n. coframe, 37
  - for the connection forms, 38
  - for the curvature forms, 38
  - for the curvature tensor, 38
  - for the Hessian, 99
  - for the Laplace-Beltrami operator, 99
  - for the Ricci tensor, 38
  - for the scalar curvature, 39
  - for the traceless Ricci tensor, 43
- umbilical point, 34
- Van der Waerden-Bortolotti covariant derivation, 35
- vector field, 10

volume

of a geodesic ball, 28

of the boundary of a geodesic  
ball, 28

weak maximum principle at infinity,  
133

weakly distance decreasing, 155

weighted spherical mean, 200

Weyl tensor, 44

Yamabe

equation(s), 39

invariant, 47

Yamabe equation

on hyperbolic space, 211