Appendix A
From Computing to Relativity Theory Through Algebraic Logic: A Joint Scientific Autobiography

Hajnal Andréka and István Németi

This is a two-in-one autobiography. If one omits the word “joint” from the title, omits István Németi from the authors, and changes “we” everywhere to “we, with István”, one gets a separate autobiography for Hajnal. Likewise, if one omits the word “joint” from the title, omits Hajnal Andréka from the authors, and changes “we” everywhere to “we, with Hajnal”, one gets a separate autobiography for István.

The sub-titles refer to the subjects and stations we find most important in our carrier so far. These are arranged more or less in chronological order, but there are large overlaps. The goals and style of our research were laid down from 1966 till 1970, when István took part in the daring project of automatizing the Hungarian power system. The research directions, both practical (programming-oriented) and theoretical (general logic, universal algebra), suggested by this work were directly followed till about 1976. After this, till about 1986 we pursued three subjects in parallel: nonstandard-time semantics for dynamic logic of programs (continuation of the programming aspects), categorical injectivity logic and partial algebras (more specific logic and algebra), and algebraic logic that we pursue from 1971 till today. In these periods, we tried to focus and get deep in some specific sub-areas, besides keeping up with the general aspects of our research. From about 1991 the emphasis in our research is also on “widening” besides on “deepening”. From about 1991 we tried to integrate and connect algebraic logic to logic, programming, algebra and in science in general as much as we could. From about 1998 we applied algebraic logic to relativity theory, physics and methodology of science, and we also connected computing with relativity theory in the form of relativistic computing.

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In what follows, numbered references such as [4] refer to our joint publication list in this volume. Other publications are also referred to, these are given explicitly in the text.

Large-Scale AI Program for the Hungarian Power System (c. 1966–1970)

István got an electric engineer's diploma from Technical University Budapest in 1966. His first job was at Institute for Power System Study (Hungarian short name was ERŐTERV for Erőmű- és Hálózattervező Vállalat). The Hungarian electrical network was planned and maintained here, it operated under the Ministry of Heavy Industries. The power system that consisted of different power-lines, power-houses, transformer-stations, was handled by several copybooks, István's job was to introduce the regularly measured data in these copybooks. This was not a very creative job for an engineer, and we do not know how István's scientific career would have turned if Géza Bogdánfy would not have been there. Bogdánfy was the head of the group dealing with this book-keeping process. Géza was watching and observing, and after a while, he sat down and wrote a computer program that modeled the power system, and from that time on the data were introduced to this program, instead of the several copybooks. Once one had this program, it could be used for many other purposes, e.g., to simulate the whole Hungarian power system and see what happened if a specific power-line or power-house was destroyed (by bad weather e.g.,) or if a specific new power-line was built. The team participating in creating this program consisted of Géza, István, Attila Bánhegyi, Károly Füle, Annamária Nitsh and Anna Simay. They constantly improved the program, always aiming at the highest quality and applicability. It was important for them that people at ERŐTERV and similar companies use the program in their everyday work. They indeed reached this goal. By the end of 1970 the program was rather big, complex, it was in use also in the late 80s. To give a feeling of the program's complexity, it was inverting matrices of size 500 times 500, while the largest ones other programs could invert at that time were of size 40 times 40, see [13, p. 302]. This was a big success, and seeing what this program could do, respectable professor Károly Szendy, head of ERŐTERV, jumped up and down in joy.

The program system used heuristic methods, real random generators,[1] and e.g., the program that inverted matrices used a repeated decomposition of matrices. At the end of this endeavor, the program was so complex that sometimes István felt as if it behaved like an intelligent being. This matched a previous meeting of István with a miracle of science. When he was drafted in the army after finishing university, he was stationed near big MIG military airplanes. These airplanes were enormous and heavy like buildings, yet from one moment to the other, they flew up and disappeared

[1]By an appropriate software, the program could ask the state of a specific part of the hardware (printer) at the moment of asking. This worked quite well for a random number.

in the sky, they were flying so fast. The creation of the large-scale program system with Géza Bogdánfy and seeing the MIG supersonic airplanes gave István a vision of science that we are still pursuing.

Software Department, Theorem Prover, General Logic, Universal Algebra (c. 1970–1976)

The experience in writing the program system sparked interest in its creators towards artificial intelligence, system's theory and mathematical logic for handling semantic aspects of programming.[2] After finishing the power system project, István joined a software department of the Ministry of Heavy Industries.[3] He wanted to pursue the semantic aspects of programming. This meant, in short, to focus on the question of "what" a program did, and not on "how" the program did what it did. The need for this came up at ERŐTERV, where for handling this need István devised a simple formal language for specifying programs, and each sub-program had to be documented in this language before insertion into the system. Missing unambiguous documentations had cost valuable computer time in the implementation.[4]

Hajnal joined István in making true his vision about science in 1971. She had obtained a thorough mathematical training as member of the legendary first special mathematical class organized for mathematical talents,[5] and then at the mathematics programme of Faculty of Science, Eötvös Loránd University. Already at high school, Hajnal was attracted to computers and she read a book about cybernetics that caught her imagination. In the last year of university, she took courses about Turing machines and recursive functions (from János Urbán), about Mealy and Moore automata (from András Ádám) and about Chomsky grammars (from Tamás Legendy). Hajnal was excited about these topics, and she was glad that Miklós Náray was willing to be her degree thesis advisor in this subject. After obtaining a mathematics diploma in 1971, she was happy to join the Software Department of NIMIGÜSZI, where she met István.

[2]They read books and papers such as [Ashby, W: Design for a brain, Chapman Hall Ltd, 1960], [Simon, H.: The architecture of complexity, Proc. Amer. Phil. Soc. 106,6 (1962)]. An intellectual circle was forming around the team along these lines, Tamás Gergely, Bertalan Hajnal, Ferenc Sebestyén, Ajándok Eőry, Pál Juhász-Nagy were some members. For example, István gave special seminars for biologists at ELTE about mathematical logic. István also was member of the science fiction club (lead by Péter Kuczka), he met there scientists like astronomer Iván Almár, biologist Tibor Gánti. Later cooperation with physicist Gyula Dávid brought up joint intellectual values that were cradled in this circle.

[3]It was the Software Department of NIMIGÜSZI (shorthand for the Hungarian Nehézipari Minisztérium Ipargazdasági és Üzemszervezési Intézete), the head was Miklós Náray. Bogdánfy and Füle also joined this department.

[4]Keep in mind that the program system was written at the dawn of the computer era when computer time was sparse and precious.

[5]Fazekas Mihály High School Budapest, 1962–1966.

Our abilities matched and complemented each other's amazingly well. Hajnal learned English from István by reading Arthur C. Clarke's science fiction novel 2001 together. István underlined the unknown words and next time of the reading Hajnal had to recall their Hungarian meanings. We also read together Cohn's Universal Algebra book. We read the definitions and theorems together, and then we closed the book and began independently to prove the theorem. Often, after a while we both gave up, and showed each other's half-proofs. This often resulted in two different proofs for the same theorem, because we both could finish the other's half-proof. Then we matched the proofs with the one given in the book, and sometimes this was a third proof. Perhaps, our delight in seeing several different proofs for the same theorem was born here.

Inside the software department, we formed a group called Program Languages, the head was István. Members of this group were Hajnal, Kálmán Balogh, László Gyuris, István Herényi, Katalin Lábodi, Péter Szeredi and Judit Szoldán. We participated both in very concrete software developments, and in more theoretical studies connected with programming technology. We often delivered studies written for concrete contracts with the Hungarian Bureau for Planning (Hungarian short name was OT for Országos Tervhivatal). Life in the Software Department was lively and intellectual. Not only programs were written for concrete softwares, but at the same time software technology was devised, new programming languages found in the literature were adopted, adapted, modified. On the theoretical side, regular seminars were held, for example for ALGOL 68, affix grammars, semigroup-theoretical automata.

Following up interest in the semantic aspects of programming, we worked on how one could give clear and unambiguous program specifications. Once one had such a specification, verifying whether the program indeed satisfied this specification was the next natural step. This lead to using theorem-prover programs for this checking. A working theorem prover was indeed written in our group.[6] We called this program semi-automatic, because it was not very efficient. In order for it to prove a theorem, we often had to break down the theorem into several lemmas and make the prover prove the lemmas first and then prove the theorem from the lemmas. It was our conviction that a working theorem-prover should be somewhere between an automatic theorem-prover and a simple proof-checker. We emphasized that this was in harmony with the fact that natural intelligent beings were not working in ivory towers, alone after having been created, but in constant communication with their environments and other intelligent entities, constantly learning, evolving, adapting. We found it evident that an artificial intelligence had to operate along these lines, too.[7]

In parallel with the above programming works, we were intensively studying mathematical logic, algebra, and set theory. István ordered (that is, made NIMIGÜSZI buy) the monograph Cylindric Algebras Part I by Henkin, L., Monk, J. D. and Tarski, A. as soon as it appeared in 1971, and we began studying it right away. The zeroth

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[6] It was written by Kata Lábodi and delivered to OT in 1973, we believe that this was the first working theorem prover in Hungary, see [29].

[7] It is satisfying for us to see nowadays the general use of theorem provers such as Isabelle/HOL and proof-checkers like the Coq proof-assistant system.

chapter of the book is in fact an introduction to universal algebra, á la Tarski, and we immensely enjoyed it. We enjoyed not only its content, but we loved its style that was reminiscent to programming.

We considered universal algebra to be the theory of complex systems. We thought that an important advantage of universal algebra was that one could use arbitrary and arbitrarily many functions and relations when modeling a complex system, it did not enforce a stricter format to its user. Universal algebra then deals with how to simplify (make a homomorphic image), focus on a part (form a subalgebra), decompose (write an algebra into a subdirect product of simpler algebras), analyze and then reassemble a system. Our view was in contrast with the prevailing view among mathematicians that universal algebra was the crown of classical algebra that was worth studying only after being familiar with all the classical structures such as groups, modules, vector spaces, etc. We thought that universal algebra was especially important for computer science because there the tendency was that unprecedented complex systems arose at no time (the reason being that to build a program one had to use only "paper and pencil" and not costly heavy materials like bricks, concrete). We supported these ideas with concrete examples from programming technology, the list evolved into a rather popular series of surveys about applications of universal algebra, model theory and categories in computer science.[8]

In this period, we worked mostly together with and under the guidance of Tamás Gergely. We both won 3-year stipends from the Academy for writing a thesis for the candidate's degree. Istváns official advisor was Tamás Gergely and Hajnal's was Bálint Dömölki. We were in leave from the Software Department, our only duty was to pass exams and then write each a thesis. This time was very precious for us: we wanted to learn and understand as much as we could. Besides learning, we developed our own ideas about what general logical systems were and we began to publish and attend conferences.[9] Via passing several exams at this period, we got into contact with Hungarian logicians and algebraists László Kalmár, Rózsa Péter,[10] András Hajnal,[11] Ervin Fried, Tamás Schmidt.

[8] This series of surveys is [36, 53, 59, 72, 77, 92, 108]. In a sense, the ideas are in the background of the 2017 book [4], too.

[9] A great experience was to give a talk and take part in Banach Semester'73 in Warsaw. We met here Helena Rasiowa, Wim Blok, Larisa Maksimova, we met János Makowsky as member of an excited team in abstract model theory, and we attended a memorable talk by William Lawvere.

[10] Rózsa Péter, alias Rózsi néni, knew Hajnal earlier from the special mathematical high-school class, and from university where Hajnal attended courses given by her. Rózsi néni accepted István immediately as Hajnal's co-worker. She wrote a letter to her friend, Helena Rasiowa, when it turned out that there was no more place for talks at the 1973 Banach Semester, and indeed we gave our joint talk with Tamás Gergely at Rasiowa's own seminar. Rózsi néni was mentor of Hajnal's (small) Ph.D. thesis [8], we gave special seminars about mathematical logic under her umbrella at the university, and the three of us had several friendly programs together.

[11] András was Hajnal's teacher at the special high-school mathematical class already. When we showed him a paper we wrote about a simple cylindric algebraic proof of the completeness theorem of first-order logic, he sent it to Donald Monk for opinion. Monk told that a similar proof was lying in Tarski's drawer, but since it was not published till that time, we could publish ours mentioning Tarski. That is what we did, see [32].

The stipend provided a half-year study in some western center in its subject. There were two main centers in the world for building artificial intelligence, one at the Machine Intelligence Department of Edinburgh University and the other at MIT. We had been following the activities at Edinburgh with great admiration. Each year they issued a thick volume with title Machine Intelligence that contained their latest research papers, and we hardly could wait for them to appear. The last one we got and read was Machine Intelligence 4. It was natural that we asked the trip to be to Edinburgh.

István began the trip, together with Tamás Gergely, by visiting cybernetician Gordon Pask's exciting interdisciplinary group in London. Then they visited the Machine Intelligence Department of Edinburgh University, Tamás for some weeks and István for some months. There they met the Edinburgh Prolog programming language. Prolog is a specific logic programming language, and the latter is a specific area of new-wave programming languages (as we called them). The "old-fashioned" programming languages, e.g., ALGOL 68, were "procedural" ones, they specified in detail some computational steps, and the purpose of these steps were not part of the program. In the new-wave programming languages, basically, you specified "what you wanted" (e.g. by the form of a statement about a specific object), and then "how to get such an object" was the result of a concrete proof from a concrete body of knowledge. In short, a "program" was a body of knowledge (a first-order theory, usually), an "input" for this program was a description of a desired object, and the "output" of the computation was how to get such an object (this could be extracted from a proof of the existence of such an object). The Edinburgh researchers were generous to print out the interpreter for the Prolog language, which Tamás Gergely brought home in a suitcase in December 1974. It was given to Kata Lábodi to make it run. Péter Szeredi helped her, and finally he developed the Hungarian Prolog out of this cooperation. István and Tamás saw Freddy II robot assemble a toy car from its scattered parts, and they met many wonderful people, Rod Burstall and Gordon Plotkin among them. Hajnal joined István in Edinburgh at the end of December (for a month). We got friends with Maarten van Emden, who worked with Robert Kowalski on the minimal model and fixpoint semantics for Horn clauses, they both are great pioneers of logic programming. Kowalski moved to London soon after István arrived, and Emden moved to Canada next year. He, and Jos and Eva, provided us with important professional opportunities, in the form of teaching visits, and a warm family in Canada.

By the end of the 3-year stipend, it began to dawn on us that we had to choose between the creation of a big artificial intelligence program (an even richer version of the one made for the power-system at ERŐTERV), and pursuing a research career in mathematics. Hungary's resources suggested the second choice. After the 3-year stipends, we both were admitted to the Mathematical Institute of the Hungarian Academy of Sciences (István in 1976 and Hajnal in 1977). We began an intensive research career, we traveled a lot, participated in and organized conferences, had scientific visits at numerous universities, we have an extended correspondence with research fellows at all parts of the world, we have a lively circle of colleagues and students, and we got into wonderful friendships.

In the next period of, roughly, 1976 to 1986, we worked on three subjects mostly in parallel: categorical injectivity logic, nonstandard-time semantics for logics of programs, and algebraic logic (cylindric, relation and dynamic algebras).

Categorical Injectivity Logic, Partial Algebras (c. 1976–1983)

Partial algebras appear in all parts of mathematics. In particular, the semantics of a program as input-output relation is best treated as a partial function by letting this function be undefined at an input where the program does not stop (i.e., enters into an infinite cycle along its execution). There was a reluctance for using partial algebras among mathematicians, because the usual algebraic notions can be understood for partial algebras in an uncannily numerous ways. For example, what is the natural meaning of an equality of two terms that involve partial functions? One way of thinking is that the equality is true exactly when both sides are defined and they are equal. Another equally natural way is that the equality is true also when both sides are undefined. Similarly, there are many natural notions for subalgebras, homomorphic images. The tendency for treating partial functions was to replace them with their "total versions". The total version took some pre-agreed value where the original partial function was not defined, otherwise the two functions agreed. However, this move distorted the nature of partial functions.

Well, the wealth of choices in partial algebra notions attracted us! Universal algebra deals with total functions (that is, functions defined everywhere) and we were quite familiar by that time with universal algebra. We felt ready for extending universal algebra to partial algebras.

Instead of using an ad-hoc direct process, we decided to use category theory as a bridge. For showing an example, let us concentrate on one basic theorem of universal algebra, Birkhoff's variety theorem. It says that the smallest equationally defined class containing a class K of algebras is the class of all algebras constructed from members of K by the use of direct products, subalgebras, and homomorphic images.[12] This theorem involves the notions of equational logic, subalgebras and homomorphic images, the "right" versions of which we were to find out in partial algebra theory. To use category theory as a bridge, first we had to "elevate" these notions to category theory.

At the categorical level, we defined the so-called injectivity logic of a category.[13] In this logic, the formulas are the morphisms of the category, the models are the objects of the category, and a morphism is "true" or "valid" in an object if it is injective relative to it (that is, each morphism from the domain of the morphism to the object factors through the morphism). Given two classes H, S of morphisms, we

[12] In short, $ModEqK = HSPK$.

[13] Independently of us, this logic was also defined in [Banaschewski, B., Herrlich, H., Subcategories defined by implications. Houston J. Math. 2 (1976), 149–171].

defined the so-called *HS*-equations as a subclass of morphisms, in such a way that these were abstract categorical versions of the usual total algebraic equations.[14]

Then we proved a general Birkhoff-type variety theorem for categories. Namely, we proved that in any category (under some conditions) the *HS*-equational hull of a class *K* of objects is *HSPK*.[15] If we choose the category to be that of total algebras, and we choose *H*, *S* to be the classes of all surjective and one-to-one homomorphisms, respectively, we obtain the usual Birkhoff variety theorem.

To obtain a variety theorem for partial algebras, we choose the category to be the category of all partial algebras and we find out what the theorem says. First of all, we have to choose our favorite notions *H* and *S* of homomorphisms and subalgebras (arbitrarily in so far as the conditions of the theorem allow), and then see what (first-order logic) formulas injectivity of the *HS*-equations correspond to. These formulas will be the partial algebraic equations that correspond to our choice of *H*, *S*. By this, we imported the variety theorem from total algebras to partial algebras in such a way that we did not have to do away with plurality of the partial algebraic notions.

We did not stop here. We made the direct product part of the theorem to depend on a parameter, too. This parameter was a class *F* of filters. Then we generalized the injectivity approach to cones of morphisms (a set of morphisms with a common domain) as formulas, we defined the *HSF*-cones and voilá! we had a Birkhoff-type theorem for *HSF* in place of *HSP*.

The *HSF*-cones, the generalized analogues for equations, were defined so that, roughly, *H* made restrictions on the domain of the cone, *S* made restrictions on the co-domains of the members of the cone, and *F* determined how many members the cone could consist of. For example, if we chose *F* to be the class of all filters, this number was 1, if we chose *F* to be the class of all ultrafilters, then this number was "finite", and if we chose *F* to be the class of all principal ultrafilters, then there was no restriction on the number. In the category of total algebras, the natural choice for *H* and *S* was the class of all surjective homomorphisms and all one-to-one homomorphisms. But, they could be chosen to be the class of all isomorphisms, too. This way, all the well-known axiomatizability theorems for equations, equational implications, universal formulas, Horn-formulas, positive formulas, infinitary quasi-equations were special cases of this one *HSF*-theorem.[16] A by-product of importing the universal algebraic notions from total algebras to partial algebras through category theory was a powerful unification of the several axiomatizability theorems of total algebras, all looking similar but treated in model theory completely separately with separate proofs.

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[14]This means the following. If we let *H* and *S* be the classes of all surjective and one-to-one morphisms in the category of total algebras, then to each *HS*-equational morphism there corresponds a set of equations, and vice versa, so that in each algebra the *HS*-equation is injective if and only if the corresponding set of equations is true in the algebra.

[15]Here, the *HS*-equational hull of *K* is the class of all objects in which all *HS*-equations injective in *K* are injective, and *HSPK* is the class of all objects that can be obtained from members of *K* by category theoretic direct products, *H*-morphism-images and *S*-morphism-subalgebras.

[16]In symbols, for example, $ModQeqK = SP^rK$, $ModUnivK = SUpK$, $ModInfqeqK = SPK$.

István's candidate's dissertation, submitted in 1976, was about this kind of generalizing the algebraic notions to partial algebras. Already in 1974 we were advisors of two degree-dissertations in this subject, those of Anna Pásztor (title "The concept of variety in the theory of categories") and of Ildikó Sain (title "Category theoretical investigations in order to generalize identities and quasi-identities"). They both continued working with us after university, they became dear colleagues. Ildikó Sain was part of almost all our later works, too, starting with 1977. Anna Pásztor moved to Germany and became Ana Pasztor, her Ph.D. advisor was Peter Burmeister in the subject of partial algebras and category theory. Ana defended her Ph.D. in 1979 under his supervision. We kept contact with Peter while the preparation of this Ph.D., this was the first Ph.D. dissertation where we cooperated with a foreign supervisor in directing a dissertation.[17] Burmeister was an expert in partial algebra theory, his Ph.D. dissertation in 1971 was about partial algebras. It was natural, that after Ana's Ph.D. dissertation, we continued working with Peter in partial algebra theory. We visited each other, and wrote several joint papers. He summarized the theory of partial algebras, based on our injectivity approach, in his monograph [Burmeister, P.: A model theoretic oriented approach to partial algebras. Introduction to theory and application of partial algebras. Part I. Akademie Verlag, Berlin 1986].

Our injectivity approach reminded René Guitart and Christian Lair of Ehresmann's sketches and of their own category theoretic work. They elaborated the connections, wrote papers about this connection.[18] A visit to them was financed by the Hungarian Academy of Sciences in 1982. It happened that Jacques Riguet, pioneer of cybernetics and theory of binary relations, was returning home from Budapest to Paris. He invited us to join him, and we took his car instead of a train. We went by car from Budapest through Vienna, Stuttgart, Munich to Paris. We have never been in this part of the world before, it was a great experience. In Paris, we lived in Riguet's home, his kind personality and warm hospitality is a staying memory. Our stay in Paris was rich in every way. First of all, we had long lively discussions with Guitart and Lair comparing our views, we gave talks, we visited several logic and computer science groups, attended a memorable talk by René Thom, we got invited to dinner into the homes of René Guitart and of Iréne Guessarian, we visited science fiction and cartoon book stores. We are grateful to Riguet for bringing this about.

Papers we wrote on the injectivity approach are [9, 43, 49, 48, 78, 76, 75, 80, 87], further papers on partial algebras are [37, 44, 62, 64, 107], and further related papers in category theory are [52, 94].

[17] Later we had several joint Ph.D. students with foreign colleagues. Maarten Marx was joint Ph.D. student of Johan van Benthem and István, Szabolcs Mikulás was joint Ph.D. student of Johan and Hajnal, Tarek Sayed was joint Ph.D. student of Mohammed Amer and István.

[18] For example, a whole section is about our work in [Guitart, R., Lair, C., Calcul syntaxique des modeles et calcul des formules internes. Diagrammes, Vol 4, Dec. 1980. 106 p.].

Nonstandard-Time Semantics for Dynamic Logic of Programs (c. 1978–1986)

We continued interest in program verification, but now on a more theoretical side. Floyd's method for proving correctness of a program consisted of inserting statements into the nodes of the block-diagram of the program. These statements were about the contents of the computer registers at that point in the execution of the program, and then one had to prove implications between these statements. This is a most natural way of proving a program correct, and the most natural first goal was to prove that all correct programs could be proved to be correct by Floyd's method.

The first result we got was a strong negative one about program verification in general: there is no recursive subset of all correct programs that contains a small "we-must-prove" core of the correct programs [50]. This meant that Floyd's method could not be complete. As usual, we did not stop here. We figured that the reason for this strong incompleteness was that a halting run of a program had to be finite, and this way the (hard) theory of natural numbers filtrated into the theory of program verification. Our suggested solution was to use any model of Peano arithmetic in place of the natural numbers for the time-scale, but then make restrictions on program runs along time-scale. This restriction was that all first-order formulas survived the "jumps" in a nonstandard time-scale, we called such runs "continuous". All standard runs (that is, runs along the natural numbers) are continuous. Now we were in the position of proving a completeness theorem: Floyd's method proved exactly those programs correct that were correct not only with respect to standard runs, but also with respect to continuous nonstandard runs. This completeness theorem gave us a method for *proving* if Floyd's method did not prove a correct program to be correct. All we had to do was to exhibit a continuous run of the program with respect to which the program was not correct. Indeed, we exhibited many correct programs that were not Floyd-provable. We aimed at finding as simple such programs as we could, and to our great joy, we could use ultraproducts, our favorite tool, to show about a nonstandard run that it was continuous.[19] We found it intriguing that the "non-existent", "theoretical", infinite runs provided practical information on concrete programs.

In Floyd's program verification method, reference to time along which the program was running appeared in no way. There were more sophisticated methods, in which time appeared in the form of time-modalities such as *First*, *Next*, *Always* (in Burstall's intermittent assertions method) and *Always-in-the-future* additionally in Pnueli's method. If one defines the language properly, and one requires in the definition of a continuous run survival of the additional formulas, too, then one can prove similar completeness theorems for Burstall's and Pnueli's program verification methods.

We went on, and we made dependence on time explicit by introducing a three-sorted language one sort of which was the sort of time, another sort was for program

[19]We worked hard to find the simplest examples and their simplest proofs for non-Floyd-provability. When we proudly presented such in a talk in Dana Scott's seminar at Pittsburgh in 1984, his remark was that he favored more sophisticated tools in mathematics.

runs, and the third sort was the sort for the data of the programs. In Floyd's method, all formulas are about this third sort. In the three-sorted language we can "talk" about processes (programs, actions) living in time explicitly, we can express partial and total correctness as well as many other properties of programs like concurrency, non-determinism, fairness. This is an ordinary first-order logic, in which we made explicit all the features relevant to program semantics. Let us call it NDL for "Nonstandard-time dynamic logic".[20] Everything we did about Floyd's, Burstall's and Pnueli's method can be done in this explicit framework. NDL was used for characterizing the "information contents" of distinguished well-known program verification methods, for comparing powers of program verification methods as well as for generating new ones.

We began elaborating the idea of NDL together with Tamás Gergely, he later pursued this line mostly with László Úry. They wrote an excellent book on NDL [Gergely, T., Úry, L., First-order programming theories, Springer, 1991]. We continued work on this subject mostly with László Csirmaz, Ana Pasztor, and Ildikó Sain, they contributed many interesting results. Both Csirmaz and Sain wrote their candidate's theses on this subject, one of Pasztor's papers on NDL is [Pasztor, A., Recursive programs and denotational semantics in absolute logics of programs, Theoretical Computer Science 70 (1990), 127–150].[21] The Madrid group Ana Gil-Luezas, Theresa Hortalá-Gonzáles and Mario Rodriguez-Artalejo took up this subject, we had many enjoyable discussions with them. Other colleagues, too, got interested and wrote papers on nonstandard semantics for DL, for example Martin Abadi, Robert Cartwright, Petr Hájek, David Harel, Daniel Leivant, Albert Meyer, Michael Richter, Manfred Szabo, Marek Suchenek, Andrzej Szalas. Chinese colleagues Zhaowei Xu, Yuefei Sui, Wenhui Zhang rediscovered nonstandard-time semantics for DL in 2015, they continue research in this line.

We presented our results on several conferences. The most memorable for us were, perhaps, Poznan 1980 (Logics of Programs and their Applications, organized by Andrzej Salwicki), Yorktown Heights 1981 (Logics of Programs Workshop, organized by Dexter Kozen), Szeged 1981 (Fundamentals of Computation Theory, organized by Ferenc Gécseg), Leeds 1988 (Many-sorted logic and its application in computer science, organized by Karl Meinke and John Tucker).

We want to write a few words about our New York trip. At the airport upon arrival Dexter Kozen met us, we were taken to Jim Thatcher's house where several other participants of the conference were staying, too. The house was one built from wood, in the midst of uninhabited woods surrounding Yorktown Heights, a creek was flowing below it. Jim Thatcher explained that he was keeping the entrance unlocked so that if a thief wanted to take something, he did not have to make damage by forcing his way in. We were given a beautiful bedroom, one wall was almost entirely made of glass through which one could see the woods. It had a separate bathroom,

[20] As the reader may guess already, in our case the qualifier "nonstandard" is a positive one.

[21] This paper explains connection of nonstandard-time semantics with the set theoretic notion of absoluteness, [231] writes more on this. In short, standard DL is not a KPU-absolute logic, NDL can be viewed to be its KPU-absolute version.

with large soft towels, everything was comfortable and beautiful. But at night we were cold, just one thin blanket on the bed. We hardly could believe that in such a luxurious place there were no extra blankets. But we did not find any. Next day, when we asked for extra blankets, instead of getting them, we were simply explained how to use an electric blanket. We had breakfast with others and then retreated for a while. When we emerged, we found nobody but a letter saying that they left for the workshop in Yorktown Heights (it began in the afternoon), but there was a car around and a map, we should take the car to follow them. We were there alone in the midst of the woods, with a car and a map, but we did not drive! Thank God, we found out that John Tucker who had had quite an adventurous trip to America the day before was also stranded with us in the woods. We do not remember how, but eventually an arrangement was made and Jerzy Tiuryn took us to the workshop.[22]

We met many people at this workshop, some for the first time, for example Krzystof Apt, Peter van Emde Boas, Erwin Engeler, David Harel, Marek Karpinski, Dexter Kozen, Albert Meyer, Grazina Mirkowska, Rohit Parikh, Vaughan Pratt, Andrzej Salwicki, Jim Thatcher, Jerzy Tiuryn, John Tucker. The atmosphere at the workshop, and that of the whole trip, was fantastic. People were generous, friendly, open, sincere, we felt we were accepted and appreciated. The atmosphere at the workshop was lively, intellectual, exuberant people vibrating with intellectual thoughts. That is an atmosphere in which we feel at home, in which we are happy. We found many of these properties characteristic in our later trips to America, too. It was easy for us to make lasting friendships that make our lives rich.

Papers we published on nonstandard-time dynamic logic of programs are [46, 54, 51, 50, 74, 84, 82, 88, 89, 111, 123], related papers are [155, 231].

Algebraic Logic, Tarski's School, Cylindric and Relation Algebras (1971–)

When István met Boolean algebras as a successful algebraic version of propositional logic, he figured that there should be a similar class of algebras for predicate logic. Indeed, he found cylindric algebras in the literature, and we began to study them as soon as we had the Henkin-Monk-Tarski 1971 monograph. We read the book from beginning till end, every passage, footnote. At a 1973 Banach Semester we already gave talks related to cylindric algebras, and we were thrilled to meet James Donald Monk at the 1978 Banach semester Universal Algebra and its Applications. The first thing we asked him was about the second part of the monograph that was promised in the book. Well, Monk was not very definite about the appearance of Part II, but he was open to our questions in the subject, and we began a cooperation that was perhaps the most important thing in our careers, it gave our scientific careers an

[22]That was an adventurous trip, too! We had our first car accident in the USA on the way to the workshop. The second of the two car-accidents in our lives was in Nashville on the way from the airport to Bjarni Jónsson's home, in 1988.

upward inclination, in every way. Henkin, Monk, and Tarski had a draft of a paper about cylindric algebras that they were going to publish, Monk began to write a more detailed version of that paper, and we were happy to second-read the manuscript. On the way, we answered their open questions, improved theorems, constructed counterexamples. This lead to a parallel paper written by us that became the second part of the Henkin-Monk-Tarski-Andréka-Németi Springer Lecture Notes book [2]. Working on this book together with Monk was exciting, heaven[23] for us. Perhaps this Springer book inspired Monk to begin to write Part II of the monograph. We continued the well-working cooperation of our reading the drafts of the chapters as they were in the make. Our contributions are acknowledged throughout Part II.

The 1978 Banach Semester on universal algebra and its applications was memorable to us not only for meeting Donald Monk and starting our collaboration with him. Here was that George Grätzer told us that he thought we should prove ourselves by solving other people's problems and answering their questions, not just proving theorems of our own. We took his advice to heart. We began by solving a problem in his 1968 Universal Algebra book. This asked if Birkhoff's variety theorem implied the axiom of choice or not (it did not [57, 66]).[24] We went on and solved many open problems published by other mathematicians. The most difficult to crack was the one in Ralph McKenzie's 1966 dissertation (published in 1970): is there an integral relation set algebra that has no permutational representation? (Yes, there is [126].) The oldest one is perhaps the problem in Jónsson's 1959 paper on modular lattices: is there a weakly representable but not representable relation algebra? (Answer: there are many [132].) We solved several problems from the 1971 and 1985 monographs of Henkin, Monk, and Tarski, solved problems from the Tarski-Givant 1987 book [Tarski, A., Givant, S., A formalization of set theory without variables. AMS Colloquium Publications, 1987], among others (e.g., [101, 157, 163, 237, 240, 245]).

In 1982, after the appearance of our joint Springer book, Leon Henkin was touring Yugoslavia, giving talks about cylindric algebras at several universities. Monk told Henkin that this was a good opportunity for meeting us. But Leon was reluctant to come to Budapest, so we met in Szeged for one day, Szeged was near the border of Hungary and Yugoslavia. That was an exciting meeting! Leon explained his method twisting, a method for creating non-representable cylindric algebras from

[23]Talking about heaven, in 1980 Hajnal and István were visiting Waterloo University in Canada. Our host, Maarten van Emden was so generous to invite Monk and arrange for all three of us to stay for some days in Walhalla Inn, Waterloo. This way we could work more effectively on the manuscript. That was the period when we were making all kinds of constructions by the use of the ultrafilter-choice functions pair, their version for ultrafilters for infinitary relations. Monk did not mind our slipping the new constructions below his door at nights.

[24]We went on and showed that not only the axiom of choice, but the axiom of foundation also has impact on truth of theorems of universal algebra. We flirted with set theory at another occassion, too: we showed that the solution of the so-called finitization problem of algebraic logic depends on set theory. Roughly, while it is not solvable in Zermelo-Fraenkel set theory with the axiom of choice, the finitization problem has positive solutions in appropriate non-well-founded set theories. Papers we wrote about the influence of the set theoretical background we work in are [57, 66, 134, 161, 165, 179].

representable ones. He crossed his two fingers for illustration, and that was an unforgettable explanation. Leon was surprised to see us. He had expected one person, Andréka-Németi, not two,[25] and one of them a female at that! Later, we met Leon several times in Berkeley where he lived, once we were staying in his home. He took us–Hajnal, István, and Ildikó Sain–to a nearby merry-go-round, after which his famous cylindric algebraic MGR-equation was named. This equation holds in all set cylindric algebras. He had devised the method twisting in order to show that this equation failed in some (abstract) cylindric algebras.[26] It was very enjoyable to listen to Leon as he told stories about small and big things, about how and why Twin Drive near his home was created, and about his children.

Getting back to the merry-go-round, his student Diane Resek proved in her 1975 Ph.D. dissertation that the cylindric algebraic equations together with infinitely many of Leon's Merry-Go-Round equations ensured representability as a cylindric-relativized set algebra. Richard Thompson showed that of these infinitely many MGR-equations only four suffice in the theorem. These theorems are very important but their proofs were about 200 pages each. This is why Monk was sorry to leave them out from Part II. We were curious to see the proofs, so we invited Richard Thompson to enlighten us. He visited for a year in 1986 (on an IREX stipendship). He was happy to talk about cylindric algebras on a weekly full-afternoon seminar, and we learnt a lot from him. The end of his visit approached, and he was still making preparations for telling the proof. We began to worry, but then on the way home after a seminar by Richard, all of a sudden, a short elegant proof emerged from all the preparations in Hajnal's head.[27] This is how finally the theorem got published with a 7-page detailed proof [106]. The class of cylindric-relativized set algebras (Crs) turned out to be rather important in the application of cylindric algebra theory to logic. For example, István proved that the equational theory of Crs is decidable [11], this lead to the decidable guarded fragment of first-order logic [169].

After meeting Henkin in 1982, we looked forward to meeting our third co-author Alfred Tarski whose work and views on science we admired. An opportunity came up when we had a one-year teaching position at Waterloo University Canada in 1983–1984. Our first visit to Berkeley was in May 1984. We could not meet Tarski, since he died at the end of the previous year, but we were received in a very warm way. We were given his room (725 Evans Hall) at the university as our office on the seventh floor. The room had a magnificent view on the bay, but it was cold. We prepared our talk on a rare sunny spot on a terrace some levels below, facing the hills. There were many questions after the talk. A person was asking rather concrete questions about the Hamate semigroup, and we had to admit that we never heard of this semigroup. It turned out that this person was Richard Thompson, and the Hamate semigroup

[25]This was dual to our previously thinking that Kirby Baker working in universal algebra meant two persons, Kirby and Baker.

[26]Leon did not stop at showing that some cylindric algebras (CAs) were not representable, not even as relativized ones. He conjectured that all cylindric algebras were obtainable from representable ones by his method twisting (together with another of his similar method, dilation). Indeed, our student András Simon proved this for the 3-dimensional case in his 1991 Ph.D. Dissertation.

[27]This shows how useful a good foundation can be.

referred to the semigroup of substitutions in a cylindric algebra. The name Hamate was his invention to call this semigroup, both to refer to Henkin-Monk-Tarski and to a move in the game go. Julia and Abraham Robinson attended the talk, they said some kind words to us afterwards. Leon and Ginett Henkin gave a party in the evening, and here is where we met Steve Givant for the first time. The three of us talked towards the end of the party, we asked and he talked about quasi-projective relation algebras, the key player of the book[28] he was finishing at that time. We offered to read and check the manuscript, and indeed he sent it to us, and we learnt the subject while reading it. This was the beginning of a long collaboration and a lifelong friendship. Another "first" on this visit was that we met William Craig. We got the opportunity of getting to know his sweet personality. We were always very happy to meet him later, in Berkeley, Budapest or Warsaw.[29] We also met Diane Resek for the first time during this visit. We met Roger Maddux in person later this year, at the Charleston universal algebra conference. We had exchanged letters, and we looked forward to meeting him. We already met Don Pigozzi[30] and Steve Comer in Hungary, and Wim Blok and Bjarni Jónsson in Warsaw. These were all important meetings for us that lead to professional friendships. We have joint papers with almost all of Tarski's algebraic logic family: scientific children and grandchildren.

The 1983-84 trip to America was full of events. It began with a 2-month visit to Montreal: at McGill University we discussed categorical logic with Michael Makkai, and at Concordia University we had discussions on NDL with Fred Szabo. Then, at Waterloo University we taught two courses on the foundation of computer science, and worked with Maarten van Emden and Areski Nait Abdallah on logic programming, we also had discussions with Stanley Burris on his universal algebra book. After courses ended in Canada, we visited universities in the USA, we gave several talks at each: Carnegie-Mellon University Pittsburgh (April 3–10, Dana Scott, Ken Manders), Colorado University at Boulder (April 26–June 20, July 18–August 15, Donald Monk, Jean Larson), University of California at Berkeley (May 3–10), Stanford Research Institute (May 8, Joseph Goguen, José Meseguer, Richard Waldinger, Leslie Lamport), University of California at Los Angeles (May 10–12), The University of Chicago and The University of Illinois at Chicago (June 20–25, Wim Blok, Saunders MacLane, John Baldwin). Then we visited George Grätzer and his group at The University of Manitoba at Winnipeg in Canada (July 3–10), and returned to the USA for the Charleston Universal Algebra Conference organized by Steve Comer (July 10–17).

Monk and Henkin finished Part II around this time, it appeared in 1985. We were still thinking a lot about cylindric algebras, an example is the following. The set of valid formulas of first-order logic with equality but no function or relation

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[28] the Tarski-Givant 1987 book.

[29] In one of his visits to Budapest, we wrote a joint paper about partial algebras [107]. He stayed in our Budapest home, and in the evenings he enjoyed dancing to live music in the nearby Déryné Bistro.

[30] Don Pigozzi's collaboration with Tarski on producing Part I of the Henkin-Monk-Tarski monograph on cylindric algebras may have inspired our similar cooperation with Monk. We first met at a universal algebra workshop held in Esztergom Hungary, organized by Ervin Fried, around 1979.

symbols is decidable. This may make one think that the set of equations valid in the minimal cylindric algebras (that is, cylindric algebras generated by the diagonals) is decidable. Indeed, there was a statement to this effect in Part I, the proof was promised to appear in the second volume. In the course of writing it, Don Monk asked if we could reconstruct the proof of this statement. We started doing it, and we were almost there, only one small step was missing. Thinking about this just one small gap, in 1984 on an airport when traveling home from the USA, István got the idea of the proof of the opposite statement: the equational theory of the minimal cylindric algebras is not even recursively enumerable! Later we learnt that Matti Rubin also got this result, we elaborated its logical meaning together with him, as follows. Cylindric algebra equations correspond to formula-schemes of first-order logic. A formula-scheme is something like "$\varphi \to \exists x\varphi$ where φ is a formula". The logical side of the above algebraic theorem is that, while the set of valid formulas of equality logic is decidable, the set of valid formula-schemes of this logic is not even recursively enumerable! If we allow a binary relation in the language, then the set of valid formulas becomes "harder" (undecidable), but the set of valid formula-schemes becomes "easier" (recursively enumerable). For details, see [101].

We got invited to the 1985 January Boolean Algebra meeting of the Oberwolfach Research Institute in Germany. There was a parallel Set Theory meeting, and we traveled to Oberwolfach together with András Hajnal. This research institute is famous for its excellent meetings where the emphasis is on the exchange of ideas between the participants. Indeed, we had plenty of discussions with the participants of both meetings, and we returned home with a wealth of new ideas, e.g., about Boffa's non-well-founded set theory. Most important, we met here Matti Rubin. We learnt that, independently of us, he also proved that the equational theory of infinite-dimensional minimal cylindric algebras is not decidable, and he also constructed uncountably many subvarieties of infinite-dimensional representable cylindric algebras. Thus, we had an ideal ground for joint research, that we began to pursue happily. He visited us in the summer. At the end of his visit, we traveled together to Szeged to participate on the Colloquium on Ordered Sets. On the train, besides talking about various mathematical ideas, he said he was looking forward to seeing his old friend Ivo Düntsch on the conference. So we, too, looked forward to meeting Ivo. There was an immediate feeling of friendship between us. Next time we met Ivo was in 1988, on the Ames algebraic logic conference where we began to think about McKenzie's problem. We succeeded in solving it as a joint effort, since then we have had many enjoyable meetings and joint work with Ivo.

István submitted his dissertation for the doctoral degree with the Academy in 1986. The two main results were a translation of first-order logic to the equational theory of 3-dimensional cylindric algebras and showing that the equational theory of Crs is decidable.[31] Hajnal submitted her dissertation for the same degree in 1991, it

[31] The main result of the Tarski-Givant book is translating first-order logic to the equational theory of relation algebras. They ask how much associativity of relation-composition is needed in this result. The answer is in Istvan's dissertation: it can be weakened till "CA_3", but it cannot be weakened till

contained results about cylindric and relation algebras, the main subject being how hard the basic operations were definable over each other.[32]

Our second one-year stay in America was in 1987-88. It began with the conference Algebras, Lattices, and Logic, which was held in the Asilomar Conference Center, California, July 6–12. Together with Ildikó Sain, we arrived at Berkeley on July 4, we stayed with the Henkins, and traveled with Leon and Ginette by car to Asilomar. On the way, Leon was showing us the attractions of the landscape, for example we stopped to see the place of the seals. This Asilomar conference was one of the many conferences to come that was joint event of the universal algebra and the algebraic logic communities. The title of Richard Thompson's talk was "High deeds in Hungary" (he spent a year before with us in Budapest). Asilomar is on the shore of the Pacific, the ocean is rather cold there. István grew up at lake Balaton,[33] water is his element. He could not help taking a swim in the ocean. Alasdair Urquhart joined him, and there were two people in the cold water. Alasdair came out after a while but István stayed, and Richard was anxiously pacing up and down the shore. When István finally came out, all blue, we began to trot home to get him warmed a bit. Richard was running along with a heavy briefcase in his hands. István spent the rest of the evening in the bathtub, in warm water, until finally he thawed. Asilomar itself was quite a cold place with no sunshine at all. We were dying for some warm sunshine, and Peter Ladkin, participant of the conference, got pity of us. He took the two of us on his small plane, and we flew east till we found a sunny stripe of land. There we landed, bathed in the sunshine for a while, and then got back to cool Asilomar. Following the conference, we spent ten days in Berkeley, then we flew from the west coast of America to the east coast: to Miami. During the two weeks we stayed there, we gave several talks in the Computer Science Department of Florida International University, and most of all we spent as much time on the beach as we could. Here we got plenty of sunshine, sizzling hot sand, warm blue Atlantic ocean, and palm trees.[34]

After visiting the two coasts, we settled in the very middle of America, in Ames Iowa. The purpose of the one-year visit was to work with Roger Maddux, and this was made possible by a two-semester teaching position at Iowa State University. In the first semester, we taught undergraduate and graduate courses in algebra and logic. The university had an excellent library, István spent lots of time in it browsing the books. Here he found, among others, Sullivan's 1979 book Black Holes. This gave an idea, and in the winter break we posted an announcement of an interdisciplinary course "Infinity and the mind", offered jointly with Ildikó Sain to graduate students in mathematics, computer science, physics and philosophy. The topics were "Under-

"*WA*". A consequence is that the free 3-dimensional cylindric algebra is not atomic, solution of a problem from Part II of the Henkin-Monk-Tarski monograph.

[32]The results also lead to answering several open problems stated earlier in the literature by Craig, Jónsson, and Monk.

[33]More precisely, he spent the summers–most important part of the year–at the lake with his grandparents.

[34]The ocean reminded István of how he perceived lake Balaton when he was a small child. Now that he grew up, he needed something bigger than a lake to have the same sensation.

standing and resolution of logical paradoxes recurring in logic itself, black holes, relativity, time travel, infinities, artificial intelligence, foundation of mathematics. Connections between mathematical and physical infinities". There was a huge interest and the course was a big success. We began with the elements of computation theory, arriving at the Church Thesis. When István wanted to argue for the plausibility of this thesis, all of a sudden the idea of relativistic computing occurred to him: one can use a black hole to "slow time infinitely": for an observer approaching the black hole only a finite amount of time may pass while "at home" a computer is computing for an infinity of time. So, we gave as a homework to find a physical device (any gadget the existence of which did not contradict the laws of physics as accepted at that time) which could compute a non-Turing computable task. This was called "the hard-core science fiction homework", and we returned to it, giving more and more clues, till the end of the course.[35] At the end of the semester, the head of the Mathematics Department of ISU invited the three of us to his office and thanked for the successful course.

We lived in Don Pigozzi's house. He was away on a sabbatical, and the house was ideal for the three of us with Ildikó. It was spacious and had a garden. Here was that Hajnal saw the Milky Way first in her life, shining across the night sky with its branches clearly visible.[36] Parallel to teaching, we participated on the algebra seminar of the department, and worked with Roger. Among others, we elaborated the method of splitting for relation algebras having the analogous method for cylindric algebras as a pre-image [121]. That turned out to be a rather useful method for constructing non-representable relation algebras. In the winter break, in January 1988, Hajnal and István gave talks at Florida International University, and this time they stayed at Miami Beach. In Ames, it was so cold that Ildikó's fingers got frozen to the metal mailbox, but we enjoyed hot sun and warm water at the beach. In the spring break, we visited Bjarni Jónsson in Nashville and after the courses ended Ildikó and István visited George Strecker in Manhattan Kansas.

We mentioned that there was a series of algebraic logic and universal algebra conferences around this time. The Asilomar conference in 1987 was followed by the Algebraic Logic and Universal Algebra in Computer Science Conference June 1–4 Ames Iowa, Algebraic Logic Colloquium August 8–14 1988 Budapest Hungary, Conference honoring Don Monk May 29–31 1990 Boulder Colorado, Interconnections between model theory and algebraic logic June 9 1990 Berkeley California, Algebraic Logic Meeting June 10–20 1990 Oakland California, The Jónsson Symposium: a Symposium on Algebras, Lattices, and Logic July 2–6 1990 Laugarvatn Iceland, Algebraic Methods in Logic and in Computer Science Sept 15–Dec. 15 1991 Warsaw Poland.

[35] When we came home to Budapest, the idea of this relativistic computing met some hostility among mathematicians. That is why we only returned to this idea when we were working more deeply in relativity theory and collaborated with physicist Gábor Etesi [188].

[36] Ames is a small town built around the university, in the midst of vast corn-fields. There was no light-pollution there.

Why did algebraic logic attract us so much? Here are some answers.
(1) Its main subject is the structure of concepts of a piece of knowledge (theory). A cylindric algebra is the "content" of a theory, without "form"[37]: we can associate a first-order language to this theory once we select a set of notions/concepts to be "basic" (or "observable" or "primary"). This gives a good ground for treating logical interpretations. We have always been more interested in the semantic aspects "what" as opposed to the syntactic one "how", stemming perhaps from Istváns programming experience. Algebraic logic gives the deepest understanding of the semantic aspects in logic, to our minds.
(2) It is a bridge between algebra and logic, and connections attract us. As a bridge, algebraic logic connects the world of "logics" and that of classes of algebras: to any logic it associates a class of algebras (and vice versa). Then it connects logical properties to algebraic ones by stating theorems of the kind: a logic has property L if and only if the corresponding class of algebras has property A. The fact that usually natural logical and natural algebraic properties correspond to each other vindicates, to our minds, the existence of this bridge. For example, interpolation property of a logic corresponds to the amalgamation property of a class of algebras. Logicians investigated interpolation property as an important logical property, without ever thinking of algebra, and algebraists investigated the amalgamation property in algebra without knowing any connection to logic.
(3) Algebraic logic has things to say about an important aspect of human thinking: abstracting and creating/devising abstract notions. It can model aspects of the methodology of science: the network of scientific theories.
(4) It is complex enough to be intellectually entertaining. A theorem or notion in cylindric algebra always has an algebraic, a geometric and a logical side to it, and it is most satisfying to see all three sides at the same time in parallel.[38] It shows many aspects, facets of one thing: algebra, geometry, set theory, algorithmic issues, and combinatorics.

Papers we wrote about cylindric algebras are [2, 11, 6, 12, 7, 32, 40, 60, 69, 70, 65, 73, 79, 86, 95, 96, 101, 106, 115, 114, 120, 153, 164, 163, 171, 170, 173, 185, 182, 183, 207, 208, 210, 226, 223, 236, 237, 244], papers on relation algebras are [97, 98, 105, 102, 116, 118, 119, 125, 121, 120, 12, 126, 133, 135, 141, 132, 148, 146, 142, 157, 3, 164, 168, 163, 173, 187, 216, 215, 220, 4, 238, 239, 241, 243, 245, 246]. Dynamic and Kleene algebras are algebraic forms of dynamic logic, papers we wrote on dynamic and Kleene algebras are [60, 71, 81, 215, 224].

[37] Using Johan van Benthem's words, it is content without wrapping.

[38] For example, a theorem in algebraic logic most often has a more algebraic presentation, and a more logic-oriented one. To search for extreme two versions satisfies our delight in seeing more than one proofs for the same theorem.

Logic Graduate School, the Amsterdam–Budapest–London Triangle (c. 1991–1998)

A colorful, intense and happy period began for us after we came home from the USA in July 1988. We felt we understood algebraic logic, and we wanted to integrate this beautiful part more into science, we wanted more people to know about it. We opened to new people, new directions, and new activities.

Right after we came home, there was the Algebraic Logic Colloquium in Budapest that we organized with much help from Miklós Ferenczi and György Serény. The plenary talks were given by Wim Blok, Steve Comer, William Craig, Steve Givant, George McNulty, Don Pigozzi, Boris Plotkin, Boris Schein and Richard Thompson. In the proceedings of the conference, that we edited together with Monk, we tried to involve people outside of Tarski's school, too.

It was at this time when our fruitful and lasting relationship with the Symbolic Logic Department of Faculty of Humanities, Eötvös Loránd University began. This department was founded by Imre Ruzsa, a strong mathematician working in philosophical logic, who wanted to stress connection with philosophy. People from the Symbolic Logic Department we met were Anna Madarászné, András Máté, László Pólos, Tibor Szécsényi. We also began cooperations with the History and Methodology of Science Department of Faculty of Science of the same university, led by George Kampis. We met exciting people here, too, besides George Kampis whom we knew already: Miklós Rédei, László Ropolyi, László E. Szabó, and Péter Szegedy. System theory, cybernetics and holistic views on human culture were in the air in this circle of people.

New winds were blowing in Hungary as the soviet empire was disintegrating and the borders were opened toward western countries. After a talk given by us in the Symbolic Logic Department, László Pólos approached us wanting to know if we would be willing to join the work and organization of a new independent university, Corvin University. The idea was attractive since teaching and having students was always part of our lives. Well, organizing a new university, finding money, people, and most of all agreement between people were not easy, and finally the idea died after several years. But in the midst of the organizational efforts, we already run our Logic Department of would-be Corvin University, the Logic Graduate School (LGS).[39] Besides Hungarian students, we had foreign ones mostly from Amsterdam, but we had students from Berkeley, Egypt, Israel and Slovakia, too. There were courses given in English by several colleagues, for example in 1995 one course was given to us by Lajos Soukup about the role of models of set theory in algebraic logic. We had an "eastern style of teaching", the teachers and students of LGS formed a close community, we were like a big family, we spent much time together also outside of classes, we made each other's lives richer. Some students from this period are Viktor Gyuris, Ben Hansen, Eva Hoogland, Ági Kurucz, Judit Madarász, Maarten Marx, Szabolcs Mikulás, Péter Rebrus, Gábor Sági, Tarek Sayed-Ahmed, András Simon.

[39]This was a mentality we learnt from Géza Bogdánfy: the first step in an enterprise is that you make yourself useful.

Besides continuing our good relations with American colleagues and friends, we began to have important European relations, too. Most of all, we met Johan van Benthem and his Amsterdam school, and we met Dov Gabbay and his London school. These two schools were visiting each other regularly to exchange ideas and results. It was our honor that they accepted us, our algebraic logic group, into this cooperation, and so formed the Amsterdam-London-Budapest triangle. The Amsterdam and London groups were strong in modal logic, temporal logic, and traditional model theory of first-order logic, we brought algebraic logic into this triangle.

Modal logic and algebraic logic had been developing separately, largely ignoring each others results and ideas that were often the same in two wrappings. Their meeting was beneficial to both of them. For example, Johan van Benthem in his talk at the 1991 algebraic logic Banach Semester posed several open problems from modal logic that were solved within few days on the spot by using methods from algebraic logic. One of them concerned completeness of the Lambek Calculus for a relational semantics (see [131]). Robin Hirsch and Ian Hodkinson from the London group brought new ideas and new techniques from game theory, they enriched algebraic logic with new directions and many deep theorems. A completely new view of cylindric and relation algebras is contained in their book [Hirsch, R., Hodkinson, I., Relation algebras by games. North-Holland, Amdterdam, 2002]. The influence of algebraic logic on modal logic can be seen in the book [Gabbay, D.M., Kurucz, A., Wolter, F., Zakharyaschev, M., Many-dimensional modal logics: theory and applications. North-Holland, Amsterdam, 2003]. Arrow logic invented by Johan van Benthem and Yde Venema, a logic of transitions, is a kind of unification of modal logic and algebraic logic.

Perhaps the most spectacular result of this cultural meeting is the guarded fragment GF of first-order logic [145, 169]. This is a large subset of first-order logic, with decidable satisfiability problem. The finite-variable hierarchy works well for this fragment, and it is expressive in the sense that many important "derivatives" of guarded formulas can be expressed with a guarded formula. For example, if a k-variable GF formula is preserved by taking submodels, then it is equivalent to a k-variable guarded formula in which only universal quantifiers occur. The GF can be extended with features that are important in computer science, e.g., fixed-point sentences can be added to GF without destroying decidability. The GF is quite popular among computer scientists.

Relativity Theory, Relativistic Computing, Methodology of Science (c. 1998–)

In the evenings, István read science fiction books for relaxing his mind before sleeping. In one evening, he read that relativity theory stemmed from the fact that light was traveling with the same speed for every observer. He was wondering whether we could construct a world, any, in which this was true. By this time, we were quite skillful in constructing models in logic. We switched off the light, and in the dark we began the play of creating worlds. We just wanted to see whether we can imagine an arbitrary model in which light was traveling with the same speed for anyone, we did not care to get a model that was confirmed by physics. In that night, we had the main ideas for this model, and the main ideas for special relativity theory.

This was proof for us that logic was useful in understanding the world around us. István did meet relativity theory in his university studies in the form of Minkowski metric, but this did not leave him with a feeling of understanding. The model we created turned out the same that is used in physics, and we were wondering whether one could construct a different model. Well, after formalizing special relativity in first-order logic, we could derive from the Light axiom together with some auxiliary axioms, that are usually assumed in physics tacitly, the model. This means that this is the only way of creating a world for special relativity. Thus, creating the same model was no coincidence.

Besides continuing previous work on algebraic logic, we turned to exploring relativity theory by using first-order logic (FOL). In our previous work, we looked at logic as a model of human cognition.[40] In our relativity work, we use first-order logic as a tool in this cognition, a tool that helps our cognition. For example, one can derive, informally, from the Light axiom that the clocks of a traveling observer tick slower than ours. Yet, one cannot derive this formally in FOL if one forgets about the Symmetry axiom. After noticing this, we realize: of course!, when we say "slower" it does matter what time-units the observers use for measuring time. One could measure time in seconds, the other in years. So, for comparing their time, we do need to assume that they use the same units for measuring time, and the Symmetry axiom ensures this.

In an axiomatic approach, one is forced to use concrete formulations of statements when proving them. Often, a statement has more than one formulations each of which expresses the same intuitive content, yet these formulations may not be provably equivalent.[41] In an informal argument, we may think of one formulation at one part of the argument, and we may think of another, non-equivalent, formulation at another part of the argument. This way, it is easy to arrive at false conclusions. We believe that this necessity of using concrete formulations in a formal axiomatic approach is a huge help for the intellect, maybe the most important benefit.

Thus, a first-order logic approach forces one to be very explicit in formulating the statements, using concrete definitions and formulating each axiom, important ones as well as "book-keeping ones" that one often uses tacitly in informal arguments. Once one has a theory put in this form, it can be used for many purposes. One avail of using such formalized theories is in teaching. Another one is in moving against compartmentalization of science. When Vienna Circle wanted to formalize science in

[40] We explain this view in [139].

[41] For example, one may express that two stationary observers/coordinate-systems (who agree in the event happened at their origin, i.e., at point $(0, 0, 0, 0)$) use the same time-units by saying that they "see" the same event at point $(1, 0, 0, 0)$ their time-unit as well. Another formulation is to say that they "see" the same events at each point of their time-axes. The two formulations are not equivalent, and have different consequences.

this way, first-order logic was not available yet. In fact, we believe that mathematical logic developed thanks to the efforts of the Vienna Circle, in great part. When one wants to formalize a larger chunk of science or knowledge, it is important to introduce structure to this formalization. Breaking up a big axiom system to many small ones and indicating their interconnections with interpretations between them is one natural way of structuring a complex theory.[42] Using small and well-understood theories is important in foundational thinking, too, see [Friedman, H., On foundational thinking 1. FOM (Foundations of Mathematics) Posting, Archives www.cs.nyu.edu, January 20, 2004].

In short, in an axiomatization, one uses a network of theories in place of just one huge theory. The individual theories in such a network may have different statuses: recording facts about the real world, explaining the "meaning" of a definition, or theorizing about what would-have-been if we found different facts. We believe that Hilbert's 6th problem about formalizing physics can be solved in this complex manner. Further, our faith is that one can make steps toward the original dream of the Vienna Circle about unity of science via using this kind of axiomatic approach. We began doing this for relativity theory, the first steps are available in [191].

We are now testing usefulness of algebraic logic by applying its methods and theorems in relativity theory. When we were already working deep in algebraic logic, Leon Henkin suggested us to apply our knowledge to some real-life scientific theory, e.g., describe its concept-algebra. At that time, this task seemed to us futuristic. But now, we are doing this with Judit Madarász and Gergely Székely. Donald Monk also suggests this task in [Monk, J.D., An introduction to cylindric set algebras, IGPL, 2000, p. 455]. The concept-algebra of a theory or model is the structure of (first-order logic) definable relations in it. The structure is given by the logical connectives. Thus, a concept algebra is a Boolean algebra together with some additional operators. We began to investigate the concept algebra of special relativity. We completely described the structure of unary and binary definable relations, these are finite. The structure of ternary definable relations is already infinite, but it is atomic. With some differences, the same is true for the concept algebra of classical Newtonian spacetime. However, there are no homomorphisms between the two algebras. For details, see [247].

We are also working on a connection between relativity theory and our original subject: computers, computability. This is elaborating the idea that István had in Ames 1988, that is, using relativistic spacetime to devise a "computer" that can compute non-Turing-computable tasks. The same idea was discovered about the same time independently by other researchers, too, and relativistic computation became one branch of unconventional computation.

[42] [Burstall, R., Goguen, J.: Putting theories together to make specifications. In: Proc. IJCAI'77 (Proceedings of the 5th International Joint Conference on Artificial Intelligence, Vol 2, pp. 1045–1058] is about using this method in computation, while [Konev, B., Lutz, C., Ponomaryov, D., Wolter, F.: Decomposing description logic ontologies. In: Proceedings of 12th Conf. on the Principles of Knowledge Representation and Reasoning, Association for the Advancement of Artificial Intelligence, 2010. pp. 236–246] is about the need of using this method in real-life computer applications.

Judit Madarász, Péter Németi and Gergely Székely joined the relativity theoretical research at various points. They are now furthering this project with enthusiasm and energy, they contribute new ideas, directions, and interesting new results.

Our papers in relativistic computing are [188, 197, 200, 201, 202, 209, 211, 212, 219, 242]. Our papers about relativity theory in general are [175, 192, 194, 195, 199, 198, 203, 204, 206, 205, 211, 213, 214, 217, 218, 221, 225, 229, 228, 230].

Appendix B
Joint Annotated Bibliography of Hajnal Andréka and István Németi

Számológép was the journal of NIMIGÜSZI, publisher: the director of NIMIGÜSZI, Budapest, editor: Péter Ihrig. It published mostly papers in Hungarian in connection with programming, computers.

CL&CL was an international journal (ISBN 963 311 039 4) published in Budapest. Its full name was Computational Linguistics and Computer Languages.

Books

[1] *Generalization of the concept of variety and quasi-variety to partial algebras through category theory.* **Dissertationes Mathematicae (Rozprawy Math.)** No. 204. PWN - Polish Scientific Publishers, Warsaw, 1983. 51 p. Andréka, H. and Németi, I.

[2] *Cylindric Set Algebras.* **Lecture Notes in Mathematics** Vol 883, Springer-Verlag, Berlin, 1981. vi+323 p. Henkin, L., Monk, J. D., Tarski, Andréka, H. and Németi, I.

[3] *Decision problems for equational theories of relation algebras.* **Memoirs of Amer. Math. Soc.** Vol. 126, No. 604, American Mathematical Society, Providence, Rhode Island, 1997. xiv+126 p. Andréka, H., Givant, S. and Németi, I.

[4] *Simple Relation Algebras.* Springer International Publishing AG, 2017. xxiv+622 p. Givant, S. and Andréka, H.
Each algebra is a subdirect product of subdirectly irreducible algebras, and the standard method of studying an algebra is by decomposing it to its subdirectly irreducible factors and then reassemble the algebra from its factors. Subdirectly irreducible relation algebras are the same as simple ones (simple means having no nontrivial congruences). In this book we analyze simple relation algebras by cutting them into pieces along an arbitrary equivalence relation, and also by constructing simple relation algebras from any others.

J. Madarász and G. Székely (eds.), *Hajnal Andréka and István Németi on Unity of Science*, Outstanding Contributions to Logic 19,
https://doi.org/10.1007/978-3-030-64187-0

[5] *Universal Algebraic Logic. (Dedicated to the Unity of Science).* Birkhauser, in preparation. Andréka, H., Gyenis, Z., Németi, I. and Sain, I.

Books Edited

[6] *Algebraic Logic.* **Colloq. Math. Soc. J. Bolyai** Vol. 54, North-Holland, Amsterdam, 1991. vi+746 p. Editors: Andréka, H., Monk, J. D. and Németi, I.

[7] *Cylindric-like algebras and algebraic logic.* **Bolyai Society Mathematical Studies** Vol. 22, Springer Verlag, Berlin, 2012. 478 p. Editors: Andréka, H., Ferenczi, M. and Németi, I.

Dissertations

[8] *Algebraic investigation of first order logic.* (In Hungarian) **Doctoral Dissertation with Eötvös Loránd University**, Budapest, 1973. 162 p. Andréka, H.

[9] *Extending the universal algebraic notions of variety and related ones to partial algebras using abstract model theory and category theory.* (In Hungarian) **Candidate's Dissertation with the Hungarian Academy of Sciences**, Budapest, 1976. 171 p. Németi, I.

A great portion of this is published as [87]. This was the starting point of later investigations in partial algebra theory and the injectivity approach to category theoretic logic.

[10] *Universal algebraic investigations in algebraic logic.* (In Hungarian) **Dissertation for Candidate's degree with the Hungarian Academy of Sciences**, Budapest, 1977. 199 p. Andréka, H.

[11] *Free algebras and decidability in algebraic logic.* (In Hungarian) **Doctoral Dissertation with the Hungarian Academy of Sciences**, Budapest, 1986. xviii+169 p. Németi, I.

Parts of this are published as [125, 153, 160, 223].

[12] *Complexity of equations valid in algebras of relations.* **Doctoral Dissertation with the Hungarian Academy of Sciences**, Budapest, 1991. 103 p. Andréka, H.

This is published as [163].

Publications (Articles, Book Chapters, Other)

1970 – 1974

[13] *Hierarchic partition of large scale systems and its application for power system study.* **Acta Technica** 71 (1971), pp. 285–303. Bogdánfy, G. and Németi, I.

[14] *Debugging large and at the same time complex systems of programs.* (In Hungarian) **Számológép** 71/3 (1971), 59–69. Németi, I.

[15] *Computer program performing hierarchic decomposition.* (In Hungarian) **Számológép** 72/1 (1972), 18pp Németi, I.

[16] *Pattern recognition program for the computer EMG 830-20.* (In Hungarian) **Számológép** 72/2 (1972), 85–93. Németi, I. and Baksza, L.

[17] *Software foundations for computer simulation of biological systems.* **IVth International Biophysics Congress Moscow**, 1972. EXXIIa2/9, EXXIIa2/10. Eöry, A. and Németi, I.

[18] *Logical foundations for the formalization and application of general system theory.* (In Hungarian) In: **System Theory Research** (Rendszerkutatás). Publisher for Economics and Law (Közgazdasági és Jogi Könyvkiadó), Budapest 1973. pp. 307–357. Gergely, T. and Németi, I.

[19] *Notes on maximal congruence relations, automata and related topics.* **Acta Cybernetica** Tom 2, Fasc 1 (Szeged 1973), pp. 71–88. Andréka, H., Horváth, S. and Németi, I.

[20] *On the equivalence of sets definable by satisfaction and ultrafilters.* **Studia Sci. Math. Hungar.** 8 (1973), pp. 463–467. Andréka, H. and Németi, I.

[21] *A theorem on the semantics of first-order predicate logic.* (In Hungarian) **Számológép** 73/1 (1973), 168–175. Andréka, H. and Németi, I.

[22] *On the minimal language for defining Boolean algebras.* (In Hungarian) **Számológép** 73/1 (1973), 25–40. Andréka, H., Németi, I. and Paizs, K.

[23] *Application of cylindric algebras to data structures (news).* (In Hungarian) **Számológép** 73/2 (1973), p. 28. Andréka, H. and Németi, I.

[24] *An algebraic introduction to logic.* (In Hungarian) **Számológép Kiskönyvtár** 73 (1973), 27–64. Andréka, H., Farkas, Zs. and Németi, I.

[25] *Program writing and program verifying programs.* (In Hungarian) **Számológép Kiskönyvtár** 73 (1973), 86–99. Andréka, H. and Németi, I.

[26] *Subalgebra systems of algebras with finite and infinite, regular and singular arities.* **Annales Univ. Budapest. Eötvös Sec. Math.** 17 (1974), pp. 103–118. Andréka, H. and Németi, I.

[27] *Sufficient and necessary condition for the completeness of a calculus.* **Zeitschr. Math. Logic u. Grundl. Math.** Bd 20 (1974), pp. 433–434. Andréka, H., Gergely, T. and Németi, I.

[28] *On some questions of n–th order logic.* (In Russian) **Kibernyetyika** 74/5, 74/6 (Kijev 1974), pp. 61–67, 77–83. Andréka, H., Gergely, T. and Németi, I.

[29] *Plans to improve our semi-automatic programverifier system.* (In Hungarian) Research Report of the Institute NIM IGÜSZI - OSZI - KSH, Budapest, August 1974. Andréka, H., Balogh, K., Lábodi, K., Németi, I. and Tóth, P.

1975 – 1979

[30] *On the role of general system theory in the cognitive process.* In: **Progress in Cybernetics and System Research**, Vol 2. Hemisphere Publishing Corporation, 1975. pp. 137–150. Gergely, T. and Németi, I.

[31] *Logical foundations for a general theory of systems.* **Acta Cybernetica** Tom 2, Fasc 3 (Szeged 1975), pp. 261–276. Gergely, T. and Németi, I.

[32] *A simple, purely algebraic proof of the completeness of some first order logics.* **Algebra Universalis** 5 (1975), pp. 8–15. Andréka, H. and Németi, I.

[33] *Many–sorted languages and their connection with higher order languages.* (In Russian) **Kibernyetyika** 75,4 (Kijev 1975), pp. 86–92. Andréka, H., Gergely, T. and Németi, I.

[34] *On some questions of higher order logic.* (In Hungarian) **Matematikai Lapok** 24 (1975), pp. 63–94. Andréka, H., Gergely, T. and Németi, I.

[35] *Remarks on free products in regular varieties and sink–complemented subalgebras.* **Studia Sci. Math. Hung.** 10 (1975), pp. 23–31, Andréka, H. and Németi, I.

[36] *Application of universal algebra in computer theory.* (In Hungarian) **Számológép Kiskönyvtár** 75(1975), 145–152. Andréka, H. and Németi, I.

[37] *On a property of the category of partial algebras.* **CL&CL** Vol XI (1976), pp. 5–10. Németi, I.

[38] *On a proof of Shelah.* **Bulletin de l'Academie Polonaise des Sciences (Series Math.)** 27 (1976), pp. 1–7. Andréka, H., Dahn, B. I. and Németi, I.

[39] *On the adequateness of predicate logic programming.* **AISB European Newsletter** Issue 23 (1976), pp. 30–32. Andréka, H. and Németi, I.

[40] *On universal algebraic construction of logics.* **Studia Logica** 36,1–2 (1977), pp. 9–47. Andréka, H., Gergely, T. and Németi, I.

[41] *On the congruence lattice of pseudosimple algebras.* In: **Contributions to Universal Algebra** (Proc. Coll. Szeged 1975), Colloq. Math. Soc. J. Bolyai Vol 17, North–Holland, Amsterdam, 1977. pp. 15–20. Andréka, H. and Németi, I.

[42] *The generalised completeness of Horn predicate logic as a programming language.* **Acta Cybernetica** Tom 4, Fasc 1 (Szeged 1978), pp. 3–10. Andréka, H. and Németi, I.

[43] *Łos lemma holds in every category.* **Studia Sci. Math. Hungar.** 13 (1978), pp. 361–376. Andréka, H. and Németi, I.

[44] *From hereditary classes to varieties in abstract model theory and partial algebras.* **Beiträge zur Algebra und Geometrie** 7 (1978), pp. 69–78. Németi, I.

[45] *Neat reducts of varieties.* **Studia Sci. Math. Hungar.** 13 (1978), pp. 47–51. Andréka, H. and Németi, I.

[46] *Completeness of Floyd logic.* **Bulletin of the Section of Logic** 7/3 (1978), pp. 115–120. Andréka, H. and Németi, I.

The Floyd-Hoare method for proving program correctness is complete with respect to the so-called continuous-traces semantics. It is not complete with respect to the standard-traces semantics of programs. Full proofs are contained in [50].

[47] *On universal algebraic logic and cylindric algebras.* **Bulletin of the Section of Logic** 7/4 (1978), pp. 152–158. Andréka, H. and Németi, I.

[48] *Injectivity in categories to represent all first order formulas.* **Demonstratio Mathematica** 12 (1979), pp. 717–732, Andréka, H. and Németi, I.

[49] *Formulas and ultraproducts in categories.* **Beiträge zur Algebra und Geometrie** 8 (1979), pp. 133–151. Andréka, H. and Németi, I.

[50] *Completeness problems in verification of programs and program schemes.* In: **Mathematical Foundations of Computer Science'79** (Proc. Conf. Olomouc Czechoslovakia 1979). Ed: Becvar, J. Lecture Notes in Computer Science Vol 74, Springer–Verlag, Berlin, 1979. pp. 208–218. Andréka, H., Németi, I. and Sain, I.

Thm.1 states a strong incompleteness property for program correctness statements with the standard semantics: there is no recursive set containing the "we must prove these program correctness statements" and contained in the standard -valid program correctness statements. Thm.2. states that the reason for this incompleteness is in the definition of standard semantics for program correctness statements: this standard semantics is not stable in Zermelo-Fraenkel Set Theory. These theorems justify the Henkin style semantics for dynamic logic introduced in the second part of the paper, for which Thm.3 states strong completeness.

[51] *Henkin-type semantics for program schemes to turn negative results to positive.* In: **Fundamentals of Computation Theory'79** (Proc. Conf. Berlin 1979). Ed: L. Budach, Akademie Verlag, Berlin, 1979. Band 2., pp. 18–24. Andréka, H., Németi, I. and Sain, I.

[52] *Reduced products in categories.* In: **Contributions to General Algebra** (Proc. Conf. Klagenfurt 1978) Verlag Johannes Heyn, 1979. pp. 25–45. Andréka, H., Makai, E., Márki, L. and Németi, I.

[53] *Applications of universal algebra, model theory, and categories in computer science.* (Survey and bibliography) **CL&CL** Vol XIII (1979), pp. 251–282. Andréka, H. and Németi, I.

[54] *Program verification within and without logic.* **Bulletin of the Section of Logic** 8/3 (1979), pp. 124–129. Andréka, H., Németi, I. and Sain, I.

This paper is an abstract for the first part of [50].

[55] *Not all representable cylindric algebras are neat reducts.* **Bulletin of the Section of Logic** 8/3 (1979), pp. 145–147. Andréka, H. and Németi, I.

[56] *Dimension-restricted free cylindric algebras and finitary logic of infinitary relations.* **Journal of Symbolic Logic** 44 (1979), p. 442. Andréka, H. and Gergely, T.

1980–1984

[57] *Does SPK $\supseteq$ PSK imply axiom of choice?.* **Comm. Math. Univ. Carolinae.** 21,4 (1980), pp. 699–706. Andréka, H. and Németi, I.

"SPK contains PSK for all classes K of algebras" holds in Zermelo–Fraenkel set theory if PK is understood as "the class of algebras isomorphic to direct products of elements of K", but the same statement implies the axiom of choice if PK is understood without isomorphism. Each of IP being a closure operator and HP being a closure operator implies the axiom of choice. These give partial answers to Problem 28 in Gratzer's 1979 Universal Algebra book.

[58] *On systems of varieties definable by schemes of equations.* **Algebra Universalis** 11 (1980), pp. 105–116. Andréka, H. and Németi, I.

[59] *Additions to survey of applications of universal algebra, model theory, and categories in computer science.* **CL&CL** Vol XIV (1980), pp. 7–20. Andréka, H. and Németi, I.

[60] *Some constructions of cylindric algebra theory applied to dynamic algebras of programs.* **CL&CL** Vol XIV (1980), pp. 43–65. Németi, I.

[61] *Model theoretical semantics for many–purpose languages and language hierarchies.* In: **Computational Linguistics** (Proc. 8th Int. Conf. Tokyo 1980) Tokyo, 1980. pp. 213–219. Andréka, H., Gergely, T. and Németi, I.

[62] *Quasi equational logic of partial algebras.* **Bulletin of the Section of Logic** 9/4 (1980), pp. 193–198. Andréka, H., Burmeister, P. and Németi, I.

[63] *On problems in cylindric algebra theory.* **Abstracts of Amer. Math. Soc.** 1 (1980), p.588. Andréka, H. and Németi, I.

[64] *Quasivarieties of partial algebras – a unifying approach towards a two–valued model theory for partial algebras.* **Studia Sci. Math. Hungar.** 16 (1981), pp. 325–372. Andréka, H., Burmeister, P. and Németi, I.

[65] *Dimension complemented and locally finite dimensional cylindric algebras are elementarily equivalent.* **Algebra Universalis** 13 (1981), pp. 157–163. Andréka, H. and Németi, I.

[66] *HSPK is an equational class, without the axiom of choice.* **Algebra Universalis** 13 (1981), pp. 164–166. Andréka, H. and Németi, I.

Birkhoff's variety theorem "HSP=ModEq" is proved in Zermelo-Fraenkel set theory without the axiom of choice. This answers Problem 31 in Grätzer's 1968 Universal Algebra book in the negative.

[67] *Similarity types, pseudosimple algebras, and congruence representation of chains.* **Algebra Universalis** 13 (1981), pp. 293–306. Andréka, H. and Németi, I.

[68] *On cylindric-relativized set algebras.* In: **Cylindric Set Algebras**, Lecture Notes in Mathematics vol 883, Springer-Verlag, Berlin Heidelberg New York, 1981, pp. 131–315. Andréka, H. and Németi, I.

[69] *Connections between algebraic logic and initial algebra semantics of CF languages.* In: **Mathematical Logic in Computer Science** (Proc. Coll. Salgótarján 1978). Eds: Dömölki, B. and Gergely, T., Colloq. Math. Soc. J. Bolyai Vol 26, North-Holland, Amsterdam, 1981, pp. 25–83. Andréka, H. and Sain, I.

[70] *Connections between cylindric algebras and initial algebra semantics of CF languages.* In: **Mathematical Logic in Computer Science** (Proc. Coll. Salgótarján 1978). Eds.: Dömölki, B. and Gergely, T., Colloq. Math. Soc. J. Bolyai Vol 26, North–Holland, Amsterdam, 1981. pp. 561–605. Németi, I.

[71] *Dynamic algebras of programs.* In: **Fundamentals of Computation Theory'81** (Proc. Conf. Szeged 1981). Ed: Gécseg, F. Lecture Notes in Computer Science Vol 117, Springer–Verlag, Berlin, 1981. pp. 281–290. Németi, I.

[72] *Some universal algebraic and model theoretic results in computer science.* In: **Fundamentals of Computation Theory'81** (Proc. Conf. Szeged 1981). Ed: Gécseg,

F. Lecture Notes in Computer Science Vol 117, Springer–Verlag, Berlin, 1981, pp. 16–23. Andréka, H. and Németi, I.

[73] *Which finite cylindric algebras are generated by a single element?.* In: **Finite Algebra and Multiple–valued Logic** (Proc. Coll. Szeged 1979). Colloq. Math. Soc. J. Bolyai Vol 28, North–Holland, Amsterdam, 1981. pp. 23–39. Andréka, H. and Németi, I.

[74] *A characterization of Floyd provable programs.* In: **Mathematical Foundations of Computer Science'81** (Proc. Conf. Strbské Pleso, Czechoslovakia 1981). Eds.: Gruska, J. and Chytil, M. Lecture Notes in Computer Science Vol 118, Springer–Verlag, Berlin, 1981. pp. 162–171. Andréka, H., Németi, I. and Sain, I.

An explicit characterization of the information content of the Floyd programverification method is given, as follows. A run (sequence of intensions of the registers) in perhaps nonstandard time of a program is called continuous if it satisfies induction over all possible dynamic logic formulas containing one free time variable. Thm.1: Assume that T is a theory about data containing the Peano's Axioms. A program p is correct for output formula ψ wrt each continuous runs in each model of T if and only if p can be proved correct wrt ψ by the Floyd inductive assertion method. Detailed proof is given.

[75] *A general axiomatizability theorem formulated in terms of cone–injective subcategories.* In: **Universal Algebra** (Proc. Coll. Esztergom 1977). Colloq. Math. Soc. J. Bolyai Vol 29, North–Holland, Amsterdam, 1981. pp. 13–35. Andréka, H. and Németi, I.

[76] *Cone–implicational subcategories and some Birkhoff–type theorems.* In: **Universal Algebra** (Proc. Coll. Esztergom 1977). Colloq. Math. Soc. J. Bolyai Vol 29, North–Holland, Amsterdam, 1981. pp. 535–578. Németi, I. and Sain, I.

[77] *Qualitative mathematics (In Hungarian).* **Magyar Tudomány** 1983/2, (1983), pp. 99–103. Andréka, H. and Németi, I.

[78] *Problems with the category theoretic notions of ultraproducts.* **Bulletin of the Section of Logic** 10,3 (1981), pp. 122–127. Buy, H. H. (Bui Huy Hien), and Németi, I.

[79] *Foundations for stepwise refinement of program specifications via cylindric algebra theory.* **Diagrammes** 8,1 (1982), pp. N1–N24. Németi, I.

[80] *On notions of factorization systems and their applications to cone–injective subcategories.* **Periodica Math. Hungar.** 13,3 (1982), pp. 229–235. Németi, I.

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Part I: Nonstandard Dynamic Logic (NDL) is introduced, by replacing standard semantics of First-order Dynamic Logic (FoDL) with an explicit time semantics. NDL is then proved to be strongly complete with respect to a proof concept $\vdash N$ (Thm.2, detailed proof, translating NDL into FOL). It is known that FoDL is rather wild, e.g., its validities are not recursively enumerable, hence the move to explicit time semantics was necessary to obtain the above completeness theorem (analogous to Henkin's Nonstandard Second-order Logic). The proof system $\vdash N$ is a new rather strong method for proving properties of programs (see Part II of the paper).
Part II: NDL, introduced in Part I, is shown to be useful for reasoning about programs and for characterizing the information contents of known program proving methods. First, natural properties of programs are proved under natural axioms about time. (Thm.3,4) Termination of the "count-down

program" is proved by using the order axioms on time, induction on data, and comprehension on intensions (Thm.7). The Naur-Floyd-Hoare inductive assertions method for proving correctness of programs $\vdash F$ is introduced (Sect. 6). Thm.9 is a semantic characterization, in terms of statements in NDL, of the information implicitly contained in the Floyd-method. This information content is time-induction over quantifier-free formulas. If in addition we can reason about ordering in time, our program-proving ability does not increase, we do not go beyond the power of Floyd's method. But if we can perform addition on time, or if we can quantify over time points in the induction, our reasoning ability is beyond the power of Floyd's method. Note that quantifying over time is roughly the same as using time-modalities. However, if our theory for the data contains the Peano axioms, full induction over time does not lend more reasoning power than the Floyd method (Thm.11). The proof concept $\vdash N$ is strictly stronger than Floyd's method (Thm.10). slightly updated

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NDL, and a lattice of program verifying methods are introduced. In this lattice, comparison of theories in NDL are based on their strengths for proving partial correctness of programs. Figure 2 contains many statements about this lattice. Thm.6: $Ia + Ts$ is strictly weaker than $Ia + To$. Intuitively: When using full induction Ia over time, it does matter whether we can compare time instances by "later than" relation, or we just have successor on time. (Used together with quantifier-free induction over time in place of full induction, this does not matter.) Thm.6 is proved in detail with figures.

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The main result is that induction over formulas containing a single universal quantifier of sort time is already strictly stronger than Floyd's method, for proving partial correctness of programs (Thm.2). It was known that Floyd's method is equivalent to induction over formulas not containing quantifiers of sort time ([82], Thm.9). The case for induction over formulas containing a single existential quantifier is left open. This paper is a continuation of [84].

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