Appendix
Equations of Motion

This appendix provides equations of motion which are relevant to the topic of this thesis. Although these equations are not directly used along the thesis, they give insight on the motion of particles under different fluid conditions.

The evaluation of the forces acting on a spherical particle arise the BBO equation, which can be expressed as,

$$\rho_p V_p \frac{dU_p}{dt} = 6\pi a\mu(U_f - U_p) + V_p(\rho_p - \rho_f)g + \rho_f V_p \frac{dU_f}{dt} + \frac{1}{2}\rho_f V_p \frac{d}{dt}(U_f - U_p) - 6\pi a^2\mu \int_0^t \left(\frac{(d/d\tau)(U_f - U_p)}{(\pi\mu(t-\tau)/\rho_f)^{1/2}} \right) d\tau \tag{A.1}$$

The right hand side terms of the former equation correspond to the drag force, gravity force, pressure gradient force, virtual mass force and Basset history force.

The Tchen equation is defined as:

$$\frac{dU_p}{dt} + aU_p + c\int_{-\infty}^{t} \frac{dU_p}{dt}(t-\tau)^{-1/2} d\tau = aU_f + b\frac{dU_f}{dt} + c\int_{-\infty}^{t} \frac{dU_p}{dt}(t-\tau)^{-1/2} d\tau - \frac{2(s-1)}{2s+1}g \tag{A.2}$$

in which:

$$a = \frac{18\nu}{(s+1/2)d^2} \tag{A.3}$$

$$b = \frac{3}{2(s+1/2)} \tag{A.4}$$

$$c = \frac{9(\nu/\pi)^{1/2}}{(s+1/2)d} \tag{A.5}$$

L. Blay Esteban, *Dynamics of Non-Spherical Particles in Turbulence*,
Springer Theses, https://doi.org/10.1007/978-3-030-28136-6

$$s = \frac{\rho_p}{\rho_f} \tag{A.6}$$

and

$$\frac{d}{dt} = \frac{\delta}{\delta t} + U_j \frac{\delta}{\delta x_j} \tag{A.7}$$

Then, the temporal derivative is evaluated along the discrete particle trajectory. The diameter of the particle is d, while ρ_p and ρ_f are the density of the particle and the fluid respectively, U_p and U_f are the velocity of the particle and of the fluid respectively and g is the gravity.

The extra force term that comes from the integration of the normal stress on the sphere surface reads as:

$$F_{press} = \frac{\pi d^3}{6} \rho_f \frac{dU_f}{dt} \tag{A.8}$$

and is incorporated in the term bdU_f/dt of Eq. A.3

On the other hand, the modified Riley equation is given as:

$$\rho_p \frac{\pi d^3}{6} \frac{dU_p}{dt} = -\frac{\pi d^2}{8} \rho_f C_D (U_f - U_p)|U_f - U_p| - \rho_f \frac{\pi d^3}{6} C_A \frac{d(U_p - U_f)}{dt} + \frac{\pi d^3}{6} (\rho_p - \rho_f) g$$
$$+ \rho_f \frac{\pi d^3}{6} \frac{DU_f}{Dt} - \frac{\pi d^2}{4} C_H \frac{\sqrt{\rho_f \mu}}{\pi} \int_{-\infty}^{t} \frac{d(U_p - U_f)}{dt} (t - \tau)^{-1/2} d\tau \tag{A.9}$$

in which the following terms are corrected by means of empirical correlations to extent the applicability of the equation to larger particulate Reynolds numbers:

$$C_D = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}) Re_p \leq 200 \tag{A.10}$$

$$C_A = 1.05 - 0.0066 / \left(A_C^2 + 0.12 \right) \tag{A.11}$$

$$C_H = 2.86 - 3.12 / \left(A_C^2 + 1 \right)^3 \tag{A.12}$$

$$A_C = \frac{|U_p - U_f|^2}{d \left| \frac{d(U_p - U_f)}{dt} \right|} A_C \leq 60 \tag{A.13}$$

with the particulate Reynolds number is

$$Re_p = \frac{|U_p - U_f| d}{\nu} \tag{A.14}$$

and

$$\frac{D}{Dt} = \frac{\delta}{\delta t} + U_j \frac{\delta}{\delta x_j} \tag{A.15}$$

meaning that the extra pressure gradient term is evaluated along the fluid motion instead of along the particle trajectory.