Appendices

A

Taxonomy of Iterative Learning Control Literature

A.1 Taxonomy

In Appendix A, we categorize the ILC literature into two different parts in order to highlight the overall trends in ILC research. The first part is related to the publications that focus on ILC applications and the second part is related to the publications that consider theoretical developments. However, it is in fact very difficult to separate the literature into these two broad groupings, thus the categorizations given in this appendix are based on authors' subjective decisions. Also, note that for this categorization effort, the first author read the abstracts of all the papers indicated. If it was possible to understand the main approach and idea of the paper from the abstract, then the paper was categorized on that basis. However, when the author could not understand how to categorize a paper from its abstract, then other parts of the paper were read in order to decide on the correct category for the paper. As already commented, however, this categorization is still based on subjective decisions and is not a technical development. Also, as mentioned in Chapter 2, the literature search for our taxonomy covers only the publications between 1998 and 2004, because the literature before 1998 was surveyed and classified in [299].

A.2 Literature Related to ILC Applications

In [299], ILC literature dealing with applications was categorized as "robotics" and "applications." In "robotics," detailed categories were given as "elastic joints," "flexible links," "cartesian coordinates," "neural networks," "cooperating manipulators," "hybrid control," and "nonholonomic." In applications, detailed categories were given as "vehicles," "chemical processing," "mechanical/manufacturing systems," "nuclear reactor," "robotics demonstrations," and "miscellaneous." In this section, we began by trying to follow the above categories, but found it difficult to restrict all the publications between 1998 and 2004 into the categories given in Table 4.2 of [299]. Thus, we made more detailed categories, including the topics "Robots," "Rotary systems," "Batch/factory/chemical processes," "Bio/artificial muscle," "Actuators," "Semiconductors," "Power electronics" and "Miscellaneous," and in each category we provide further sub-categories.

A.2.1 Robots

In [299], robotics was the most active area of ILC application. Since 1998, this continues to be the case. Robotic applications of ILC have included:

- General robotic applications, including rigid manipulators and flexible manipulators [109, 166, 449, 408, 175, 329, 222, 422, 448, 21, 223, 36, 213, 533, 203, 542, 541, 174, 555, 109, 225, 319, 204, 240, 95].
- Mechatronics design [462, 487].
- Robot applications with adaptive learning [423].
- Robot applications with Kalman filters [323]
- Impedance matching in robotics [316, 472, 49, 32].
- Table tennis [289].
- Underwater robots [403, 404, 234].
- Acrobat robots [535, 483].
- Cutting robots [224].
- Mobile robots [304, 71].
- Gantry robots [183].

A.2.2 Rotary Systems

Rotational motion is generally disturbed by position-dependent or time-periodic external disturbances. Thus, control of rotary systems is a good candidate for ILC applications. Papers related to this area include:

- The vibration suppression of rotating machinery [266].
- Switched reluctance motors (SRM) [402, 401, 399, 400].
- Permanent-magnet synchronous motors (PMSM) [250, 500, 373, 262, 375, 249, 499, 374].
- Linear motors [441].
- (Ultrasonic) induction motors [398, 285, 284].
- AC servo motors [407].
- Electrostrictive servo motors [206].

A.2.3 Batch/Factory/Chemical process

The number of ILC applications in process control has increased significantly since 1998. The literature includes:

- Tracking control of product quality in agile batch manufacturing processes [493, 256].
- Chemical reactors [294, 293, 264].
- Water heating systems [497].
- Laser cutting [459].
- Chemical processes [151, 495, 45, 189, 445].
- Batch processes [494, 496, 88].
- Industrial extruder plants [352].
- Problem of a moving liquid container [170, 383].

A.2.4 Bio/Artificial Muscle

Bioengineering and biomedical applications are not yet a popular ILC application area, but slowly the number of applications is increasing, as evidenced by the following:

- Biomedical applications such as dental implants [211, 212].
- FNS applications [117, 489].
- Human operators [14].
- Artificial muscle [205].
- Pneumatic systems [53].
- Biomaterial applications [469].

A.2.5 Actuators

ILC applications to non-robotic/non-motor actuators are closely related to the mechanical hard-nonlinearity compensation problem. Related publications are as follows:

- A proportional-valve-controlled hydraulic cylinder system [50].
- Electromechanical valves [198, 361, 199].
- The hysteresis problem of a piezoelectric actuator [280].
- Linear actuators [261, 444].

A.2.6 Semiconductor

It is quite interesting to see that ILC is widely applied in the semiconductor production process. Between 2001 and 2003, the following literature was published in semiconductor applications of ILC: [537, 255, 94, 113, 89, 112, 87, 392, 111, 110]. For a more detailed discussion of the application of ILC to semiconductor manufacturing processes, refer to [114].

A.2.7 Power Electronics

Examples of ILC applications to electrical power systems can be found in the following:

- Electronic/industrial power systems [550, 549, 551, 478].
- Inverters [2, 41].

A.2.8 Miscellaneous

Many miscellaneous applications of ILC are described in the following papers:

- Traffic [201].
- Magnetic bearings [101, 76].
- Aerospace [67, 73].
- Linear accelerators [248].
- Dynamic load simulators [477].
- Hard disk drives [512, 513].
- Temperature uniformity control [254, 292, 258].
- Visual tracking [274].
- Quantum mechanical systems [364].
- Piezoelectric tube scanners [197].
- Smart microwave tubes [1, 405].

Figure A.1 plots the number of papers focused on the use of ILC in applications. As shown in this figure, ILC has most-dominantly been applied to the area of robotics. However, notably, ILC has also been widely used in rotational motion control systems, in the process control industry, and for semiconductor manufacturing processes.

We also note that to check the practical uses of ILC, we searched United States patent abstracts using the keywords "Iterative" AND "Learning."[1] From this search, we found ILC-related patents in motor control [371], process control [168], disk drive control [72], and network communication [230].

A.3 Literature Related to ILC Theories

Since the spectrum of the theoretical developments is so broad and individual papers often treat several different topics, assigning a given paper to a specific category can be quite subjective. The approach in this monograph is to try to separate papers that considered ILC as a specific topic from those that connected ILC analysis with other control theory topics. The general categories are defined as "General (Structure)," "General (Update Rules)," "Typical ILC Problems," "Robustness," "Optimal and Optimization," "Adaptive," "Fuzzy and Neural," "Mechanical Nonlinearity Compensation," "ILC

[1] http://patft.uspto.gov/netahtml/search-bool.html

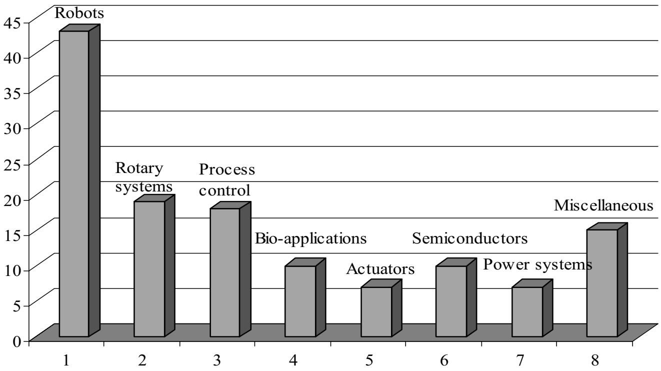

Fig. A.1. Publication numbers for the application-focused ILC literature

for Other Repetitive Systems and Control Schemes," and "Miscellaneous." The first three categories are related to unique ILC problems (i.e., ILC's own issues not related to other control theories). The next four categories (robust, optimal, adaptive, fuzzy/neural) are for papers that combine or use results from these specific fields to advance the theoretical developments of ILC. The next two categories consider special cases where ILC has been applied to develop theoretical solutions to these special problem classes (mechanical nonlinearity and repetitive control) and the final category collects miscellaneous contributions.

A.3.1 General (Structure)

In this category, we include literature related to "ILC structure," "convergence analysis," "stability analysis," and "basic theoretical works."

- Structure [346, 351, 363, 180, 454, 347].
- Equivalence of ILC to one-step minimum prediction control or feedback control [302, 466, 162, 345, 163, 164, 161, 165].
- Analysis in the point of passivity (dissipativity) [31, 26, 25, 33, 315, 24].
- Analysis in the point of positivity [182, 145].
- Divergence observation [277].
- Steady-state oscillation condition and its utilization [238].
- Strongly positive system [11, 12].

A.3.2 General (Update Rules)

In this category, we include literature that discusses "ILC update rules" and their "performance comparisons."

- Update rules such as D-type ILC, P-type ILC, I-type ILC, PD-type ILC, and PID-type [502, 409, 63, 536, 410, 547].

- Fractional ILC [60].
- Using current-cycle feedback [369].
- Anticipatory ILC [473].
- Update in Hilbert space [35, 34].
- Performance guaranteed ILC, convergence speed improvement, or performance improvement [504, 195, 515, 509, 510, 70, 492, 128, 246, 130].
- Linearization [190].
- Automated tuning [278].
- Comparison of ILC update rules [498, 503, 320, 514].
- Discussion on convergence and/or robustness [331, 251, 335, 517].

A.3.3 Typical ILC Problems

In this category, we include ILC problems such as nonminimum phase, initial condition reset, higher-order approach, 2-D analysis, and frequency-domain analysis. From Table 4.1 of [299], it is shown that these typical ILC problems had been popularly studied before 1997. But, we can observe that many publications are still devoted to these topics.

- Nonminimum phase and/or noncausal filtering [158, 216, 157, 333, 218, 318, 413, 286, 414, 154, 242, 92, 219, 241, 465].
- Inverse model-based or pseudo-inverse-based ILC [156, 155, 317].
- Initial reset condition [426, 353, 75, 355, 131, 431, 429, 430, 435, 438, 434].
- Higher-order ILC [6, 74, 239, 516, 365, 328].
- 2-D approach/analysis [129, 123, 301, 99, 124, 385, 147, 348, 148, 135, 169, 145, 122].
- Frequency-domain analysis and/or synthesis based on frequency-based filtering [377, 127, 327, 265, 326, 330].

A.3.4 Robustness Against Uncertainty, Time Varying, and/or Stochastic Noise

This category includes robustness problems such as disturbance rejection, stochastic affects, H_∞ approaches, etc. Arif et al. [17] used the following update rule: $u_{k+1}(t) = u_k(t) + \Gamma_1 \dot{e}_k(t) + \Gamma_2 \dot{e}_{k+1}(t)$, where $\dot{e}_{k+1}(t)$ is the predicted error, to improve the ILC convergence speed for time-varying linear systems with unknown but bounded disturbances. In [455], time-periodic disturbances and unstructured disturbances were compensated using a simple recursive technique that does not use Lyapunov equation (refer to [508] for disturbance compensation using Lyapunov functions). For general ideas about robust ILC, refer to [296] for the linear case and see [475, 529, 77, 368, 520, 450, 452] for the nonlinear case. Other related papers include:

- Disturbance rejection with feedback control [86].
- Disturbance rejection with an iteration-varying filter [325].

- Nonlinear stochastic systems with unknown dynamics and unknown noise statistics [55].
- Stochastic ILC [397, 396, 395, 54], and with error prediction [17].
- With measurement noise [313, 324].
- H_∞ approach [360].
- μ-synthesis approach to ILC [115, 116].
- Model-based ILC [23, 382].
- ILC based on the backstepping idea [455].
- Polytope uncertainty approach to ILC [267].

A.3.5 Optimal, Quadratic, and/or Optimization

Optimal ILC is considered one of the main ILC theoretical areas and it has a well-established research history. Norm-optimal ILC is due to [143] as commented in [185]. Recently there have been several different quadratic cost function-based ILC algorithms. Papers in this category include:

- Optimal ILC [9, 181, 185, 186, 187, 342, 394, 341, 458, 5].
- Optimization-based ILC [188, 171].
- Linear quadratic optimization-based method [340, 167].
- Quadratic cost function-based method [91, 237, 253].
- Numerical optimization [288].

A.3.6 Adaptive and/or Adaptive Approaches

Adaptive control-based ILC is very popular and many theoretical works in ILC are related to Lyapunov functions and/or adaptive control concepts. In this category, we only include the literature which focuses on purely theoretical adaptive ILC.

- General works [424, 133, 134, 349, 539, 295, 79, 80, 96, 56, 132, 350, 322].
- Model reference [260].
- Model reference with basis functions [366, 485].
- State-periodic adaptive learning control [3].

A.3.7 Fuzzy or Neural Network ILC

In the ILC literature, it has been shown that learning gains can be determined from neural network or fuzzy logic schemes [298]. Specific results include:

- Fuzzy ILC or fuzzy ILC for initial setting [78, 479, 85, 406, 546, 7, 370].
- Feedforward controller (LFFC) using a dilated B-spline network [65, 464].
- Artificial neural networks/neural networks, or ILC application to neural networks [534, 446, 83, 193, 208, 221, 97, 100, 227, 226, 275, 490, 523, 442, 480, 84].

A.3.8 ILC for Mechanical Nonlinearity Compensation

Many ILC publications show that mechanical hard nonlinearities can be compensated for successfully if they have some sort of periodicity in the time, state, or frequency-domain. The main idea of hard-nonlinearity compensation is to analyze stability in the iteration domain as done in [424].

- ILC without a priori knowledge of the control direction, for non-Lipschitz plants [508, 527, 229].
- ILC with input saturation [505, 519].
- Input singularity [507, 526].
- Deadzone [530, 228, 393].
- Coulomb friction [118, 119, 152, 121, 3].
- Using the Smith predictor for time delay and disturbance systems [207, 511].
- ILC for systems with delay [427, 354, 359].

A.3.9 ILC for Other Repetitive Systems and Other Control Schemes

Though classical control theories have been used for ILC performance improvement, it is also possible to use ILC theory for the performance improvement of other control schemes. Using the general idea of ILC, the performances of several other types of control strategies have been improved, including: repetitive control, PID, optimal control, neural network, etc. [149, 209, 48, 20, 344, 468, 93, 217]; and model-based predictive control [252, 257].

A.3.10 Miscellaneous

Papers that we cannot separate into the categories given above include:

- Different tracking control tasks [506].
- Slowly varying trajectory and/or direct learning control (DLC) for non-repeatable reference trajectory, or DLC for MIMO systems [15, 19, 521, 525, 524, 4].
- LMI-based ILC [144, 421, 150, 146, 386].
- Monotone ILC [62, 303, 306, 310, 308, 357, 358].
- Hamiltonian control systems [140, 138, 139, 141].
- MIMO linear time-varying systems [443].
- Observer-based ILC [450].
- Blended multiple model ILC [451].
- Composite energy function ILC [501, 524].
- Cascaded nonlinear systems [380, 378].
- Nonlinear systems with constraints [57].
- Maximum phase nonlinear systems [90].

- Unknown relative degree [425]; and arbitrary or higher relative degree [81, 436, 545, 22].
- Decentralized iterative learning control [488].
- Internal model-based ILC [453, 61, 64, 457, 528].
- Distributed parameter systems [379, 381].
- ILC with prescribed input–output subspace [173, 176]; with desired input in an appropriate finite-dimensional input subspace [419, 420]; and with bounded input [120].
- Sampled-data ILC [428, 432, 440, 439, 433, 434, 437].
- Experience/information database [16, 18, 15].
- Fourier series-based learning controller [376].
- Learning variable structure control [532].
- With weighted local symmetrical integral feedback controller [69].

Figure A.2 plots the number of papers related to theoretical developments in ILC. As seen in this figure, ILC theory has been advanced by being connected to existing control theories such as robust, adaptive, optimal and neural/fuzzy control. However, the ILC structure and update problems, which are investigated within ILC's own framework, dealing with ILC problems such as the non-minimum phase systems, the initial reset problem, the higher-order issue, 2-D analysis, and convergence/performance improvement, have been more widely studied. Figure A.2 reveals that much research has been devoted to ILC's own theoretical and structural problems. It is also interesting to point out that while other control schemes have been used to help improve ILC, in the same way the ILC concept has been used for the performance improvement of other control schemes.

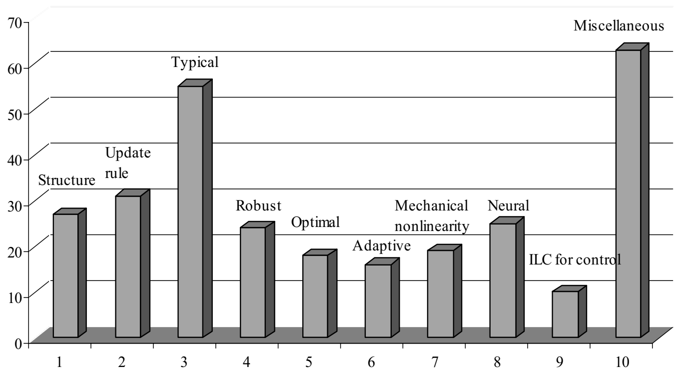

Fig. A.2. Publication numbers for the theory-focused ILC literature

A.4 Discussion

In this appendix we have categorized and discussed the iterative learning control literature published between 1998 and 2004. From the categorization of application-related literature, we have found that ILC applications have been extended from robotics and process control to more specific semiconductor manufacturing and bioengineering applications. However, applications remain dominated by manipulator-based robotics, rotary systems, and process control problems, which are basically time- or state-periodic in either the desired trajectory or the external disturbances. Although some of the publications have shown that ILC can be used in the areas of aerospace, non-robotic actuator control, biomedical applications, visual tracking, artificial muscles, and other emerging engineering problems, successful industrial applications have not yet been reported in these areas.

From the survey of theory-focused literature, it is seen that ILC theory has been developed in two different areas: research on ILC's own features and research on ILC systems fused with other control theories. Most of the recent theoretical work has been related to performance improvement with various types of uncertainties and/or instabilities. However, although many recent theoretical achievements have provided beautiful mathematical formulations of ILC, much of the theoretical development remains far away from actual application considerations.

B

Maximum Singular Value of an Interval Matrix

In this appendix the maximum singular value of an interval matrix is calculated and the effectiveness of the proposed method is illustrated through numerical examples. Calculations are given for both square and non-square matrices. The maximum singular value of an interval matrix can be used to check the monotonic convergence of uncertain ILC systems.

B.1 Maximum Singular Value of a Square Interval Matrix

To develop our algorithm we make use of Hertz's idea for finding extreme eigenvalues of a symmetric interval matrix [192]. Let us consider a real square non-symmetric interval matrix given as

$$A^I = [a^I_{ij}], \quad a^I_{ij} = [\underline{a_{ij}}, \overline{a_{ij}}], i, j = 1, \ldots, n, \tag{B.1}$$

where a^I_{ij} is an element of the interval matrix A^I, $\underline{a_{ij}}$ is the lower boundary of the interval a^I_{ij}, and $\overline{a_{ij}}$ is the upper boundary of the interval a^I_{ij}. If the lower and the upper boundary matrices are defined as $\underline{A} = [\underline{a_{ij}}]$ and $\overline{A} = [\overline{a_{ij}}]$, respectively, the interval matrix can then be written as $A \in A^I = [A^o - \Delta, A^o + \Delta]$, where the center matrix and the radius matrix are defined, respectively, as

$$A^o = \frac{1}{2}(\overline{A} + \underline{A}); \ \Delta = \frac{1}{2}(\overline{A} - \underline{A}). \tag{B.2}$$

In fact, the upper boundary of the singular values of an interval matrix can be found as (in descending order) $\sigma_i(A^I) = \sqrt{\lambda_i((A^I)^T \otimes A^I)}$ where $\otimes$ represents multiplication of interval matrices (see Section 4.1), σ is the singular value, and λ is the eigenvalue. However, as commented in [106], the results of this method will be quite conservative. Thus, we propose an exact calculation that will not be conservative.

To begin, consider the following relationship between singular values and eigenvalues:

$$\sigma_i(A) = \text{Positive}\left(\lambda_i \begin{bmatrix} 0 & A^T \\ A & 0 \end{bmatrix}\right), \tag{B.3}$$

where Positive(·) considers only the positive part of (·). Define

$$H = \begin{bmatrix} 0 & A^T \\ A & 0 \end{bmatrix}.$$

Obviously, H is a symmetric matrix. Including interval uncertainties, let us use also define the symmetric interval matrix

$$H^I = \begin{bmatrix} 0 & (A^I)^T \\ A^I & 0 \end{bmatrix}.$$

Now, we make use of the existing result from [192] to find the maximum singular value of A^I. In what follows, the main idea and results are briefly summarized. From [192], since H and H^I are symmetric matrices, we have the relationship:

$$\lambda = x^T H x = x^T \begin{bmatrix} 0 & A^T \\ A & 0 \end{bmatrix} x, \tag{B.4}$$

where x is an eigenvector corresponding to λ and $x^T x = 1$. Let us divide x into two parts given as $x^T = [y^T, z^T]$. Then, $y_i = x_i$, $i = 1, \ldots, n$ and $z_i = x_{n+i}$, $i = 1, \ldots, n$, and from (B.4), we obtain $\lambda = 2\left(\Sigma_{i=1}^n \Sigma_{j=1}^n a_{ji} y_i z_j\right)$. Therefore, the value of λ depends on signs of y_i and z_j. That is, the maximum of λ occurs at one of the vertex points of a_{ij}, which is given as

$$a_{ij} = \begin{cases} a_{ij} = \overline{a_{ij}} & \text{if} \quad y_i z_j \geq 0, \\ a_{ij} = \underline{a_{ij}} & \text{if} \quad y_i z_j < 0. \end{cases} \tag{B.5}$$

Now, since y and z are length-n vectors, we have total number of 2^n different sign patterns for y and 2^n different sign patterns for z. For example, when $n = 3$, the sign patterns of y and z could be $+++$, $++-$, $+-+$, $+--$, $-++$, $-+-$, $--+$, $---$. In this case, we have a total of $2^3 \times 2^3 = 64$ combinations as shown in Table B.1 and Table B.2. However, Table B.1 and Table B.2 produce the same vertex matrices set for A^I. Therefore, for our purpose, it will be enough to check a total of 2^5 vertex matrices corresponding to Table B.1. These vertex matrices can be found easily. For example, in Table B.1, for the sign pattern $+--$ of y and for the sign pattern $+-+$ of z, the sign of the vertex matrix is generated from zy^T as

$$\begin{bmatrix} + \\ - \\ + \end{bmatrix} \begin{bmatrix} + & - & - \end{bmatrix} = \begin{bmatrix} + & - & - \\ - & + & + \\ + & - & - \end{bmatrix}, \tag{B.6}$$

Table B.1. 32 Sign Patterns with Sign$(y_1) = +$ for a 3×3 Matrix

y	z	y	z
$+++$	$+++$	$+-+$	$+++$
	$++-$		$++-$
	$+-+$		$+-+$
	$+--$		$+--$
	$-++$		$-++$
	$-+-$		$-+-$
	$--+$		$--+$
	$---$		$---$
$++-$	$+++$	$+--$	$+++$
	$++-$		$++-$
	$+-+$		$+-+$
	$+--$		$+--$
	$-++$		$-++$
	$-+-$		$-+-$
	$--+$		$--+$
	$---$		$---$

which means that the corresponding vertex matrix is

$$\begin{bmatrix} \overline{a_{ij}} & \underline{a_{ij}} & \underline{a_{ij}} \\ \underline{a_{ij}} & \overline{a_{ij}} & \overline{a_{ij}} \\ \overline{a_{ij}} & \underline{a_{ij}} & \underline{a_{ij}} \end{bmatrix} . \tag{B.7}$$

In the following algorithm, based on the above discussion, for an $n \times n$ interval matrix, a generalized method is developed.

Algorithm B.1. *Algorithm for estimating the maximum singular value of an interval matrix:*

- ***Step 1***: *Produce a set of* ± 1 *vectors with* $y_1 = 1$ *of length* n *given by*

$$Y = \{y \in R^n : y_1 = 1,\ |y_j| = 1,\ \text{for } j = 2, \ldots, n\} .$$

- ***Step 2***: *Produce a set of* ± 1 *vectors of length* n *given by*

$$Z = \{z \in R^n :\ |z_j| = 1,\ \text{for } j = 1, \ldots, n\} .$$

- ***Step 3***: *Make an* $n \times n$ *diagonal matrix* T_y *defined by* $(T_y)_{ii} = y_i$ *and* $(T_y)_{ij} = 0$ *for* $i \neq j,\ i, j = 1, \ldots, n$ *where* $y \in Y$.
- ***Step 4***: *Make an* $n \times n$ *diagonal matrix* T_z *defined by* $(T_z)_{ii} = z_i$ *and* $(T_z)_{ij} = 0$ *for* $i \neq j,\ i, j = 1, \ldots, n$ *where* $z \in Z$.

Table B.2. 32 Sign Patterns with Sign(y_1) = $-$ for a 3×3 Matrix

y	z	y	z
$-++$	$+++$	$--+$	$+++$
	$++-$		$++-$
	$+-+$		$+-+$
	$+--$		$+--$
	$-++$		$-++$
	$-+-$		$-+-$
	$--+$		$--+$
	$---$		$---$
$-+-$	$+++$	$---$	$+++$
	$++-$		$++-$
	$+-+$		$+-+$
	$+--$		$+--$
	$-++$		$-++$
	$-+-$		$-+-$
	$--+$		$--+$
	$---$		$---$

- ***Step 5***: *Produce a matrix set*

$$\mathcal{S}^v := \{A_{yz} : A_{yz} = A^o + T_y \Delta T_z, \ \forall\ y \in Y \ \text{ and } \ \forall\ z \in Z\}.$$

- ***Step 6***: *Find the maximum singular values of all elements of the finite set* $\mathcal{S}^v$ *and select the largest one as the maximum singular value of the interval matrix* A^I.

B.2 Maximum Singular Value of Non-square Interval Matrix

The results of the preceding section can easily be extended to the general non-square interval matrix case. Let us consider an $m \times n$ interval matrix A^I. Then, H^I is an $(m+n) \times (m+n)$ interval matrix. Now, introducing a length-n vector y and a length-m vector z, using the same procedure as done in the square matrix case, we have $\sigma(A) = 2\left(\Sigma_{i=1}^{n}\Sigma_{j=1}^{m} a_{ji}y_i, z_j\right)$. Then, there are total number of 2^{m+n-1} possible combinations of vertex matrices to be considered. For example, for a 3×2 matrix, we have a total of $2^3 \times 2^1$ combinations as shown in Table B.3. In Table B.3, for example, for the sign pattern $+-$ of y and for the sign pattern $+-+$ of z, the sign pattern of the vertex matrix is generated from zy^T to be

Table B.3. 16 Sign Patterns for a 3 × 2 Non-square Matrix

y	z	y	z
$++$	$+++$	$+-$	$+++$
	$++-$		$++-$
	$+-+$		$+-+$
	$+--$		$+--$
	$-++$		$-++$
	$-+-$		$-+-$
	$--+$		$--+$
	$---$		$---$

$$\begin{bmatrix} + \\ - \\ + \end{bmatrix} [+ \quad -] = \begin{bmatrix} + & - \\ - & + \\ + & - \end{bmatrix}, \tag{B.8}$$

which means that the corresponding vertex matrix is

$$\begin{bmatrix} \overline{a_{ij}} & \underline{a_{ij}} \\ \underline{a_{ij}} & \overline{a_{ij}} \\ \overline{a_{ij}} & \underline{a_{ij}} \end{bmatrix}. \tag{B.9}$$

B.3 Illustrative Examples

B.3.1 Example 1: Non-square Case

Let us test the non-square case first. For the non-square case, the following example is adopted from [106]:

$$A \in A^I = \begin{bmatrix} [2,3] & [1,1] \\ [0,2] & [0,1] \\ [0,1] & [2,3] \end{bmatrix}. \tag{B.10}$$

Using the results given in Section B.2, the maximum singular value of A^I is found to be 4.54306177572459, which is quite close to the value 4.543062 given in [106]. This result shows that the suggested method in this monograph can find the exact (without conservatism) upper boundary of the maximum singular value of an interval matrix. Note that the suggested scheme in this monograph does not require any assumptions.

B.3.2 Example 2: Square Case

Next, for an example with a square matrix and to represent an exception of Deif's method [106], the following center is used:

$$A^o = \begin{bmatrix} -3.33 & -2.24 & 0.06 \\ 1.03 & -0.34 & 1.09 \\ -2.02 & -1.02 & 2.27 \end{bmatrix}, \tag{B.11}$$

with an associate radius matrix taken as:

$$\Delta = \begin{bmatrix} 1.32 & 0.86 & 4.38 \\ 0.84 & 2.97 & 1.42 \\ 1.61 & 3.06 & 0.55 \end{bmatrix}. \tag{B.12}$$

Using the suggested method, the maximum singular value of A^I is found to be 9.8549, but from Deif's method, it is found to be 9.7408. For demonstration purposes, random tests are performed. Figure B.1 shows the results of a Monte-Carlo-type random test where the maximum singular values of a large number of matrices taken from A^I were computed. In the figure the dash-dot line is the calculated maximum singular value from the suggested method (9.8549) and the solid line is the maximum singular value from Deif's method. Clearly there exist exceptions in the case of Deif's method, while the suggested method bounds the maximum singular values without any exception.

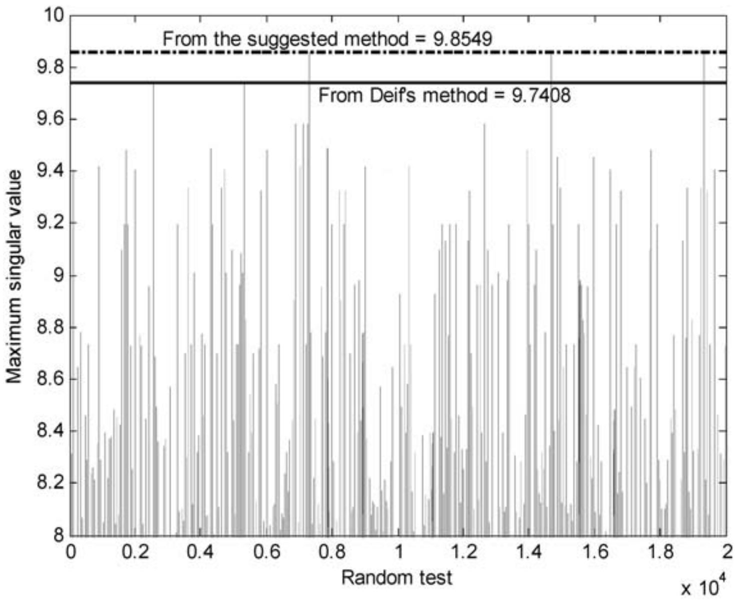

Fig. B.1. Maximum singular values of randomly selected matrices and the calculated maximum singular values from the suggested method (dash-dot line) and from Deif's method (solid line)

B.4 Summary

In this appendix, algorithms for calculating the maximum singular value for square and non-square interval matrices were developed. Using an existing example from [106], whose result was developed based on perturbation theory under some restrictive assumptions, it was verified that the proposed method in this appendix could find the exact maximum singular value. Furthermore, by example it was also shown that the existing method [106] does not find the maximum singular value in some cases while the suggested method finds the maximum singular value without any exception.

C

Robust Stability of Interval Polynomial Matrices

In this appendix, the concept of an interval polynomial matrix system is introduced and a robust stability condition for such systems is derived. This robust stability condition can be used for testing the asymptotic stability of uncertain (interval) higher-order ILC systems.

C.1 Interval Polynomial Matrices

As commented in [372, 270], matrix polynomials (or polynomial matrices) [160, 13] are important in the theory of higher-order vector differential equations, multi-input, multi-output control systems [51], and n-D circuits. For the last two decades, the robust stability problem for polynomial matrices has been steadily studied [372, 243, 232, 476, 481]. In fact, for robust analysis, the interval concept has been quite popular, as shown in [314, 37, 42, 215, 179, 178]. Interval polynomial matrices occur in discrete-time, multivariable problems where physical constants in the plant are subject to interval uncertainty. Under the interval uncertainty concept, after Kharitonov [235] provided an analytical solution for the stability of the continuous interval polynomial, a great amount of literature has been devoted to the study of robustness for interval matrices and interval polynomials. For instance, interval matrices, matrix polytopes, interval polynomial matrices, and polynomial matrix polytopes have been well defined and studied [37, 42, 220, 543, 544, 215, 476, 481]. Also, it has been well known that the stability of continuous interval polynomial matrices or interval matrix polynomials can be checked by Kharitonov polynomials [243, 232, 476, 481]. However, relatively very few research efforts have been devoted to discrete interval polynomial matrices [243] or discrete interval matrix polynomials [233]. Two recent results are Henrion [191], who suggested an LMI condition for the robust stability of polynomial matrix polytopes and polytope type polynomial matrices, and Psarrakos [372], who provided a stability radius for discrete polynomial matrices. In Henrion's work, an LMI condition was used for polynomial matrices when coefficients of the polynomial matrix vary dependently. In Psarrakos's work, it was required to calculate the norm of the inverse of the nominal polynomial matrix, which could be quite conservative. In this appendix, a new analytic Schur stability checking method is developed, which is algebraically simple and less conservative than the existing results. In this newly developed approach, Markov matrices of the polynomial matrix are exploited by using the inverse of the polynomial matrix.

To ensure the consistency of notations and definitions, based on [243, 232, 476, 191, 481], let us repeat the definitions of interval polynomial matrices. When the (i, j)-th element of matrix $P(z)$ is denoted by

$$p_{ij}(z) = a_{ij0} + a_{ij1}z + \cdots + a_{ijm}z^m, \; i, j = 1, \ldots, m, \tag{C.1}$$

where m is the degree of polynomial $p_{ij}(z)$, matrix $P(z)$ is called a polynomial matrix. When the coefficients of a polynomial lie in intervals like

$$\underline{a_{ijk}} \leq a_{ijk} \leq \overline{a_{ijk}}, \; k = 0, \ldots, m; i, j = 1, \ldots, n, \tag{C.2}$$

where n is the dimension of square $P(z)$; $\overline{(\cdot)}$ is the maximum extreme value of $(\cdot)$ and $\underline{(\cdot)}$ is the minimum extreme value of $(\cdot)$, these polynomial matrices are called interval polynomial matrices (denoted by $P^I(z)$) [243]. Note that polytopic polynomial matrices [37, 476] or polynomial matrix polytopes [191] should be distinguished from interval polynomial matrices. As commented in [191], these polynomial matrix polytopes are linear combinations of a set of given polynomial matrices.

C.2 Definitions and Preliminaries

Let us consider a real monic polynomial matrix of the form:

$$P(z) = I_{m\times m}z^n + A_1 z^{n-1} + \cdots + A_{n-1}z + A_n, \tag{C.3}$$

where $I_{m\times m}$ is the $m \times m$ identity matrix; the coefficient matrices $A_i, \; i = 1, \ldots, n$ are $m \times m$ real square matrices, i.e., $A_i \in \mathbf{R}^{m\times m}$; and z is a point in complex plane, i.e., $z \in \mathbf{C}$. The following definitions are then used to discuss the stability of the polynomial matrix $P(z)$.

Definition C.1. *[243, 476, 336] The roots λ^* of $\det(P(\lambda)) = 0$ are called eigenvalues of $P(z)$[1]. Thus, when we define a set $S_\lambda = \{\lambda \mid \det(P(\lambda)) = 0\}$, if $\max_{\lambda\in S_\lambda} |\lambda| < 1$, then the polynomial matrix $P(z)$ is robust D-stable. In this appendix, robust D-stability is also called Schur stability without notational confusion.* ■

[1] This notation is not unusual. For a similar discussion, refer to [247, 105, 372, 194, 13].

Definition C.2. *When the elements of a matrix A are intervals such as $a_{ij} \in [\underline{a_{ij}}, \overline{a_{ij}}]$, this matrix is called an interval matrix A^I. The modulus matrix ($|A|_m$) of an interval matrix is defined as*

$$|A|_m = \left[a_{ij}^m : a_{ij}^m \in \max\{|\underline{a_{ij}}|, |\overline{a_{ij}}|, i, j = 1, \ldots, n\} \right],$$

where a_{ij}^m are elements of modulus matrix $|A|_m$. For a non-interval matrix A, we define $|A|_m = |A| = [|a_{ij}|]$, which takes the absolute value of each element of A. ■

For the derivation of the Schur stability of the interval polynomial matrix $P^I(z)$, the following lemmas are needed.

Lemma C.3. *[51] If $P(z)$ is a real polynomial matrix and $\det(P(z))$ is not identically zero then it is invertible (i.e., non-singular) and its inverse is a real-rational matrix.* ■

Lemma C.4. *[98, 418] For an $m \times m$ matrix R, if $\rho(R) < 1$ (ρ means spectral radius), then $\det(I \pm R) \neq 0$.* ■

Lemma C.5. *If $P(z)$ is invertible,[2] then $[P(z)]^{-1}$ can be expanded as $\sum_{k=0}^{\infty} T_k z^{-k}$ (i.e., $[P(z)]^{-1} = \text{span}\{Iz^{-i}, i = 0, \ldots, \infty\}$).* ■

Proof (of Lemma C.5). By Lemma C.3, there exists $[P(z)]^{-1}$ whose elements are real-rational functions of z, denoted $p_{ij}^{-1}(z) = \sum_{k=0}^{\infty} t_{ij_k} z^{-k}$. That is, each element of $[P(z)]^{-1}$ can be expanded into its Markov parameters. Clearly then, we can write $T_k = [t_{ij_k}]$. ■

Note that T_k are called the *Markov matrices* of the inverse polynomial matrix $[P(z)]^{-1}$.

Lemma C.6. *[338] For any square matrices, R, T, and V, if $|R|_m \leq V$, then the following inequalities are true:*

$$\rho(RT) \leq \rho(|R|_m |T|_m) \leq \rho(V|T|_m),$$

where the subscript m means the modulus matrix. ■

C.3 Stability Condition for Interval Polynomial Matrices

The key idea of the suggested method is to utilize Markov matrices in the region $|z| \geq 1$ of the complex plane. The method will be developed based on the matrix determinant. Our results are organized into three subsections. The first two consider the stability of polynomial matrices. The results from these first two subsections are then used in the third subsection to develop our main result.

[2] The assumption that $P(z)$ is invertible is practically meaningful. From [51], "all polynomial matrices are invertible for almost all z unless the determinant of $P(z)$ is 0 for all z." Thus, it is the basic assumption that $P(z)$ is invertible.

C.3.1 The Stability of Polynomial Matrices: Part 1

Let us begin this subsection by rewriting the polynomial matrix (C.3) as

$$\begin{aligned} P(z) &= z^{n-1}(zI + A_1 + A_2 z^{-1} + \cdots + A_n z^{-n+1}) \\ &= z^{n-1}(zI + S(z)) = z^{n-1}Q(z), \end{aligned} \tag{C.4}$$

where $S(z) := A_1 + A_2 z^{-1} + A_3 z^{-2} + \cdots + A_n z^{-n+1}$, and $Q(z) := zI + S(z)$. By taking the determinant of both sides of (C.4), we have:

$$\det[P(z)] = \det[z^{n-1}I \cdot Q(z)] = \det[z^{n-1}I] \cdot \det[Q(z)]. \tag{C.5}$$

Here, observe that $z = 0$ is the solution such that $\det[z^{n-1}I] = 0$. Furthermore, $z = 0$ is not defined in $Q(z)$, because denominators become zeros. Thus, for the polynomial $Q(z)$, the complex plane without the origin is considered. Define $\mathbf{C}^* = \mathbf{C} - \{0\}$. Then, to determine the stability of $P(z)$ from $\det[Q(z)]$, the following lemmas can be developed:

Lemma C.7. *In* $\mathbf{C}^*$*, from (C.4),* $P(z)$ *is stable if and only if* $Q(z)$ *is stable.* ■

Proof. In $\mathbf{C}^*$, $\det[z^{n-1}I] \neq 0$. Hence, only z such that $\det[zI + S(z)] = 0$ makes $\det[P(z)]$ zero. Thus, from the following latent solutions:

$$S_{z^*} = \{z|\det[Q(z)] = 0,\ z \in \mathbf{C}^*\};\ S_{z^{**}} = \{z|\det[P(z)] = 0,\ z \in \mathbf{C}^*\},$$

the following set equality is true: $S_{z^*} = S_{z^{**}}$. Hence, $P(z)$ is stable if and only if $Q(z)$ is stable. ■

Lemma C.8. *In* $\mathbf{C}^*$*, the polynomial matrix* $Q(z)$ *is stable if and only if* $\det[Q(z)] \neq 0$*, for all* $|z| \geq 1$*.* ■

Proof. It is certain that, in the complex plane, there exists a z such that $\det[Q(z)] = 0$. Thus, the condition "$\det[Q(z)] \neq 0,\ \forall\ |z| \geq 1$" is equivalent to the condition "there exists a z such that $\det[Q(z)] = 0$ only in the disk of $|z| < 1$." Thus, by Definition C.1, $Q(z)$ is stable. For the "only if" condition, assume that $Q(z)$ is stable. Also assume there exists any z, $|z| \geq 1$ such that $\det[Q(z)] \neq 0$. Then, from $S_\lambda = \{\lambda \mid \det(P(\lambda)) = 0\}$, we have $\max_{\lambda \in S_\lambda} |\lambda| \geq 1$. This contradicts the fact that $Q(z)$ is stable. Hence, $\det[Q(z)] \neq 0,\ \forall\ |z| \geq 1$ is the stability condition. ■

Therefore, based on the results of Lemma C.7 and Lemma C.8, it is concluded that in $\mathbf{C}^*$, $P(z)$ is stable if and only if $\det[Q(z)] \neq 0$, for all $|z| \geq 1$. The following theorem is then suggested:

Theorem C.9. *If* $\det(A_n) \neq 0$*, then* $P(z)$ *is stable in* $z \in \mathbf{C}$ *if and only if* $\det[Q(z)] \neq 0$*, for all* $|z| \geq 1$*.* ■

Proof. The proof can be completed by substituting $z = 0$ into $P(z)$. If $z = 0$ is substituted into $P(z)$, then $P(z = 0) = A_n$. Thus, if $\det(A_n) \neq 0$, then $z = 0$ is not a latent solution such that $\det[P(z)] = 0$. Hence, since latent solutions of $\det[P(z)] = 0$ in $\mathbf{C}$ are equivalent to latent solutions of $\det[P(z)] = 0$ in $\mathbf{C}^*$, the stability of $P(z)$ in $\mathbf{C}^*$ means the stability of $P(z)$ in $\mathbf{C}$. ∎

The above theorem shows that the polynomial matrix $P(z)$ is stable if and only if $\det[Q(z)] \neq 0$ for all $|z| \geq 1$ with $\det(A_n) \neq 0$. Next, given this necessary and sufficient condition for the stability of $P(z)$, the remaining problem with respect to the stability of a polynomial matrix is to find an equivalent condition for $\det[Q(z)] \neq 0$. This will be discussed in the following subsection.

C.3.2 The Stability of Polynomial Matrices: Part 2

Now we consider the Markov matrices of polynomial matrices, as a vehicle for discussing the stability of $\det[Q(z)] \neq 0$. Then in the next subsection, based on the boundary condition of the sum of these Markov matrices, a robust stability condition for interval polynomial matrices is developed.

To define the Markov matrices of a polynomial matrix, the following lemma is needed:

Lemma C.10. *If* $\det(P(z))$ *is not identically zero, then the polynomial matrix* $Q(z) = zI + S(z)$ *is nonsingular and* $[Q(z)]^{-1}$ *can be expanded using Markov matrices in* $\mathbf{C}^*$ *as*

$$[Q(z)]^{-1} = \sum_{k=0}^{\infty} T_k z^{n-k-1}.$$

∎

Proof. By multiplying $z^{1-n}I$ by $P(z)$, we have:

$$z^{1-n}I \cdot P(z) = (zI + A_1 + A_2 z^{-1} + \cdots + A_n z^{-n+1}) = Q(z). \quad \text{(C.6)}$$

Here, since $P(z)$ is nonsingular from Lemma C.3 and $z^{1-n}I$ is nonsingular in $\mathbf{C}^*$, then clearly $z^{1-n}I \cdot P(z)$ is nonsingular. Also, from Lemma C.5, $[P(z)]^{-1} = \sum_{k=0}^{\infty} T_k z^{-k}$. Thus, the following relationship can be established easily:

$$[zI + S(z)]^{-1} = [z^{1-n}P(z)]^{-1} = z^{n-1}[P(z)]^{-1} = \sum_{k=0}^{\infty} T_k z^{n-k-1}. \quad \text{(C.7)}$$

This completes the proof. ∎

For convenience, let us write $\sum_{k=0}^{\infty} T_k z^{n-k-1}$ as

$$\sum_{k=0}^{\infty} T_k z^{n-k-1} = \sum_{k=0}^{n-2} T_k z^{n-k-1} + \sum_{k=n-1}^{\infty} T_k z^{n-k-1}, \quad \text{(C.8)}$$

and replace the second term of the right-hand side by

$$\sum_{k=n-1}^{\infty} T_k z^{n-k-1} = \sum_{i=0}^{\infty} R_i z^{-i}, \tag{C.9}$$

where $R_0 = T_{n-1}, R_1 = T_n, R_2 = T_{n+1}, \ldots, R_i = T_{n+i-1}$. With this notation, the process for the calculation of the Markov matrices of $Q(z)$ is summarized in the following lemma:

Lemma C.11. *If* $\det(P(z))$ *is not identically zero, the inverse of* $Q(z)$ *is expressed as* $[Q(z)]^{-1} = \sum_{i=0}^{\infty} R_i z^{-i}$ *in which the Markov matrices are calculated by*

$$R_k = -\sum_{i=1}^{k-1} A_i R_{k-i}, \;\; k \geq 2, \tag{C.10}$$

with $R_0 = 0$ *and* $R_1 = I$*, and* $A_i = 0_{m\times m}$ for $i \geq n+1$. ∎

Proof. From Lemma C.10, the following relationship is given:

$$[Q(z)]^{-1} = \sum_{k=0}^{\infty} T_k z^{n-k-1}.$$

Also, based on Lemma C.10, in $\mathbf{C}^*$, there exists an inverse of $Q(z)$. Thus, the following equalities are true:

$$\begin{aligned}
Q(z)[Q(z)]^{-1} &= Q(z) \cdot \sum_{k=0}^{\infty} T_k z^{n-k-1} \\
\Leftrightarrow I &= Q(z) \cdot \sum_{k=0}^{\infty} T_k z^{n-k-1} \\
&= \left(zI + A_1 + A_2 z^{-1} + \cdots + A_n z^{-n+1}\right) \left(\sum_{k=0}^{\infty} T_k z^{n-k-1}\right) \\
&= \left(zI + A_1 + A_2 z^{-1} + \cdots + A_n z^{-n+1}\right) \\
&\quad \times \left(\sum_{k=0}^{n-2} T_k z^{n-k-1} + \sum_{k=n-1}^{\infty} T_k z^{n-k-1}\right) \\
&= \left(zI + A_1 + A_2 z^{-1} + \cdots + A_n z^{-n+1}\right) \\
&\quad \times \left(\sum_{k=0}^{n-2} T_k z^{n-k-1} + \sum_{i=0}^{\infty} R_i z^{-i}\right).
\end{aligned} \tag{C.11}$$

Here, using the fact that the left-hand side and right-hand side of (C.11) should be equal for all z, after some manipulations it is easy to show that

$T_k = 0_{m\times m}, i = 0, \dots, n-2$ and the following formula is easily derived for R_k:

$$R_k = -\sum_{i=1}^{k-1} A_i R_{k-i},\ k \geq 2,\ A_i = 0_{m\times m} \text{ at } i \geq n+1, \tag{C.12}$$

with $R_0 = 0$ and $R_1 = I$. Recall that $R_k, k = 0, \dots, \infty$, are coefficient matrices of the right-hand side of (C.9). Therefore,

$$[zI + S(z)]^{-1} = [Q(z)]^{-1} = \sum_{i=0}^{\infty} R_i z^{-i}, \tag{C.13}$$

where R_i are determined in (C.12). ■

In Lemma C.11, we provided a formula for calculation of the Markov matrices of the inverse of a polynomial matrix. Now, based on Lemma C.11, it is easy to see that $R_k \to 0$ if and only if $Q(z)$ is stable. For more detail, let us change R_k to

$$\begin{aligned} R_k &= -\sum_{i=1}^{k-1} A_i R_{k-i} \\ &= -A_1 R_{k-1} - A_2 R_{k-2} \cdots - A_n R_{k-n}. \end{aligned} \tag{C.14}$$

Then, the following relationship is obtained:

$$\begin{bmatrix} R_k \\ R_{k-1} \\ \vdots \\ R_{k-n+2} \\ R_{k-n+1} \end{bmatrix} = \begin{bmatrix} -A_1 & -A_2 & \dots & -A_{n-1} & -A_n \\ I_{m\times m} & 0_{m\times m} & \dots & 0_{m\times m} & 0_{m\times m} \\ 0_{m\times m} & I_{m\times m} & \dots & 0_{m\times m} & 0_{m\times m} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{m\times m} & 0_{m\times m} & \dots & I_{m\times m} & 0_{m\times m} \end{bmatrix} \begin{bmatrix} R_{k-1} \\ R_{k-2} \\ \vdots \\ R_{k-n+1} \\ R_{k-n} \end{bmatrix}. \tag{C.15}$$

Denoting the above equation as $\overline{R}_k = \mathcal{C}\overline{R}_{k-1}$, if $\rho(\mathcal{C}) < 1$, then $\|\overline{R}_k\| \to 0$ as $k \to \infty$ which implies $\overline{R}_k \to 0$ as $k \to \infty$. Hence $R_k \to 0$ as $k \to \infty$ if and only if $\rho(\mathcal{C}) < 1$, which is an equivalent condition for the stability of $Q(z)$. Thus, if $\rho(\mathcal{C}) < 1$, we can then calculate the absolute summation of R_k (denoted Σ_{R_k}) according to

$$\Sigma_{R_k} = I + \sum_{k=2}^{\infty} |R_k|_m \tag{C.16}$$

and, using this summation, the following lemma can be adopted to bound the modulus matrix of $Q(z)^{-1}$ (i.e., $|Q(z)^{-1}|_m$).

Lemma C.12. *[98] In* $|z| \geq 1$*, the following inequality is satisfied* $|Q(z)^{-1}|_m \leq \Sigma_{R_k}$. ■

C.3.3 The Stability of Interval Polynomial Matrices

In the remainder of this section, a stability condition for interval polynomial matrices is developed. For clarity of notation, from this point forward the superscript o is used to denote the nominal value of any variable or parameter. In particular:

Definition C.13. *A^o denotes the nominal matrix of A^I:*

$$A^o = \left[a_{ij}^o : a_{ij}^o = \frac{\underline{a_{ij}} + \overline{a_{ij}}}{2} \right].$$

■

Likewise, $P(z)$, $S(z)$, and $Q(z)$ defined in (C.3) and (C.4) are now denoted by $P^o(z)$, $S^o(z)$, and $Q^o(z)$, respectively.

Now, let us add interval radius matrices ΔA_i to $S^o(z)$ to get the interval polynomial matrices $S^I(s)$ as follows:

$$S^I(z) = A_1^o + \Delta A_1 + (A_2^o + \Delta A_2)z^{-1} + \cdots + (A_n^o + \Delta A_n)z^{-n+1},$$

from which the interval coefficient matrices are defined element-wise as $A_i^o - |\Delta A_i|_m \leq A_i^I \leq A_i^o + |\Delta A_i|_m$. We also define:

$$Q^I(z) = zI + S^o(z) + \Delta S(z), \tag{C.17}$$

where $\Delta S(z) = \Delta A_1 + \Delta A_2 z^{-1} + \cdots + \Delta A_n z^{-n+1}$ and define the summation of the modulus interval matrices $|\Delta A_k|_m$ as

$$\triangle_M = \sum_{k=1}^{n} |\Delta A_k|_m. \tag{C.18}$$

In fact, in (C.16), it is not possible to estimate Σ_{R_k}, but if $Q^o(z)$ is stable, then Σ_{R_k} is bounded from (C.15). Let us suppose that the upper boundary of Σ_{R_k} is known as Σ^*. Then, we can find a condition for robustly stability of $Q^I(z)$. For this purpose, first, let us rewrite $\det[Q^I(z)]$, using $Q^I(z) = Q^o(z) + \Delta S(z)$, to be

$$\begin{aligned}\det\left[Q^o(z) + \Delta S(z)\right] &= \det\left[Q^o(z)\left(I + \left(Q^o(z)\right)^{-1}\Delta S(z)\right)\right] \\ &= \det\left[Q^o(z)\right]\det\left[I + \left(Q^o(z)\right)^{-1}\Delta S(z)\right] \\ &= \det\left[Q^o(z)\right]\det\left[I + \left(Q^o(z)\right)^{-1}\Delta S(z)\right]. \end{aligned} \tag{C.19}$$

Then, based on Lemma C.8 and with the assumption of stable $Q^o(z)$, we have $\det\left[Q^o(z)\right] \neq 0$ for $|z| \geq 1$. Also, if $\rho\left(\left(Q^o(z)\right)^{-1}\Delta S(z)\right) < 1$, then, from Lemma C.4, we can say that

$$\det\left[I + (Q^o(z))^{-1}\, \Delta S(z)\right] \neq 0.$$

Thus, since $\det[Q^o(z)] \neq 0$ at $|z| \geq 1$, if

$$\rho\left((Q^o(z))^{-1}\, \Delta S(z)\right) < 1,$$

then the interval polynomial matrix, $Q^I(z)$, is stable. Therefore, we can conclude that $Q^I(z)$ is stable if $\rho\left((Q^o(z))^{-1}\, \Delta S(z)\right) < 1$. Next, let us investigate $\rho\left((Q^o(z))^{-1}\, \Delta S(z)\right) < 1$. Using Lemma C.6 and Lemma C.12, the following inequalities are true:

$$\begin{aligned}\rho\left((Q^o(z))^{-1}\, \Delta S(z)\right) \leq \rho\left(\left|(Q^o(z))^{-1}\right|_m |\Delta S(z)|_m\right) &\leq \rho\left(\Sigma_{R_k} |\Delta S(z)|_m\right) \\ &\leq \rho\left(\Sigma^* |\Delta S(z)|_m\right).\end{aligned}$$

Furthermore, using $|z^{-k+1}|_m \leq 1$, when $|z| \geq 1$ and $k \geq 1$, the following relationships are true:

$$\begin{aligned}|\Delta S(z)|_m = \left|\sum_{k=1}^{n} \Delta A_k z^{-k+1}\right|_m &\leq \sum_{k=1}^{n} |\Delta A_k|_m \left|z^{-k+1}\right|_m \\ &\leq \sum_{k=1}^{n} |\Delta A_k|_m = \triangle_M,\end{aligned}$$

where $\triangle_M$ was defined in (C.18). Then, by Lemma C.6, the following inequality is satisfied:

$$\rho\left((Q^o(z))^{-1}\, \Delta S(z)\right) \leq \rho(\Sigma^* \triangle_M).$$

Therefore, the following lemma can be developed.

Lemma C.14. *If* $\rho(\Sigma^* \triangle_M) < 1$*, then the interval polynomial matrix,* $Q^I(z)$*, is stable.* ■

Using this result, the following theorem is finally developed for the robust stability of $P^I(z)$:

Theorem C.15. *If (i)* $Q^o(z)$ *is stable, (ii)* $\rho(\Sigma^* \triangle_M) < 1$*, and (iii)* $\det(A_n^I) \neq 0$*, then the interval polynomial matrix* $P^I(z)$ *is robustly stable.* ■

Proof. From Lemma C.14, if $Q^o(z)$ is stable and $\rho(\Sigma^* \triangle_M) < 1$, then $Q^I(z)$ is robust stable. Also, from Theorem C.9, if $\det(A_n^I) \neq 0$, it is obvious that $P^I(z)$ is robustly stable. ■

In the sequel, a method for analytically finding Σ^* is provided. Without notational confusion, $|\cdot|$ is the modulus matrix defined earlier. From (C.15),

$$\begin{aligned}|\overline{R}_{n+1}| + |\overline{R}_{n+2}| + \cdots + |\overline{R}_{n+p}| &= |\mathcal{C}\overline{R}_n| + |\mathcal{C}\overline{R}_{n+1}| + \cdots + |\mathcal{C}\overline{R}_{n+p-1}| \\ &= |\mathcal{C}\overline{R}_n| + |\mathcal{C}^2\overline{R}_n| + \cdots + |\mathcal{C}^p\overline{R}_n| \\ &\leq \left(|\mathcal{C}| + |\mathcal{C}^2| + \cdots + |\mathcal{C}^p|\right) |\overline{R}_n|, \end{aligned} \tag{C.20}$$

where we used the inequality $|AB| \leq |A||B|$. Here, if $\mathcal{C}$ is diagonalizable as $\mathcal{C} = \mathcal{X}\Lambda\mathcal{X}^{-1}$, we then change (C.20) as follows:

$$\begin{aligned}\left(|\mathcal{C}| + |\mathcal{C}^2| + \cdots + |\mathcal{C}^p|\right) |\overline{R}_n| &= \left(|\mathcal{X}\Lambda\mathcal{X}^{-1}| + |\mathcal{X}\Lambda^2\mathcal{X}^{-1}| + \cdots + |\mathcal{X}\Lambda^p\mathcal{X}^{-1}|\right) |\overline{R}_n| \\ &\leq |\mathcal{X}| \left(|\Lambda| + |\Lambda^2| + \cdots + |\Lambda^p|\right) |\mathcal{X}^{-1}||\overline{R}_n|, \end{aligned} \tag{C.21}$$

which yields the following general formula:

$$\sum_{i=1}^{p} |\overline{R}_{n+i}| \leq |\mathcal{X}| \left(\sum_{i=1}^{p} |\Lambda^i|\right) |\mathcal{X}^{-1}||\overline{R}_n|. \tag{C.22}$$

Now, taking $p \to \infty$, if $Q^o(z)$ is stable, then we have

$$\begin{aligned}\lim_{p\to\infty} \sum_{i=1}^{p} |\overline{R}_{n+i}| &= |\mathcal{X}| \lim_{p\to\infty} \left(\sum_{i=1}^{p} |\Lambda^i|\right) |\mathcal{X}^{-1}||\overline{R}_n| \\ &\leq |\mathcal{X}|\mathrm{diag}\left(\frac{|\lambda_l|}{1-|\lambda_l|}\right) |\mathcal{X}^{-1}||\overline{R}_n|, \end{aligned} \tag{C.23}$$

where diag$(\cdot)$ is a diagonal matrix composed of diagonal terms $(\cdot)$. Since $\mathcal{X}$, λ_l, $\mathcal{X}^{-1}$, and $\overline{R}_n$ are known, we can estimate the boundary of $\lim_{p\to\infty} \sum_{i=1}^{p} |\overline{R}_{n+i}|$. Writing $\mathcal{T} := |\mathcal{X}|\mathrm{diag}\left(\frac{|\lambda_l|}{1-|\lambda_l|}\right) |\mathcal{X}^{-1}|$ and $\mathcal{F}_n := \mathcal{T}|\overline{R}_n|$, and taking the first m rows of $\mathcal{F}_n$, which is denoted as $\mathcal{D}_n$ (i.e., $\mathcal{D}_n := \mathcal{F}_n(1:m, 1:m)$), we have the following inequality:

$$\Sigma_{R_k} \leq \sum_{i=1}^{n} |R_i| + \mathcal{D}_n. \tag{C.24}$$

Therefore, since $\sum_{i=1}^{n} |R_i|$ and $\mathcal{D}_n$ are calculated, we can analytically estimate the upper boundary of Σ_{R_k}. However, (C.24) could be conservative. To accurately estimate the upper boundary of Σ_{R_k}, by introducing an operator $\mathcal{F}_q := \mathcal{T}|\overline{R}_q|$, where $q \gg n$, and writing $\mathcal{D}_q := \mathcal{F}_q(1:m, 1:m)$, we have a more accurate upper boundary of Σ_{R_k} given as

$$\Sigma_{R_k} \leq \sum_{i=1}^{q} |R_i| + \mathcal{D}_q := \Sigma^*. \tag{C.25}$$

This argument about an accurate upper boundary of Σ_{R_k} is summarized in the following theorem:

Theorem C.16. *If $Q^o(z)$ is stable, in (C.25), as $q \to \infty$, $\mathcal{D}_q \to 0$; hence $\Sigma^* \to \Sigma_{R_k}$.* ■

Proof. Since $\mathcal{T}$ is fixed, from (C.15), $|\overline{R}_q| \to 0$ as $q \to \infty$ if and only if $\rho(\mathcal{C}) < 1$. Therefore, since $|\overline{R}_q| = 0$ if and only if $\overline{R}_q = 0$, the proof is immediate. ■

Theorem C.16 shows that an accurate upper boundary of Σ_{R_k} (i.e., Σ^*) can be estimated by taking a very large q in (C.25).

C.4 Illustrative Examples

In this section, we test the conservatism of the suggested algorithm. From the existing literature, however, it is difficult to find a benchmark example for the robust stability of discrete polynomial matrices. Most examples are continuous cases [243, 232, 476, 191, 372, 481], except examples 3.5 and 4.3 of [372]. However, example 3.5 of [372] is a special case that allows the analytical calculation of $P(\lambda)^{-1}$. But, in general it is very difficult to calculate $P(\lambda)^{-1}$ analytically. Thus, in this appendix, for comparison purposes we use example 4.3 of [372], which uses a numerical range for $\|P(\lambda)^{-1}\|_2$.

C.4.1 Example 1

The main disadvantage of the suggested method in [372] is that it calculates $\|P(\lambda)^{-1}\|_2$ analytically. Although [372] provides a method for this, the method is quite complicated and the result could be very conservative (see Theorem 4.1 of [372]). In example 4.3 of [372], the stability radius of a polynomial matrix was calculated under some conditions. Let us use the example of [372], given as

$$P^o(z) = Iz^3 + A_1^o z^2 + A_2^o z + A_3^o, \tag{C.26}$$

where the coefficient matrices are Hermitian and satisfy the conditions $0 \leq \lambda_{min}(A_1^o) \leq \lambda_{max}(A_1^o) \leq 1/3$, $-1/9 \leq \lambda_{min}(A_2^o) \leq \lambda_{max}(A_2^o) \leq 1/9$, and $-1/27 \leq \lambda_{min}(A_3^o) \leq \lambda_{max}(A_3^o) \leq 1/9$. Since [372] does not provide the coefficient matrices, we selected the following matrices, which satisfy the conditions of example 4.3 of [372]:

$$A_1^o = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.3333 \end{bmatrix}; \; A_2^o = \begin{bmatrix} -0.1111 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.1111 \end{bmatrix}$$

$$A_3^o = \begin{bmatrix} -0.0370 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.1111 \end{bmatrix}.$$

In [372], the analytical perturbation radius of $P^o(z)$ is calculated as 0.0141, i.e., $\|[0_{3\times3} \;\; \Delta_1 \;\; \Delta_2 \;\; \Delta_3]\|_2$ could be 0.0141. Thus, since $\|[0_{3\times3} \;\; \Delta_1 \;\; \Delta_2 \;\; \Delta_3]\|_2 \leq 0.0141$, the polynomial matrix system is robustly stable. Let us use our method to compute this analytical stability radius. From the companion form $\mathcal{C}$, we find that the nominal system is stable and has non-repeated eigenvalues. From Theorem C.16 and (C.25), selecting $q = 100$, we found that

$$\Sigma^* = \sum_{i=1}^{q} |R_i| + \mathcal{D}_q = \begin{bmatrix} 1.1739 & 0 & 0 \\ 0 & 1.1152 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}.$$

Also, for the uncertainty, we provided 10 percent intervals to A_1^o, A_2^o, and A_3^o, from which $\|[0_{3\times 3} \;\; \Delta_1 \;\; \Delta_2 \;\; \Delta_3]\|_2 = 0.0369$. From (C.18), we calculated

$$\triangle_M = \begin{bmatrix} 0.0148 & 0 & 0 \\ 0 & 0.011 & 0 \\ 0 & 0 & 0.0556 \end{bmatrix}$$

Finally, using the calculated $\sum_{i=1}^{q} |R_i| + \mathcal{D}_q$ and $\triangle_M$, we found that $\rho(\Sigma^* \triangle_M) = 0.0833$; hence the interval polynomial system is robustly stable with 10 percent uncertainty, which cannot be concluded in [372].

C.4.2 Example 2

Let us first consider the following general non-symmetric polynomial matrix:

$$P^o(z) = I_{2\times 2} z^3 + A_1^o z^2 + A_2^o z + A_3^o, \tag{C.27}$$

where the coefficient matrices are given as

$$A_1^o = \begin{bmatrix} 0.4 & -0.3 \\ 0.4 & 0.1 \end{bmatrix}; \; A_2^o = \begin{bmatrix} 0.3 & 0.3 \\ 0.4 & -0.5 \end{bmatrix}$$

$$A_3^o = \begin{bmatrix} 0.0 & 0.5 \\ -0.1 & 0.15 \end{bmatrix}.$$

From the corresponding $\mathcal{C}$ matrix, the eigenvalues are calculated as $-0.0535 + 0.8125i$; $-0.0535 - 0.8125i$; -0.8696; -0.5086; 0.7612; 0.2240. Thus, the nominal system is stable. Also, since $\det(A_3^I)$ is not zero, the suggested method can be used. It is assumed that there exists element-wise interval uncertainty in the nominal matrices given as

$$\Delta A_1 = \begin{bmatrix} 0.0432 & 0.0324 \\ 0.0432 & 0.0108 \end{bmatrix}; \; \Delta A_2 = \begin{bmatrix} 0.0324 & 0.0324 \\ 0.0432 & 0.0540 \end{bmatrix}$$

$$\Delta A_3 = \begin{bmatrix} 0.0 & 0.0540 \\ 0.0108 & 0.0162 \end{bmatrix}.$$

To apply Theorem C.16, we selected $q = 50$, from which we found

$$\Sigma^* = \sum_{i=1}^{50} |R_i| + \mathcal{D}_{50} = \begin{bmatrix} 3.1502 & 1.8140 \\ 1.5848 & 4.1860 \end{bmatrix}.$$

Using these matrices, we found $\rho(\Sigma^* \triangle_M) = 0.9979$ which shows that the system is robustly stable (though almost marginally stable). For comparison purposes, we used Theorem 3.1 of [372]. It is, however, difficult to find the infimum of $\frac{1}{\sqrt{\sum_{k\in J} |\lambda|^{2k} \|P(\lambda)^{-1}\|_2}}$ for all $\lambda \in \partial\Omega$, which is a required computation (for notation, refer to [372]). Hence we performed random simulation tests and found that $\lambda = -1$ is the best (this is not an analytical solution, instead we did 2000 random tests to find the best λ). Using $\lambda = -1$, we calculated $\frac{1}{\sqrt{\sum_{k\in J} |\lambda|^{2k} \|P(\lambda)^{-1}\|_2}} = 0.1175$, and calculated $\|[\Delta_0 \;\; \Delta_1 \;\; \Delta_2 \;\; \Delta_3]\| = 0.1174$ which is also almost marginally stable. Hence, from this test, we found that when the exact minimum of $\frac{1}{\sqrt{\sum_{k\in J} |\lambda|^{2k} \|P(\lambda)^{-1}\|_2}}$ of Theorem 3.1 of [372] is found, the stability index of [372] is almost equal to the stability index of our method. However, as commented in [372], it is tough to find the exact minimum of $\frac{1}{\sqrt{\sum_{k\in J} |\lambda|^{2k} \|P(\lambda)^{-1}\|_2}}$ (so they used a numerical range for the approximation, but the result is quite conservative as shown in Example 1 above).

C.5 Summary

In this appendix, a new method for checking the Schur stability of interval polynomial matrices has been suggested and illustrated. The proposed method checks the stability in a simple manner and the derivation process of the method has a good analytical basis. From comparison with an existing method, it was found that the suggested method is less conservative, as demonstrated in Example 1 above, and computationally very simple. Furthermore, we have found that our method provides almost the same stability condition as [372] when the exact minimum of Theorem 3.1 of [372] is found.[3] This implies that the analytical solution proposed in this appendix can be very useful.

[3] However, actually it is very tough to find this minimum. So, [372] developed Theorem 4.1, which results in a conservative stability radius.

D

Power of an Interval Matrix

Interval computation techniques are popularly used for robust stability analysis of uncertain systems described in terms of interval parameters and interval matrices. In the past two decades, a great amount of research effort has been devoted to the analysis of interval matrices. However, there is no available result for determining the impulse response bounds of discrete-time, linear, time-invariant systems with interval uncertainty in their state-space description. Indeed, in control engineering [245, 52] and in signal processing [297], the impulse response plays an important role. As shown in [553], the interval impulse response could be effectively used for robust controller design. However for all these works, if there exist model uncertainties in a system's state-space model, the uncertain ranges of the impulse response should be carefully estimated.

Note that if the interval uncertainty in the state-space model is known then the key problem of determining the impulse response of such an uncertain system is to find the power of an interval matrix. In this appendix we will show how the power of an interval matrix can be computed. This result can then be used for a number of important problems. For our purposes, for example, it can be used for designing the learning gain matrix in ILC problems (although, the method presented here requires much more computation compared with the eigen-decomposition method developed in Chapter 6). Beyond ILC, in robust control the power of an interval matrix can be effectively used in the analysis of controllability, observability, or impulse response for uncertain interval systems. However, very limited effort has been devoted to calculating the boundaries of the power of an interval matrix. Some existing results can be found in [290, 202, 460, 172] where the convergence problem of the powers of an interval matrix was studied and it was proved that the power of an interval matrix converges to zero if the maximum spectral radius of the interval matrix is less than 1. However, the question of the boundaries of the power of an interval matrix at a specified order has not been fully addressed (though, some useful analysis of the power of an interval matrix at a specified order can be found in [244], where it was concluded that computing the boundaries of the power of an interval matrix is an NP-hard problem).

D.1 Sensitivity Transfer Method

In this section, a new method is developed for the calculation of the power of an interval matrix. This method first computes the sensitivity of the perturbation of the nominal A matrix and then applies this sensitivity to the power of the matrix A^k. The set of the power of the interval matrix can be written as

$$\mathcal{A}^k = \{P \mid P = \underbrace{AAA\cdots A}_{k},\ A \in A^I\}. \tag{D.1}$$

Then, from the relationship $A^k = \underbrace{AA\cdots A}_{k}$, we can have

$$\frac{\partial A^k}{\partial a_{ij}} = \frac{\partial A}{\partial a_{ij}}(\underbrace{A\cdots A}_{k-1}) + A\frac{\partial A}{\partial a_{ij}}(\underbrace{A\cdots A}_{k-2}) + \cdots + (\underbrace{A\cdots A}_{k-1})\frac{\partial A}{\partial a_{ij}}. \tag{D.2}$$

Here, by observing that $\frac{\partial A}{\partial a_{ij}} = I_{ij}$ where I_{ij} is a matrix whose i^{th} row and j^{th} column element is 1 and the other elements are all zeroes, we have

$$\frac{\partial A^k}{\partial a_{ij}} = I_{ij}(\underbrace{A\cdots A}_{k-1}) + AI_{ij}(\underbrace{A\cdots A}_{k-2}) + \cdots + (\underbrace{A\cdots A}_{k-1})I_{ij}. \tag{D.3}$$

Thus, we have the perturbed sensitivity (∂A^k) of A^k by the uncertain change (∂a_{ij}) of a_{ij} such as

$$\partial A^k = \partial a_{ij}\left(I_{ij}(\underbrace{A\cdots A}_{k-1}) + AI_{ij}(\underbrace{A\cdots A}_{k-2} + \cdots + (\underbrace{A\cdots A}_{k-1})I_{ij}\right). \tag{D.4}$$

For convenience, let us use the following notation:

$$\prod_{ij} := \left(I_{ij}(\underbrace{A\cdots A}_{k-1}) + AI_{ij}(\underbrace{A\cdots A}_{k-2}) + (\underbrace{A\cdots A}_{2})I_{ij}(\underbrace{A\cdots A}_{k-3} + \cdots + (\underbrace{A\cdots A}_{k-1})I_{ij}\right)$$

which simplifies (D.4) to $\partial A^k = \partial a_{ij}\prod_{ij}$. Hence, we find that when there is a perturbation amount of ∂a_{ij} in a_{ij}, there is a perturbation effect on A^k by the amount of ∂A^k that is related to the sensitivity transfer matrix $\prod_{ij}$. Here, noticing that each element of A perturbs A^k, we develop a method for bounding the uncertainty of A^k. Using the notation $P \in \mathcal{P}^1 = \mathcal{A}^k = [\underline{P}, \overline{P}]$, we make the following proposition:

Proposition D.1. *Given the order of power k, the upper and lower boundaries associated with the elements of P occur at the power of one of the vertex matrices A^v.* ■

Proof. Let us pick arbitrary i_1 and j_1, and fix all a_{pq}, where $p, q = 1, \ldots, n$, and $p \neq i_1$ or $q \neq j_1$, to specified values $a_{pq} = a^*_{pq} \in [\overline{a_{pq}}, \underline{a_{pq}}]$. Then, from $\partial A^k = \partial a_{i_1 j_1} \prod_{i_1 j_1}$, the k^{th} row and l^{th} column element of ∂A^k are determined by $\partial a_{i_1 j_1}$ and $\left(\prod_{i_1 j_1}\right)_{kl}$. Noticing that $\partial a_{i_1 j_1} = [-\triangle a_{i_1 j_1}, \triangle a_{i_1 j_1}]$, the positive (negative) maximum of ∂A^k occurs at $\triangle a_{i_1 j_1}$ $(-\triangle a_{i_1 j_1})$ if $\left(\prod_{i_1 j_1}\right)_{kl}$ is positive. Otherwise, the positive (negative) maximum of ∂A^k occurs at $-\triangle a_{i_1 j_1}$ $(\triangle a_{i_1 j_1})$. However, the sign of $\left(\prod_{i_1 j_1}\right)_{kl}$ is not determined. Hence, for arbitrary fixed i_1 and j_1, we can conclude that the positive (negative) maximum of the k^{th} row and l^{th} column element of ∂A^k occurs at one of the vertex points of $a^I_{i_1 j_1} = [a^o_{i_1 j_1} - \triangle a_{i_1 j_1}, a^o_{i_1 j_1} + \triangle a_{i_1 j_1}]$. Now let us pick another arbitrary i_2 and j_2. Then by the same reasoning given above, the positive (negative) maximum of the k^{th} row and l^{th} column element of ∂A^k occurs at one of vertex points of $a^I_{i_2 j_2} = [a^o_{i_2 j_2} - \triangle a_{i_2 j_2}, a^o_{i_2 j_2} + \triangle a_{i_2 j_2}]$, but $a_{i_1 j_1} \in \{\underline{a_{i_1 j_1}}, \overline{a_{i_1 j_1}}\}$. Finally, since we can repeat the above discussion for all a_{pq}, the positive (negative) maximum of the k^{th} row and l^{th} column element of ∂A^k occurs at the power of one of vertex matrices. ■

Proposition D.1 shows that the lower and upper boundaries of the power of an interval matrix can be found by checking all the vertex matrices. It is important to highlight that Proposition D.1 uses the finite vertex matrix set to find the boundary of the power of interval matrix set. However, from $\partial A^k = \partial a_{ij} \prod_{ij}$, it is required to check all the vertex matrices of A^I to find the maximal positive or negative perturbation of elements of A^k, i.e., $(A^k)_{ij}$, $\forall A \in A^I$. Hence, the computational amount could be huge. That is, in order to find the maximum and minimum of A^k, where $A \in A^I$, we have to check 2^{n^2} vertex matrices, where n is the size of the square A matrix, for each element of A^k. Thus, the total computational amount is $2^{n^2} \times 2^n = 2^{n^2+n}$. In the sequel, we will show that under some conditions, we do not need to check all the vertex matrices. Instead, only some specified vertex matrices need to be used for the calculation of the power of interval matrix. However, even without this result, although the computational effort to check all the vertex matrices may be high, because the impulse response of an LTI system is generally used for design purposes in an off-line manner, Proposition D.1 is still useful.

To see how to reduce the computational load, let us define the center matrix of P and radius matrix of P as $P^c = \frac{\underline{P}+\overline{P}}{2}$; $P^r = \frac{\overline{P}-\underline{P}}{2}$. Then the following result can be derived:

Proposition D.2. *If the sign of the k^{th} row and l^{th} column element of the sensitivity transfer matrix $\prod_{ij}$ does not change by ∂a_{ij}, the maximum positive* *and negative perturbations of the* k^{th} *row and* l^{th} *column element of* $\mathcal{A}^k$*, occur at the power of the following vertex matrices of* A^I*, respectively,*

$$(A^v)|_{kl}^{+} = \left\{ A^v \mid A^v = [(A^o)_{ij} + s_{kl}^{ij} \triangle a_{ij}, i, j = 1, \ldots, n] \right\} \tag{D.5}$$

$$(A^v)|_{kl}^{-} = \left\{ A^v \mid A^v = [(A^o)_{ij} - s_{kl}^{ij} \triangle a_{ij}, i, j = 1, \ldots, n] \right\}, \tag{D.6}$$

where $s_{kl}^{ij} = \text{sign} \left(\prod_{ij} \right)_{kl}$. ■

Proof. From $\partial A^k = \partial a_{ij} \prod_{ij}$, the positive (negative) maximum of ∂A^k occurs at $\triangle a_{ij}$ $(-\triangle a_{ij})$ if $\left(\prod_{ij} \right)_{kl}$ is positive. Otherwise, the positive (negative) maximum of ∂A^k occurs at $-\triangle a_{ij}$ $(\triangle a_{ij})$. This implies that the positive (negative) maximum disturbance of $(A^k)_{kl}$ occurs at $(A^o)_{ij} + s_{kl}^{ij} \triangle a_{ij}$ $((A^o)_{ij} - s_{kl}^{ij} \triangle a_{ij})$. ■

Remark D.3. Proposition D.2 was developed based on an assumption that the signs of $\prod_{ij}$ do not change. In Section D.3, some sufficient conditions are established which can be used for checking the sign variation. However, it should be noted that if the sufficient conditions given in the appendix are not satisfied, Proposition D.1 should be used. Hence, Proposition D.1 and Proposition D.2, having their own advantages and disadvantages, complement each other. Therefore, the two procedures based on the sensitivity transfer idea presented in this appendix are practically valuable in the interval model conversion problem.

Remark D.4. In Proposition D.1 and Proposition D.2, we considered general non-symmetric square interval matrices. However, we can extend these results to symmetric interval matrices. This work is direct by repeating (D.2), (D.3), and (D.4).

In this section, a new method called the sensitivity transfer method was developed to overcome the conservatism of the method suggested in Chapter 6 (called eigenpair-decomposition method). However, the computational effort of the method given in Chapter 6 is significantly less than the method developed in this appendix. Using either method, however, once the boundary of the power of an interval matrix is found then it is straightforward to find the boundaries of the impulse response. In other words, the boundaries of h_k^I, i.e., $[\underline{h}_k, \overline{h}_k]$, can be simply calculated by matrix multiplication $CP^{k-1}B$, where $P^{k-1} \in \mathcal{A}^{k-1}$, because the upper and lower boundary matrices of $\mathcal{A}^{k-1}$ have been estimated. In the next section, the effectiveness of the suggested method is illustrated through numerical examples.

D.2 Illustrative Examples

To verify the usefulness of the new method, Monte-Carlo-type random tests are performed. The results obtained from the random tests are considered as the "true" range of the impulse response of the interval LTI system, for comparison to the bounds computed by the suggested methods.

D.2.1 Example 1

Consider the following uncertain discrete-time LTI interval system:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t), \end{aligned} \tag{D.7}$$

where $B = [2, 0.5]^T; C = [1, 0]$ and the following two different nominal A matrices are tested:

- Case 1 (symmetric and unstable): $a_{11} = -1.05$; $a_{12} = 0.55$; $a_{21} = 0.55$; $a_{22} = 0.85$
- Case 2 (non-symmetric and stable): $a_{11} = 0.75$; $a_{12} = -0.40$; $a_{21} = 0.25$; $a_{22} = 0.55$.

In Case 1, the nominal eigenvalues are -1.1977 and 0.9977. Thus, the nominal system is initially unstable. In Case 2, the nominal eigenvalues are $0.6500 + 0.3000i$ and $0.6500 - 0.3000i$. Hence, the nominal system is stable. In both cases, it is supposed that there is 10 percent interval model uncertainty in the A matrix parameters. Thus, in Case 1, A^I is

$$A^I = \begin{bmatrix} [-1.155, -0.945] & [0.495, 0.605] \\ [0.495, 0.605] & [0.765, 0.935] \end{bmatrix}$$

and in Case 2, $\mathcal{A}^I$ is

$$A^I = \begin{bmatrix} [0.675, 0.825] & [-0.44, -0.36] \\ [0.225, 0.275] & [0.495, 0.605] \end{bmatrix}.$$

Figure D.1 shows the test result of Case 1. Since the system is unstable, the impulse responses diverge as k increases. In this figure, four different test results are shown: the $\times$-dot dashed lines are the upper/lower boundaries computed from `Intlab` [179]; the $\circ$-dashed lines show the upper/lower boundaries computed from the eigenpair-decomposition method; the $\diamond$-solid lines represent the upper/lower boundaries computed from the sensitivity transfer method; and the thick solid vertical bars represent the range obtained from the random test results. Clearly, the sensitivity transfer method accurately bounds the upper/lower boundaries of the impulse responses, while even if the eigenpair-decomposition method is better than `Intlab`, it is much more conservative than the sensitivity transfer method. Figure D.2 shows the test results of Case 2. From this figure, it is also seen that the sensitivity transfer method accurately bounds the upper/lower boundaries of the impulse responses. In the early phase, `Intlab` performs better than the eigenpair-decomposition-based method. However, as k increases, the eigenpair-decomposition method performs better than `Intlab`. Note that for the sensitivity transfer method, we used Proposition D.2 based on Lemma D.5 and Lemma D.6. However, the condition of Lemma D.6 was satisfied, for all k, l, i, j in Proposition D.2, only for the power $k = 1, \ldots, 4$ of Case 1 and the condition of Lemma D.5 was satisfied, for all k, l, i, j in Proposition D.2, only for the power $k = 1, \ldots, 4$ of Case 2.[1] Hence, for the higher-order power of the interval matrix, we used Proposition D.1. Now, from Figure D.1 and Figure D.2, it is clear that the sensitivity transfer method suggested in this appendix very accurately bounds the impulse responses of the uncertain interval system in both stable and unstable systems. However, the computational amount is huge. Thus we can say that the eigenpair-decomposition method and the sensitivity transfer method complement each other.

D.2.2 Example 2

For the usefulness of Proposition D.2 and Lemma D.5, let us consider the following nominal matrix, which was created using MATLAB® commands *rand* and *sign*:

$$A^o = \begin{bmatrix} 1 & -1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 \\ -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 \end{bmatrix}$$

Let us suppose that there exist ± 0.001 interval uncertainties in all elements, and that we want to find exact upper and lower boundaries of $A^5, A \in A^I$. If we use Proposition D.1, we need to check $2^{5^2} = 2^{25}$ vertex matrices. Indeed, in this case, the computational time could be huge. However, from Lemma D.5, we find that the signs of all elements of $\prod_{ij}$ do not change for all i, j. Hence, for the upper and lower boundary matrices of A^5, it is enough to use 25 vertex matrices. From these vertex matrices, we calculate the upper boundary and lower boundary matrices of $A^5,\ A \in A^I$ to be

$$\overline{P} = \begin{bmatrix} 125.7868 & -124.2167 & -0.7030 & -68.4220 & 69.5497 \\ -130.1691 & 131.8351 & -0.6711 & 91.6420 & -90.3863 \\ 29.2777 & -28.6957 & -0.8434 & -4.7393 & 5.2731 \\ -62.3879 & 63.6182 & 11.2053 & 55.4559 & -22.5556 \\ 97.7063 & -96.2963 & 11.2414 & -40.4952 & 73.4820 \end{bmatrix}$$

[1] From numerous numerical tests, we have found that Lemma D.5 and Lemma D.6 are particularly effective for a stable system and a lower-order impulse response. Also it is important to emphasize that Proposition D.2 does not require that Lemma D.5 and Lemma D.6 should be satisfied for all k, l, i, j. Instead, Proposition D.2 shows that if Lemma D.5 and Lemma D.6 hold for part of k, l, i, j, the corresponding elements of the power of an interval matrix can be estimated from the particular intervals of A^I. In such a case, the computational effort could be further reduced.

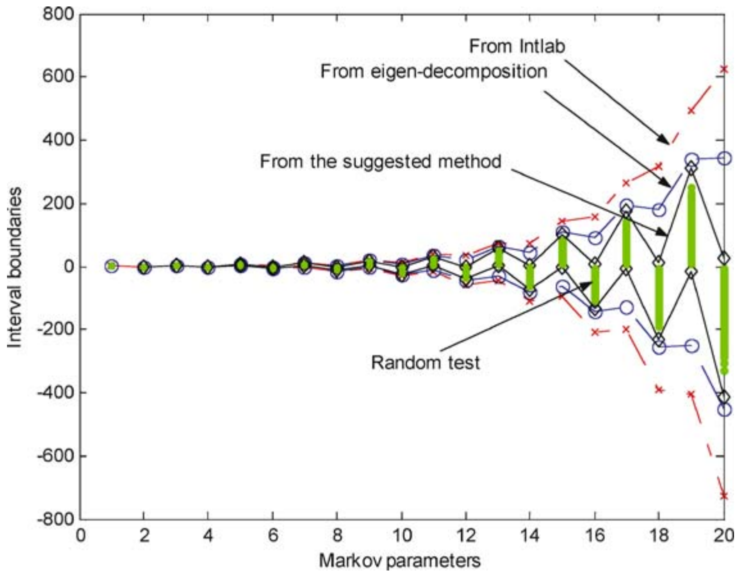

Fig. D.1. Impulse responses of Case 1. Plots are from `Intlab`, from the eigenpair-decomposition method, and from the suggested sensitivity transfer method. The vertical thick bars are the random test results.

$$\underline{P} = \begin{bmatrix} 124.2167 & -125.7868 & -1.2970 & -69.5800 & 68.4517 \\ -131.8351 & 130.1691 & -1.3291 & 90.3600 & -91.6163 \\ 28.7237 & -29.3057 & -1.1574 & -5.2613 & 4.7271 \\ -63.6139 & 62.3842 & 10.7953 & 54.5459 & -23.4456 \\ 96.2963 & -97.7063 & 10.7594 & -41.5052 & 72.5200 \end{bmatrix}.$$

D.3 Condition for Proposition D.2

In this section we provide sufficient conditions for Proposition D.2. Let us write the sensitivity transfer matrix $\prod_{ij}$ as

$$\prod_{ij} = \sum_{p=1}^{k} A^{p-1} I_{ij} A^{k-p}, \tag{D.8}$$

where $A \in A^I$. For convenience, let us write A as $A = A^o + \Delta$ where $\Delta \in \Delta A^I$. Then, using $A^k = (A^o + \Delta)^k$, and writing $\mathcal{O}^k := (A^o + \Delta)^k - (A^o)^k$, we obtain:

$$\prod_{ij} = \sum_{p=1}^{k} (A^o + \Delta)^{p-1} I_{ij} (A^o + \Delta)^{k-p}$$

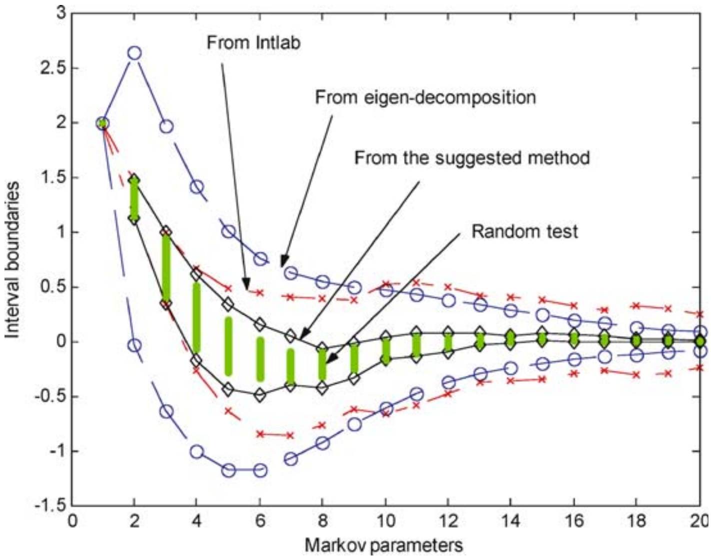

Fig. D.2. Impulse responses of Case 2. Plots are from `Intlab`, from the eigenpair-decomposition method, and from the suggested sensitivity transfer method. The vertical thick bars are the random test results.

$$= \sum_{p=1}^{k} \left[\mathcal{O}^{p-1} + (A^o)^{p-1}\right] I_{ij} \left[\mathcal{O}^{k-p} + (A^o)^{k-p}\right]. \tag{D.9}$$

Then, rearranging (D.9) yields

$$\prod_{ij} - \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} = \sum_{p=1}^{k} \{\mathcal{O}^{p-1} I_{ij} \mathcal{O}^{k-p} + \mathcal{O}^{p-1} I_{ij} (A^o)^{k-p} + (A^o)^{p-1} I_{ij} \mathcal{O}^{k-p}\}. \tag{D.10}$$

Defining the absolute matrix such as $|A| := [|a_{ij}|]$, we have

$$\left|\prod_{ij} - \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p}\right| = \left|\sum_{p=1}^{k} \{\mathcal{O}^{p-1} I_{ij} \mathcal{O}^{k-p} + \mathcal{O}^{p-1} I_{ij} (A^o)^{k-p} + (A^o)^{p-1} I_{ij} \mathcal{O}^{k-p}\}\right|$$

$$\leq \sum_{p=1}^{k} \{\left|\mathcal{O}^{p-1}\right| I_{ij} \left|\mathcal{O}^{k-p}\right| + \left|\mathcal{O}^{p-1}\right| I_{ij} \left|(A^o)^{k-p}\right| + \left|(A^o)^{p-1}\right| I_{ij} \left|\mathcal{O}^{k-p}\right|\}$$

$$\leq \sum_{p=1}^{k} \left\{ \left[(|A^o| + |\Delta|)^{p-1} - |A^o|^{p-1} \right] I_{ij} \left[(|A^o| + |\Delta|)^{k-p} - |A^o|^{k-p} \right] \right.$$
$$+ \left[(|A^o| + |\Delta|)^{p-1} - |A^o|^{p-1} \right] I_{ij} \left| (A^o)^{k-p} \right|$$
$$\left. + \left| (A^o)^{p-1} \right| I_{ij} \left[(|A^o| + |\Delta|)^{k-p} - |A^o|^{k-p} \right] \right\}, \tag{D.11}$$

where we used the inequality $|\mathcal{O}^k| \leq [|A^o| + |\Delta|]^k - |A^o|^k$, which can be derived after several algebraic manipulations. Now, defining $\Delta^* := \overline{A} - A^o = A^o - \underline{A}$ and using inequality $|\Delta| \leq \Delta^*$, we obtain

$$\left| \prod_{ij} - \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right| \leq$$
$$\sum_{p=1}^{k} \left\{ \left[(|A^o| + \Delta^*)^{p-1} - |A^o|^{p-1} \right] I_{ij} \left[(|A^o| + \Delta^*)^{k-p} - |A^o|^{k-p} \right] \right.$$
$$+ \left[(|A^o| + \Delta^*)^{p-1} - |A^o|^{p-1} \right] I_{ij} \left| (A^o)^{k-p} \right|$$
$$\left. + \left| (A^o)^{p-1} \right| I_{ij} \left[(|A^o| + \Delta^*)^{k-p} - |A^o|^{k-p} \right] \right\}. \tag{D.12}$$

Finally, denoting the right-hand side of (D.12) as R^* and writing

$$L := \left| \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right|,$$

we can make the following lemma.

Lemma D.5. *If $L \geq R^*$ element-wise, the signs of $\prod_{ij}$ do not change element-wise.* ∎

Proof. From

$$\left| \prod_{ij} - \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right| \leq R^* \leq L$$
$$= \left| \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right|,$$

we have the inequality:

$$\left| \prod_{ij} - \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right| \leq \left| \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right|.$$

Hence element-wise, if $\sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \geq 0$, then

$$0 \leq \prod_{ij} \leq 2 \left(\sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right),$$

else if $\sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} < 0$, then

$$-2 \left(\sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} \right) \leq \prod_{ij} < 0.$$

Therefore, the signs of $\prod_{ij}$ are the same to the signs of $\sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p}$. This completes the proof. ∎

When the commutative property $A^o \Delta = \Delta A^o$ holds, a less conservative condition can be derived. Note the commutative property is satisfied when A is a symmetric interval matrix, and the symmetric interval matrix system has been an important research topic as shown in [192, 389]. For this result, we use $(A^o + \Delta)^m = \sum_{u=0}^{m} {}_mC_u (A^o)^{m-u} \Delta^u$ where ${}_mC_u = \frac{m!}{u!(m-u)!}$. Now, from the following relationship:

$$\begin{aligned}
\prod_{ij} &= \sum_{p=1}^{k} \left[\sum_{u=0}^{p-1} {}_{p-1}C_u (A^o)^{p-1-u} \Delta^u \right] I_{ij} \left[\sum_{v=0}^{k-p} {}_{k-p}C_v (A^o)^{k-p-v} \Delta^v \right] \\
&= \sum_{p=1}^{k} \left[(A^o)^{p-1} + \sum_{u=1}^{p-1} {}_{p-1}C_u (A^o)^{p-1-u} \Delta^u \right] I_{ij} \\
&\quad \times \left[(A^o)^{k-p} + \sum_{v=1}^{k-p} {}_{k-p}C_v (A^o)^{k-p-v} \Delta^v \right],
\end{aligned} \tag{D.13}$$

we have

$$\begin{aligned}
\prod_{ij} - \sum_{p=1}^{k} (A^o)^{p-1} I_{ij} (A^o)^{k-p} &= \sum_{p=1}^{k} \Bigg\{ \left[(A^o)^{p-1} \right] I_{ij} \left[\sum_{v=1}^{k-p} {}_{k-p}C_v (A^o)^{k-p-v} \Delta^v \right] \\
&\quad + \left[\sum_{u=1}^{p-1} {}_{p-1}C_u (A^o)^{p-1-u} \Delta^u \right] I_{ij} \left[(A^o)^{k-p} \right] \\
&\quad + \left[\sum_{u=1}^{p-1} {}_{p-1}C_u (A^o)^{p-1-u} \Delta^u \right] I_{ij} \\
&\quad \times \left[\sum_{v=1}^{k-p} {}_{k-p}C_v (A^o)^{k-p-v} \Delta^v \right] \Bigg\} \\
&= \sum_{p=1}^{k} \sum_{v=1}^{k-p} {}_{k-p}C_v \left[(A^o)^{p-1} I_{ij} (A^o)^{k-p-v} \Delta^v \right]
\end{aligned}$$

$$+\sum_{p=1}^{k}\sum_{u=1}^{p-1} {}_{p-1}C_u \left[(A^o)^{p-1-u}\Delta^u I_{ij}(A^o)^{k-p}\right]$$
$$+\sum_{p=1}^{k}\sum_{u=1}^{p-1}\sum_{v=1}^{k-p}({}_{p-1}C_u)({}_{k-p}C_v)$$
$$\times\left[(A^o)^{p-1-u}\Delta^u I_{ij}(A^o)^{k-p-v}\Delta^v\right]. \quad \text{(D.14)}$$

Using the commutative property (notice that I_{ij} is symmetric), we simplify (D.14) as

$$\prod_{ij} - \sum_{p=1}^{k}(A^o)^{p-1}I_{ij}(A^o)^{k-p} = \sum_{p=1}^{k}\sum_{v=1}^{k-p} {}_{k-p}C_v \left[I_{ij}(A^o)^{k-v-1}\Delta^v\right]$$
$$+\sum_{p=1}^{k}\sum_{u=1}^{p-1} {}_{p-1}C_u \left[\Delta^u I_{ij}(A^o)^{k-u-1}\right]$$
$$+\sum_{p=1}^{k}\sum_{u=1}^{p-1}\sum_{v=1}^{k-p}({}_{p-1}C_u)({}_{k-p}C_v)$$
$$\left[\Delta^{u+v}I_{ij}(A^o)^{k-u-v-1}\right]. \quad \text{(D.15)}$$

Hence, we obtain the following inequality:

$$\left|\prod_{ij} - \sum_{p=1}^{k}(A^o)^{p-1}I_{ij}(A^o)^{k-p}\right| \le \sum_{p=1}^{k}\left\{\sum_{v=1}^{k-p} {}_{k-p}C_v \left|I_{ij}(A^o)^{k-v-1}\right|(\Delta^*)^v\right.$$
$$+\sum_{u=1}^{p-1} {}_{p-1}C_u(\Delta^*)^u\left|I_{ij}(A^o)^{k-u-1}\right|$$
$$+\sum_{u=1}^{p-1}\sum_{v=1}^{k-p}({}_{p-1}C_u)({}_{k-p}C_v)(\Delta^*)^{u+v}$$
$$\left.\times\left|I_{ij}(A^o)^{k-u-v-1}\right|\right\}. \quad \text{(D.16)}$$

Now, denoting the right-hand side of (D.16) as S^*, we can state the following lemma for the case of a symmetric interval matrix.

Lemma D.6. *For a symmetric interval matrix, if $L \ge S^*$ element-wise, the signs of $\prod_{ij}$ do not change.* ∎

The following remark is provided for some special cases.

- In Proposition D.2, in the case of $A > 0$ element-wise, for all $A \in A^I$, or $A < 0$ element-wise, for all $A \in A^I$, the signs of $\prod_{ij}$ do not change.

- When A^I is a symmetric interval matrix and it satisfies the property that, regardless of the magnitude of the elements of A^I, the sign of (AA) = the sign of (A) for all $A \in A^I$, then the signs of $\prod_{ij}$ do not change.

D.4 Summary

Computing the boundaries of the power of an interval matrix is a hard problem, if not NP-hard. In this appendix, we provided a solution for computing the bounds of the power of an interval matrix using the idea of sensitivity transfer. Through rigorous analysis, we are able to show that the exact boundaries of the power of an interval matrix can be found from vertex matrices.

Furthermore, in some special cases when the considered interval matrix has some structural constraints, the exact boundaries of the power of an interval matrix can be calculated from a set of selected vertex matrices. Numerical examples were presented to illustrate the proposed algorithm.

We believe that the results of this appendix can be widely used in solving many robust control problems such as the robust stability, robust controllability/observability, and others.

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